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# Self-injection-locked optical parametric oscillator based on microcombs

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Narrow-linewidth yet tunable laser oscillators are one of the most important tools for precision metrology, optical atomic clocks, sensing, and quantum computing. Commonly used tunable coherent oscillators are based on stimulated emission or stimulated Brillouin scattering; as a result, the operating wavelength band is limited by the gain media. Based on nonlinear optical gain, optical parametric oscillators (OPOs) enable coherent signal generation within the whole transparency window of the medium used. However, the demonstration of OPO-based Hertz-level linewidth and tunable oscillators has remained elusive. Here, we present a tunable coherent oscillator based on a multimode coherent OPO in a high-Q microresonator, i.e., a microcomb. Single-mode coherent oscillation is realized through self-injection locking (SIL) of one selected comb line. We achieve coarse tuning up to 20 nm and an intrinsic linewidth down to sub-Hertz level, which is three orders of magnitude lower than the pump. Furthermore, we demonstrate that this scheme results in the repetition rate stabilization of the microcomb. These results open exciting possibilities for generating tunable coherent radiation where stimulated emission materials are difficult to obtain, and the stabilization of microcomb sources beyond the limits imposed by the thermorefractive noise in the cavity.

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## 1. INTRODUCTION

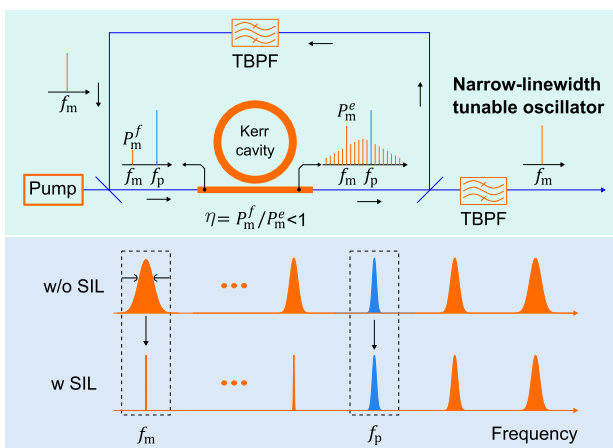
Narrow-linewidth and tunable lasers are essential for applications that require low-phase-noise and wavelength versatility such as precision spectroscopy, quantum optics, optical atomic clocks, sensing, high performance communications, and lidar, to name a few. The most widely used approaches to implement narrow-linewidth lasers have relied on spectral purification of a free-running diode or a solid-state or fiber laser—for example, active frequency locking of the laser to a high-Q cavity with an electronic servo element [1–4] or optical feedback of a laser with an external optical element such as a grating, mirror, cavity, or waveguide [5–9]. Recent advances in photonic heterogeneous integration have enabled coherent lasers with a linewidth at the 1 Hz level on a chip [10]. The gain of most conventional narrow-linewidth lasers is mainly provided by stimulated emission; thus, their operating wavelength bands, i.e., maximum tuning ranges, are fundamentally limited by the optical transitions allowed in the gain medium. In principle, narrow-linewidth and tunable coherent oscillators can be attained at extended wavelength regions by making use of nonlinear optical frequency conversion processes, such as harmonic generation, sum/difference frequency generation, or four-wave mixing [11,12]. However, this remains a challenging endeavor because one or two narrow-linewidth pump

lasers are required, and the phase matching condition needs to be stringently satisfied over a large frequency range. Stimulated Brillouin scattering is another well-known mechanism for implementing narrow-linewidth lasers [13,14]; however, the emission wavelength is fundamentally constrained by the pump.

Unlike ordinary lasers, the gain of optical parametric oscillators (OPOs) originates from an optically driven nonlinear polarization; thus, the operating wavelength band is only limited by the absorption characteristics of the nonlinear medium used and the ability to engineer the phase matching among the waves involved in the process [15]. Nowadays, OPOs are widely exploited for tunable light generation ranging from UV to terahertz, particularly in the wavelength regions where ordinary lasers hardly reach [16]. The fundamental parametric process can be described as a new pair of photons, referred to as the signal photon(s) ( $\omega_s$ ) and the idler photon(s) ( $\omega_i$ ), created from one (or two) pump ( $\omega_p$ ) photon(s) mediated by a  $\chi^{(2)}$  (or  $\chi^{(3)}$ ) nonlinear medium. According to energy conservation, the sum of the frequencies of signal and idler waves follows the pump frequency (or its double in the  $\chi^{(3)}$  case), while their relative frequency could undergo a random diffusion process [17]. Over the past decades, several approaches have been pursued in view of generating a narrow-linewidth signal (or idler) via OPOs. The most commonly applied approaches include utilizing frequency-selective elements such as intracavity etalons or

gratings [18–24]. Self-injection seeding is an alternative technique for spectral narrowing in pulse pumped OPOs [25,26]. These all-optical approaches permit signal spectral narrowing for several orders with the compatibility of broadband wavelength tuning; however, the narrowing effect is usually insufficient to reach a signal wave with a linewidth smaller than the pump. Another strategy is to stabilize the signal or idler to a frequency reference, such as a FP cavity [27–30], frequency comb [31], atomic resonance [32], or narrow-linewidth laser [33,34]. These approaches could give superior spectral coherence but need a sophisticated servo system.

In this work, we present a narrow-linewidth and tunable coherent oscillator based on self-injection locking (SIL) of a multimode continuous-wave OPO, i.e., a Kerr microcomb [35]. To be more specific, a microcomb with a single-soliton state is employed here [36]. However, other coherent states such as Turing rolls or other soliton states [37,38] could also be leveraged. The schematic architecture of the oscillator is shown in Fig. 1. Like standard SIL configuration, the system is composed of a microcomb and an external optical feedback loop. To ensure that a single-frequency narrow-linewidth oscillator is generated, a tunable bandpass filter (TBPF) is inserted into the feedback loop for selecting one comb line, which can be arbitrary except for the pump. Optical gain can be involved in the feedback loop to compensate the loss, but in principle the feedback loop can be purely passive. This configuration is thus fundamentally different from laser cavity microcombs [39–41], where the gain plays a crucial role for the comb generation. It is worth noting that the terminology of SIL in microcombs has been utilized in different contexts, including mode-locked state generation [42] and pump diode laser SIL [43–46]. OPO SIL has been discussed in the scenario of optical frequency division where the signal and the idler are subharmonics of the pump [47,48]. Here we explore and demonstrate the feasibility of dramatic spectral narrowing of a CW OPO with SIL and realize the first sub-Hertz intrinsic linewidth oscillator based on parametric gain. We unveil a rich dynamics akin to what has been found in ordinary lasers with SIL [49] and show there exists a dynamic regime where the frequency noise of the comb line can be consistently reduced to be three orders lower than the pump,



**Fig. 1.** Architecture of the self-injection-locked microcomb-based coherent oscillator. A microcomb is generated in a Kerr-nonlinearity microcavity with a continuous wave pump. The  $m$ th comb line (counted from the pump,  $f_m$ ) is selected with a tunable bandpass filter (TBPF) and re-injected into the microcavity. With proper feedback power ratio ( $\eta$ ), the linewidth of the  $m$ th comb line (the coherent oscillator) is dramatically reduced; meanwhile, some other comb lines are narrowed accordingly [see Fig. 5(a)].

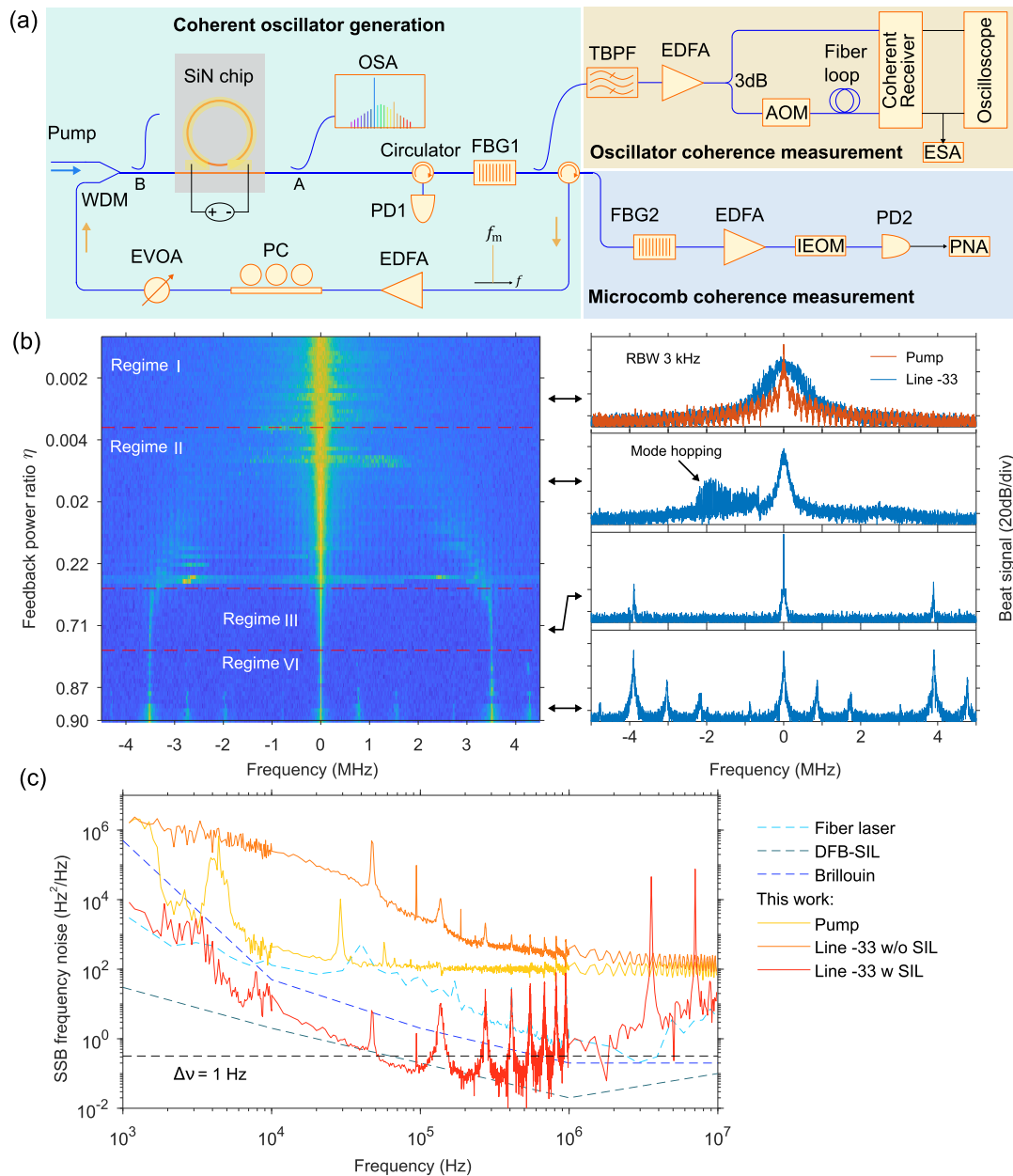
regardless of the feedback phase. Our work lays the foundation for understanding SIL dynamics in OPOs. In addition, this system allows us to select an arbitrary comb line (except for the pump) for SIL by simply setting the TBPF. Therefore, it enables simultaneous narrow-linewidth emission and continuous tuning in an extremely broad wavelength range. Furthermore, as the oscillator is one comb line of the soliton microcomb, its spectral purification can spontaneously facilitate microcomb stabilization beyond the limits imposed by the thermorefractive noise in the cavity [50].

## 2. EXPERIMENTAL STUDY OF SIL DYNAMICS

In this section, we present the experimental study of the SIL dynamics. The setup is shown in Fig. 2(a). An integrated silicon nitride (SiN) microresonator is used for microcomb generation. The height and the width of the SiN waveguide are 740 nm and 1800 nm, respectively, which result in a group velocity dispersion coefficient of  $\beta_2 = -70$  ps<sup>2</sup>/km for the TE<sub>00</sub> mode. The radius of the microresonator is 227  $\mu$ m, so the corresponding FSR is  $\sim 100$  GHz. The gap between ring and bus is 450 nm. Both the average intrinsic and external quality factors for the TE<sub>00</sub> mode family are  $\sim 9 \times 10^6$ . Two lensed fibers are used for light coupling into and out of the on-chip SiN bus waveguide. The coupling loss per facet is  $\sim 2$  dB. A narrow-linewidth external cavity diode laser ( $\lambda = 1536.9$  nm) after being amplified by an erbium-doped fiber amplifier (EDFA) is employed as the pump. A single-soliton microcomb is generated at the TE<sub>00</sub> mode family via fast thermo-optic tuning. In this initial experiment, we select the comb line  $-33$  ( $\lambda = 1562.8$  nm) by means of a fiber bragg grating (FBG) filter for SIL, but in principle any other mode could be used.

To investigate the SIL dynamics, an EDFA (small-signal gain  $\sim 35$  dB) and an electronic variable optical attenuator (EVOA) are included into the feedback loop for feedback strength control. To make the feedback efficient, a fiber polarization controller is inserted into the feedback loop. The optical loss of the feedback loop is around 10 dB when the EDFA is off. The total length of the feedback fiber loop is  $\sim 50$  m, including  $\sim 27$  m fiber in the EDFA. The feedback strength can be quantified as the on-chip power ratio of the feedback field re-entering the microresonator ( $P_m^f$ ) and emitting one ( $P_m^e$ ), i.e.,  $\eta = P_m^f / P_m^e$  (see Fig. 1). In the experiment,  $\eta$  can be measured from the comb line power ratio at nodes A and B (Fig. 2) when the feedback loop is closed. It is important to emphasize that  $\eta < 1$  is satisfied all the time in this work [see Fig. 2(b)], that is to say, the EDFA here is not employed as the “gain source” for the compound cavity laser but instead it serves as the loss compensator of the feedback loop. The equivalent Q-factor of the feedback loop is  $\sim -3 \times 10^8 / \ln \eta$ . Akin to opto-electronic oscillator, the fiber feedback loop could potentially be replaced by a high-Q microresonator [51].

To analyze the spectral characteristics of the single-mode coherent oscillator, the selected comb line is amplified and filtered for out-of-loop characterization. Subsequently, the oscillator is divided into two paths by a 3 dB coupler. One beam is frequency shifted by an AOM (80 MHz) and then delayed through a  $\sim 1.5$  km single-mode fiber. The two beams are then sent into a coherent receiver. As a result, a pair of delayed self-heterodyne beating signals with center frequency 80 MHz are obtained, which can be recorded for extracting the frequency noise power spectral density (PSD) of the oscillator. To get a qualitative evaluation of



**Fig. 2.** Experimental study of the dynamics of the self-injection-locked microcomb-based oscillator. (a) Experimental setup: WDM, wavelength division multiplex; FBG, fiber Bragg grating; EDFA, erbium-doped amplifier. IEOM, intensity electro-optic modulator; ESA, electric spectrum analyzer; PNA, (ESA-based) phase noise analyzer; OSA, optical spectrum analyzer; AOM, acoustic optical modulator; EVOA, electronic variable optical attenuator. FBG1 is applied for pump (1536.4 nm) rejection while FBG2 is for selecting one comb line (1562.8 nm) for SIL. The 3 dB bandwidth of the FBG filters is  $\sim 100$  GHz, and their center-wavelength tuning range is  $\sim 10$  nm. (b) The evolution of the laser (comb line -33) delayed heterodyne beating spectra with a feedback power ratio. The beating spectrum of the pump is also plotted as a reference. (c) Single sideband (SSB) frequency noise of the oscillator in this work and three other types of lasers. The data for DFB-SIL and Brillouin laser are taken from Refs. [14,59]. As a base line, the frequency noise corresponding to 1 Hz Lorentzian linewidth is given [62].

the spectral purity, this beating signal is also directly monitored by an ESA.

Figure 2(b) shows that the beating signal spectrum evolves with the feedback strength. From top to bottom,  $\eta$  increases monotonically (in a nonlinear manner for better display) by tuning the EVOA. Considering the phase of the fiber feedback loop is not a constant because of ambient temperature fluctuation [52], here we do not investigate the dynamics with a specific feedback phase since it is hard to measure and control in practice.

Similar to semiconductor lasers with optical feedback [49], this system contains a wealth of nonlinear dynamics. We found the dynamics of this system can also be classified as four distinct feedback regimes [53,54] according to the comb line's noise performance [see Fig. 2(b)]. Regime I corresponds to very weak feedback strength. The spectral width can be slightly broadened or narrowed from time to time, inferred as the variation of the feedback phase. With increasing feedback strength, spectral line narrowing is observed most of time. However, mode-hopping

starts to occur frequently with a given feedback strength, reflected as a burst of spikes in the beating spectrum [see Fig. 2(b)]. This regime is termed as regime II.

When the feedback reaches to a certain level (regime III), the mode hopping ceases altogether, and the line is persistently narrowed. This means the spectral narrowing is no longer sensitive to the feedback phase. Therefore, this regime allows us to obtain an extremely narrow-linewidth oscillator under relaxed conditions. In the following, the SIL state refers to this regime unless otherwise specified.

If the feedback strength is increased further (regime IV), a so-called coherence-collapsed state is obtained [53,54]. Not only does the original beating note spectrum get broadened, but new frequency components emerge. If the feedback ratio goes close to unit, the feedback power is too strong, and the soliton would vanish. A further detailed discussion of the dynamics would be lengthy and beyond the scope of this work. In this work, we mainly focus on the discovered regime III.

Figure 2(c) shows the measured frequency noise of the pump and comb line  $-33$  for the cases with and without SIL, which is measured by the correlated self-heterodyne method [55]. For the case without SIL, comb line  $-33$  features a much higher frequency noise than the pump mainly due to the thermorefractive noise of the microcavity [50,56,57]. However, its frequency noise got reduced by three to four orders of magnitude after SIL, resulting in a sub-Hertz intrinsic linewidth, which is two orders lower than the pump. The results indicate both the pump noise and thermorefractive noise can be greatly suppressed through SIL. Although it is hard to provide an analytical model for understanding the SIL dynamics in this multimode nonlinear system, particularly in regime III, the noise reduction through SIL can be intuitively explained as large cavity induced noise dilution [58].

The spectral coherence of the oscillator is comparable to other types of narrow-linewidth lasers, i.e., the fiber laser (NKT Photonics, Koheras), self-injection locked DFB laser [59], and Brillouin laser [14] [see Fig. 2(c)]. As a common feature of feedback, the beating spectrum as well as the optical frequency noise PSD exhibits peaks at frequencies close to the multiples of the FSR (3.6 MHz) of the feedback loop. However, these spikes can be greatly suppressed by the microresonator if their frequencies are tens of MHz, i.e., a meter-level long feedback loop is utilized.

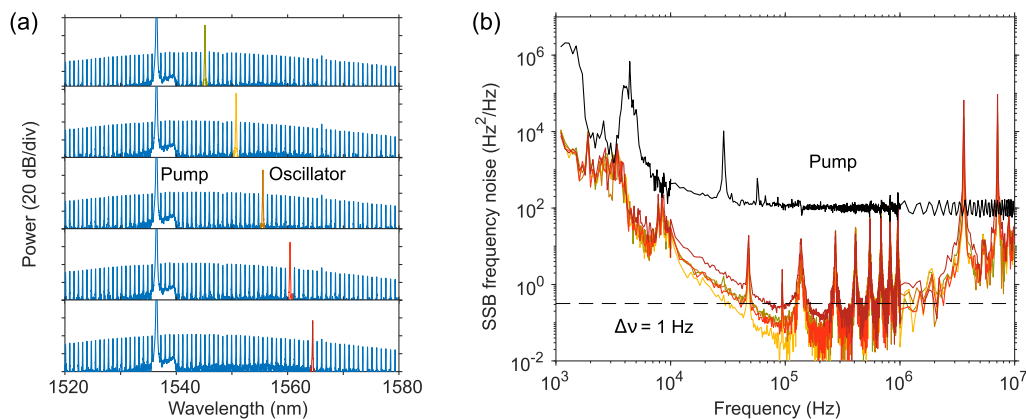
The above demonstrated self-injection-locked microcomb can be easily explored as a wavelength-tunable coherent oscillator. The emission wavelength could be selected by using a different comb

line for SIL. As shown in Fig. 3, we demonstrate that the Hertz-level oscillator is not exclusively attainable for a specific comb line. Instead it can be applicable to any comb lines with wavelengths ranging from 1545 nm to 1565 nm, limited by the working range of our filters. Although not shown here, it should be possible to obtain continuous wavelength tuning via simultaneously tuning the pump frequency and the cavity resonance over one FSR of the microresonator. Nevertheless, the long-term stability of the coherent oscillator cannot be guaranteed, because the whole system operates in atmospheric environment without any servo control loop. This issue could be solved in the future with a better packaged device or better yet with a fully integrated system. Further improvements could be attained by locking the oscillator to a stable passive optical reference, such as on-chip stable cavities [60].

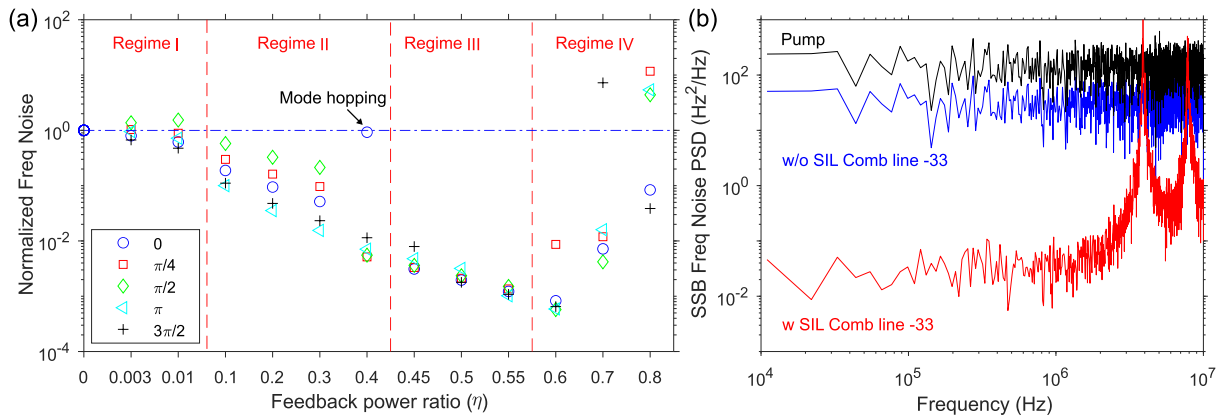
### 3. NUMERICAL STUDY OF SIL DYNAMICS

For a better understanding of the SIL dynamics of the system, we performed a numerical study. Different from semiconductor lasers where the noise is dominated by spontaneous emission, the noise sources of the current system include the pump phase and intensity noise, thermorefractive noise of the microresonator, shot noise, and technical noise [61]. To grasp the essential dynamics and facilitate understanding, we only consider the pump phase noise here. The other noise sources can be included through a similar approach. The detailed simulation method is described in Supplement 1.

First, we numerically investigate the SIL dynamics at different feedback strengths. In this case, the pump phase noise is simplified as a single tone modulation of the pump phase at 1.2 MHz. Figure 4(a) shows the simulated frequency noise induced by pump phase noise, which is normalized to the case without feedback, i.e.,  $\eta = 0$ . A variety of feedback strengths and feedback phases are considered. Consistent with the experimental observations, four dynamics regimes can be clearly distinguished according to the noise reduction performance and sensitivity to the feedback strength. In regime I, the noise could be reduced or enlarged according to the feedback phase. In regime II, the noise is usually suppressed, but the suppression ratio is phase dependent, and mode hopping can also be observed. With increasing feedback strength, the aforementioned regime III is replicated, where the noise reduction is nearly independent of the feedback phase. In IV, the system starts to enter chaotic states or remains at the stationary state according to the feedback phase, and with increasing feedback



**Fig. 3.** Wavelength tunability demonstration. The narrow-linewidth oscillators were realized at different wavelengths via selecting different comb lines for SIL. (a) Optical spectra. (b) Corresponding frequency noise PSDs.



**Fig. 4.** Numerical study of SIL of the soliton microcomb. (a) Frequency noise reduction (or amplification) as a function of feedback strength and phase. (b) Comparison of frequency noise PSD.

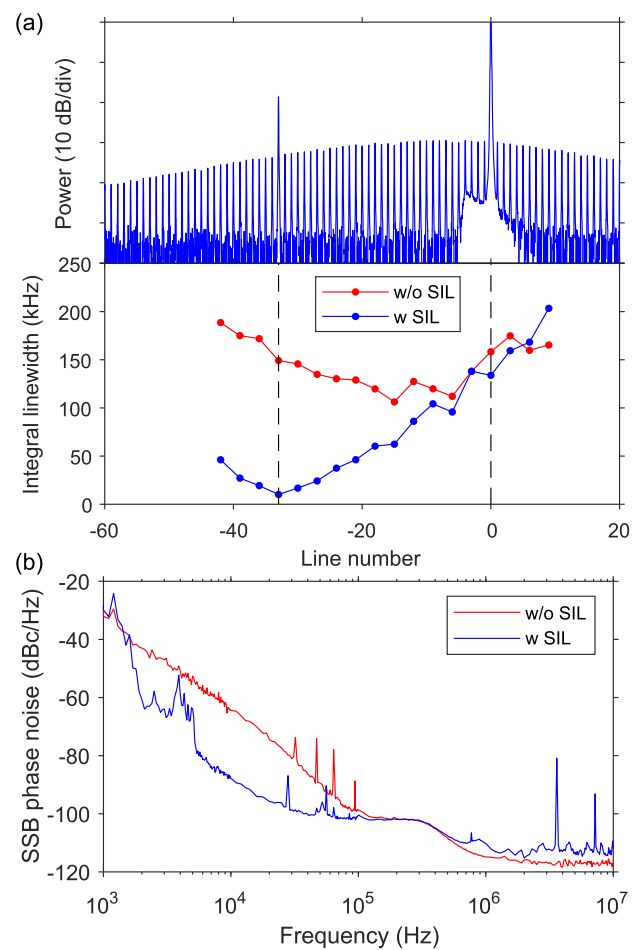
strength, it becomes completely chaotic regardless of the feedback phase.

Second, we performed a frequency noise PSD simulation comparing the system operating at the regime III and without the feedback case. The frequency noise of the pump is modeled as a flat PSD, i.e., white noise. It is noted that comb line  $-33$  features a slightly lower frequency noise than the pump without SIL, which can be attributed to the Raman effect [61]. After SIL, the frequency noise is reduced by three orders at the low-offset region [see Fig. 4(b)]. The characteristic spikes caused by feedback can also be seen, indicating that the numerical simulation captures the key features of the SIL dynamics. In the simulation, we only considered a constant feedback power ratio and did not include the gain dynamics of the EDFA, indicating that the essential role of the EDFA used in the feedback loop is a loss compensator.

#### 4. COHERENT OSCILLATOR FACILITATED SOLITON MICROCOMB STABILIZATION

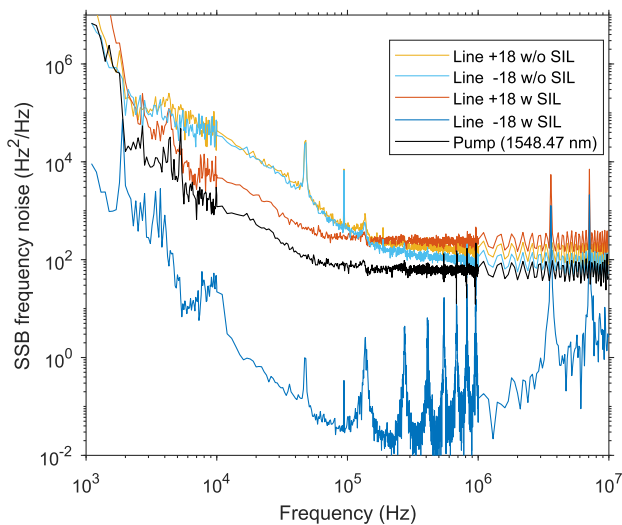
The above narrow-linewidth oscillator is one comb line of the soliton microcomb; therefore, its phase should be highly correlated with the other comb lines according to the elastic tape model [61]. The measured integral linewidth ( $\beta$ -line separation algorithm [62]) of the comb lines for the cases with and without SIL are compared and presented in Fig. 5(a). Not surprisingly, the pump linewidth is not changed because it is predetermined by the external pump source; however, the other comb lines get an effective linewidth reduction benefited from the SIL of one comb line, particularly for its neighbor lines. This phenomenon can be understood by noting that, in a frequency comb, there are only two degrees of freedom. Recent works are actively investigating the stabilization of microcombs using two low noise pumps by optical frequency division [46,63–66]. Here, the comb line with SIL is derived from the microcomb and, together with the pump, determines the two degrees of freedom of the comb.

Besides the spectral measurement of comb lines, we also measure the stability of the repetition rate. For this purpose, the repetition rate is electro-optic downconverted into a low frequency range ( $\sim 470$  MHz) with a 25 GHz intensity modulation, and then its phase noise is measured with a phase-noise analyzer. As shown in Fig. 5(b), its single-sideband phase noise is reduced at low-offset frequencies while increasing at a higher frequency offset ( $> 100$  kHz) after SIL. This phenomenon can be understood as the



**Fig. 5.** Coherent oscillator facilitated soliton microcomb stabilization. (a) Optical spectrum of the single-soliton microcomb with SIL and the measured integral linewidth of comb lines according to the frequency noise PSD. (b) Phase noise of the down converted repetition rate of the soliton microcomb.

repetition rate phase noise being mainly determined by the relative frequency stability between the pump and the narrow-linewidth oscillator. At lower offset frequencies, the relative frequency noise is mainly limited by the linewidth of the comb line due to the thermorefractive noise; therefore, comb linewidth narrowing can lead to lower repetition rate phase noise, while at higher offset



**Fig. 6.** Frequency noise of a pair comb lines symmetrically located around the pump, with one side being selected for SIL.

frequency region, the linewidth of the pump becomes a limiting factor as comb line  $-33$  features a much lower frequency noise (or higher at the spike frequencies).

To make it clear, we change the pump wavelength to 1548.47 nm and select comb line  $-18$  for SIL, making its mirror comb line, i.e., line  $18$  placed within our measurable range (C-band). As shown in Fig. 6, the frequency noise of comb line  $18$  is reduced at low-offset frequencies while increasing at a higher frequency offset after SIL. Hence, in order to exploit the narrow-linewidth oscillator for extremely pure microwave generation, a narrower linewidth pump laser is required.

## 5. DISCUSSION

In summary, we have demonstrated a narrow-linewidth yet tunable oscillator through SIL of a single comb line from a soliton microcomb. A dynamic regime suitable for parametric oscillator SIL is discovered both experimentally and numerically. Considering the gain of microcombs is based on parametric amplification and octave-spanning spectra can be directly generated in a monolithic nonlinear microresonator [67,68], our approach hence has the potential to generate extremely coherent electromagnetic radiation at wavelength regions where attaining optical amplification by stimulated emission of radiation is challenging while using pump lasers in the near infrared. Due to the existence of fiber-chip coupling loss and filter insertion loss, an optical amplifier is still utilized in the feedback loop; however, these losses could be overcome by developing a fully integrated low loss photonic delay line [69–71] or a high-Q microring filter. In terms of the output power, owing to the low conversion efficiency of the single-soliton microcomb, the power of this laser currently is limited to the sub-milliwatt (0.2 mW) level, which can be boosted further with other types of OPOs featuring much higher output power [72,73] or parametric amplifiers [74–76]. Moreover, our scheme should be also applicable to  $\chi^{(2)}$  media based OPOs, which usually feature even higher conversion efficiency and output power compared to the  $\chi^{(3)}$  case.

In addition to the realization of a single-tone pure oscillator, we also demonstrate that this scheme has the potential for thermorefractive noise suppression and microcomb stabilization, critical for ultralow noise microwave generation.

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**Disclosures.** E.L. and V.T.C. have submitted a patent application based on this work. Ó.B.H. and V.T.C. are co-founders of Iloomina AB, a company seeking to commercialize silicon photonics technologies. The remaining authors declare no conflicts of interest.

**Data availability.** Data underlying the results presented in this paper are available in Ref. [77].

**Supplemental document.** See Supplement 1 for supporting content.

## REFERENCES

1. S. A. Webster, M. Oxborrow, and P. Gill, "Subhertz-linewidth Nd: YAG laser," *Opt. Lett.* **29**, 1497–1499 (2004).
2. H. Stoehr, F. Mensing, J. Helmcke, *et al.*, "Diode laser with 1 Hz linewidth," *Opt. Lett.* **31**, 736–738 (2006).
3. M. Notcutt, L.-S. Ma, J. Ye, *et al.*, "Simple and compact 1-Hz laser system via an improved mounting configuration of a reference cavity," *Opt. Lett.* **30**, 1815–1817 (2005).
4. T. Kessler, C. Hagemann, C. Grebing, *et al.*, "A sub-40-mHz-linewidth laser based on a silicon single-crystal optical cavity," *Nat. Photonics* **6**, 687–692 (2012).
5. W. Liang, V. Ilchenko, D. Eliyahu, *et al.*, "Ultralow noise miniature external cavity semiconductor laser," *Nat. Commun.* **6**, 7371 (2015).
6. X. Huang, Q. Zhao, W. Lin, *et al.*, "Linewidth suppression mechanism of self-injection locked single-frequency fiber laser," *Opt. Express* **24**, 18907–18916 (2016).
7. D. K. Shin, B. M. Henson, R. I. Khakimov, *et al.*, "Widely tunable, narrow linewidth external-cavity gain chip laser for spectroscopy between 1.0–1.1  $\mu\text{m}$ ," *Opt. Express* **24**, 27403–27414 (2016).
8. J. Ji, H. Wang, J. Ma, *et al.*, "Narrow linewidth self-injection locked fiber laser based on a crystalline resonator in add-drop configuration," *Opt. Lett.* **47**, 1525–1528 (2022).
9. L. Dang, L. Huang, L. Shi, *et al.*, "Ultra-high spectral purity laser derived from weak external distributed perturbation," *Opto-Electron. Adv.* **6**, 210149 (2023).
10. A. Malik, C. Xiang, L. Chang, *et al.*, "Low noise, tunable silicon photonic lasers," *Appl. Phys. Rev.* **8**, 031306 (2021).
11. J. Ling, J. Staffa, H. Wang, *et al.*, "Self-injection locked frequency conversion laser," *Laser Photon. Rev.* **17**, 2200663 (2023).
12. J. C. Hill, W. K. Holland, P. D. Kunz, *et al.*, "Intra-cavity frequency-doubled VECSEL system for narrow linewidth Rydberg EIT spectroscopy," *Opt. Express* **30**, 41408–41421 (2022).
13. A. Debut, S. Randoux, and J. Zemmouri, "Linewidth narrowing in Brillouin lasers: theoretical analysis," *Phys. Rev. A* **62**, 023803 (2000).
14. S. Gundavarapu, G. M. Brodnik, M. Puckett, *et al.*, "Sub-hertz fundamental linewidth photonic integrated Brillouin laser," *Nat. Photonics* **13**, 60–67 (2019).
15. M. H. Dunn and M. Ebrahimzadeh, "Parametric generation of tunable light from continuous-wave to femtosecond pulses," *Science* **286**, 1513–1517 (1999).
16. I. Breunig, D. Haertle, and K. Buse, "Continuous-wave optical parametric oscillators: recent developments and prospects," *Appl. Phys. B* **105**, 99–111 (2011).
17. R. Graham and H. Haken, "The quantum-fluctuations of the optical parametric oscillator. I," *Z. Phys.* **210**, 276–302 (1968).
18. B. Jacobsson, M. Tiihonen, V. Pasiskevicius, *et al.*, "Narrowband bulk Bragg grating optical parametric oscillator," *Opt. Lett.* **30**, 2281–2283 (2005).
19. M. Vainio, C. Ozanam, V. Ulvila, *et al.*, "Tuning and stability of a singly resonant continuous-wave optical parametric oscillator close to degeneracy," *Opt. Express* **19**, 22515–22527 (2011).
20. Y. Peng, X. Wei, G. Xie, *et al.*, "A high-power narrow-linewidth optical parametric oscillator based on PPMgLN," *Laser Phys.* **23**, 055405 (2013).
21. S. Das, "Pump tuned wide tunable noncritically phase-matched ZnGeP<sub>2</sub> narrow line width optical parametric oscillator," *Infrared Phys. Technol.* **69**, 13–18 (2015).

22. J. Saikawa, M. Fujii, H. Ishizuki, *et al.*, "High-energy, narrow-bandwidth periodically poled Mg-doped LiNbO<sub>3</sub> optical parametric oscillator with a volume Bragg grating," *Opt. Lett.* **32**, 2996–2998 (2007).
23. P. Zeil, N. Thilmann, V. Pasiskevicius, *et al.*, "High-power, single-frequency, continuous-wave optical parametric oscillator employing a variable reflectivity volume Bragg grating," *Opt. Express* **22**, 29907–29913 (2014).
24. M. Henriksson, M. Tiuhonen, V. Pasiskevicius, *et al.*, "Mid-infrared ZGP OPO pumped by near-degenerate narrowband type-I PPKTP parametric oscillator," *Appl. Phys. B* **88**, 37–41 (2007).
25. M. Rahm, G. Anstett, J. Bartschke, *et al.*, "Widely tunable narrow-linewidth nanosecond optical parametric generator with self-injection seeding," *Appl. Phys. B* **79**, 535–538 (2004).
26. T. Xing, L. Wang, S. Hu, *et al.*, "Widely tunable and narrow-bandwidth pulsed mid-IR PPMgLN-OPO by self-seeding dual etalon-coupled cavities," *Opt. Express* **25**, 31810–31815 (2017).
27. A. Ly, C. Siour, and F. Bretenaker, "30-Hz relative linewidth watt output power 1.65  $\mu\text{m}$  continuous-wave singly resonant optical parametric oscillator," *Opt. Express* **25**, 9049–9060 (2017).
28. I. Ricciardi, S. Mosca, M. Parisi, *et al.*, "Sub-kilohertz linewidth narrowing of a mid-infrared optical parametric oscillator idler frequency by direct cavity stabilization," *Opt. Lett.* **40**, 4743–4746 (2015).
29. O. Mhibik, T.-H. My, D. Pabœuf, *et al.*, "Frequency stabilization at the kilohertz level of a continuous intracavity frequency-doubled singly resonant optical parametric oscillator," *Opt. Lett.* **35**, 2364–2366 (2010).
30. Z.-T. Zhang, C.-F. Cheng, Y. Sun, *et al.*, "Cavity ring-down spectroscopy based on a comb-locked optical parametric oscillator source," *Opt. Express* **28**, 27600–27607 (2020).
31. E. Kovalchuk, T. Schuldt, and A. Peters, "Combination of a continuous-wave optical parametric oscillator and a femtosecond frequency comb for optical frequency metrology," *Opt. Lett.* **30**, 3141–3143 (2005).
32. S. Zaske, D.-H. Lee, and C. Becher, "Green-pumped CW singly resonant optical parametric oscillator based on Mgo:PPLN with frequency stabilization to an atomic resonance," *Appl. Phys. B* **98**, 729–735 (2010).
33. I.-H. Bae, H. S. Moon, S. K. Kim, *et al.*, "Optical injection locking of a singly resonant continuous-wave optical parametric oscillator," *Opt. Lett.* **38**, 597–599 (2013).
34. Z.-T. Zhang, Y. Tan, J. Wang, *et al.*, "Seeded optical parametric oscillator light source for precision spectroscopy," *Opt. Lett.* **45**, 1013–1016 (2020).
35. P. Del'Haye, A. Schliesser, O. Arcizet, *et al.*, "Optical frequency comb generation from a monolithic microresonator," *Nature* **450**, 1214–1217 (2007).
36. T. Herr, V. Brasch, J. D. Jost, *et al.*, "Temporal solitons in optical microresonators," *Nat. Photonics* **8**, 145–152 (2014).
37. X. Xue, Y. Xuan, Y. Liu, *et al.*, "Mode-locked dark pulse Kerr combs in normal-dispersion microresonators," *Nat. Photonics* **9**, 594–600 (2015).
38. Ó. B. Helgason, F. R. Arteaga-Sierra, Z. Ye, *et al.*, "Dissipative solitons in photonic molecules," *Nat. Photonics* **15**, 305–310 (2021).
39. H. Bao, A. Cooper, M. Rowley, *et al.*, "Laser cavity-soliton microcombs," *Nat. Photonics* **13**, 384–389 (2019).
40. M. Rowley, P.-H. Hanzard, A. Cutrona, *et al.*, "Self-emergence of robust solitons in a microcavity," *Nature* **608**, 303–309 (2022).
41. W. Wang, S. T. Chu, B. E. Little, *et al.*, "Dual-pump Kerr micro-cavity optical frequency comb with varying FSR spacing," *Sci. Rep.* **6**, 28501 (2016).
42. P. Del'Haye, K. Beha, S. B. Papp, *et al.*, "Self-injection locking and phase-locked states in microresonator-based optical frequency combs," *Phys. Rev. Lett.* **112**, 043905 (2014).
43. A. S. Raja, A. S. Voloshin, H. Guo, *et al.*, "Electrically pumped photonic integrated soliton microcomb," *Nat. Commun.* **10**, 680 (2019).
44. B. Shen, L. Chang, J. Liu, *et al.*, "Integrated turnkey soliton microcombs," *Nature* **582**, 365–369 (2020).
45. A. S. Voloshin, N. M. Kondratiev, G. V. Lihachev, *et al.*, "Dynamics of soliton self-injection locking in optical microresonators," *Nat. Commun.* **12**, 235 (2021).
46. H. Taheri, A. B. Matsko, L. Maleki, *et al.*, "All-optical dissipative discrete time crystals," *Nat. Commun.* **13**, 848 (2022).
47. D.-H. Lee, M. E. Klein, J.-P. Meyn, *et al.*, "Self-injection-locking of a CW-OPO by intracavity frequency-doubling the idler wave," *Opt. Express* **5**, 114–119 (1999).
48. D.-H. Lee, M. E. Klein, J.-P. Meyn, *et al.*, "Phase-coherent all-optical frequency division by three," *Phys. Rev. A* **67**, 013808 (2003).
49. K. Petermann, *Laser Diode Modulation and Noise* (Springer, 1991), Vol. 3.
50. T. E. Drake, J. R. Stone, T. C. Briles, *et al.*, "Thermal decoherence and laser cooling of Kerr microresonator solitons," *Nat. Photonics* **14**, 480–485 (2020).
51. L. Maleki, "The optoelectronic oscillator," *Nat. Photonics* **5**, 728–730 (2011).
52. L.-S. Ma, P. Jungner, J. Ye, *et al.*, "Delivering the same optical frequency at two places: accurate cancellation of phase noise introduced by an optical fiber or other time-varying path," *Opt. Lett.* **19**, 1777–1779 (1994).
53. R. Tkach and A. Chraplyvy, "Regimes of feedback effects in 1.5- $\mu\text{m}$  distributed feedback lasers," *J. Lightwave Technol.* **4**, 1655–1661 (1986).
54. N. Schunk and K. Petermann, "Numerical analysis of the feedback regimes for a single-mode semiconductor laser with external feedback," *IEEE J. Quantum Electron.* **24**, 1242–1247 (1988).
55. Z. Yuan, H. Wang, P. Liu, *et al.*, "Correlated self-heterodyne method for ultra-low-noise laser linewidth measurements," *Opt. Express* **30**, 25147–25161 (2022).
56. G. Huang, E. Lucas, J. Liu, *et al.*, "Thermorefractive noise in silicon-nitride microresonators," *Phys. Rev. A* **99**, 061801 (2019).
57. F. Lei, Z. Ye, and V. Torres-Company, "Thermal noise reduction in soliton microcombs via laser self-cooling," *Opt. Lett.* **47**, 513–516 (2022).
58. R. Kazarinov and C. Henry, "The relation of line narrowing and chirp reduction resulting from the coupling of a semiconductor laser to passive resonator," *IEEE J. Quantum Electron.* **23**, 1401–1409 (1987).
59. B. Li, W. Jin, L. Wu, *et al.*, "Reaching fiber-laser coherence in integrated photonics," *Opt. Lett.* **46**, 5201–5204 (2021).
60. J. Guo, C. A. McLemore, C. Xiang, *et al.*, "Chip-based laser with 1-hertz integrated linewidth," *Sci. Adv.* **8**, eabp9006 (2022).
61. F. Lei, Z. Ye, Ó. B. Helgason, *et al.*, "Optical linewidth of soliton microcombs," *Nat. Commun.* **13**, 3161 (2022).
62. G. Di Domenico, S. Schilt, and P. Thomann, "Simple approach to the relation between laser frequency noise and laser line shape," *Appl. Opt.* **49**, 4801–4807 (2010).
63. T. Tetsumoto, T. Nagatsuma, M. E. Fermann, *et al.*, "Optically referenced 300 GHz millimetre-wave oscillator," *Nat. Photonics* **15**, 516–522 (2021).
64. T. Wildi, A. Ulanov, N. Englebort, *et al.*, "Sideband injection locking in microresonator frequency combs," *APL Photon.* **8**, 120801 (2023).
65. I. Kudelin, W. Groman, Q.-X. Ji, *et al.*, "Photonic chip-based low noise microwave oscillator," *arXiv*, arXiv:2307.08937 (2023).
66. S. Sun, B. Wang, K. Liu, *et al.*, "Integrated optical frequency division for stable microwave and mmwave generation," *arXiv*, arXiv:2305.13575 (2023).
67. P. Del'Haye, T. Herr, E. Gavartin, *et al.*, "Octave spanning tunable frequency comb from a microresonator," *Phys. Rev. Lett.* **107**, 063901 (2011).
68. H.-J. Chen, Q.-X. Ji, H. Wang, *et al.*, "Chaos-assisted two-octave-spanning microcombs," *Nat. Commun.* **11**, 2336 (2020).
69. H. Lee, T. Chen, J. Li, *et al.*, "Ultra-low-loss optical delay line on a silicon chip," *Nat. Commun.* **3**, 867 (2012).
70. X. Wang, L. Zhou, R. Li, *et al.*, "Continuously tunable ultra-thin silicon waveguide optical delay line," *Optica* **4**, 507–515 (2017).
71. D. Lin, S. Shi, W. Cheng, *et al.*, "A high performance silicon nitride optical delay line with good expansibility," *J. Lightwave Technol.* **41**, 209–217 (2022).
72. F. Lei, Z. Ye, K. Twayana, *et al.*, "Hyperparametric oscillation via bound states in the continuum," *Phys. Rev. Lett.* **130**, 093801 (2023).
73. Ó. B. Helgason, M. Girardi, Z. Ye, *et al.*, "Surpassing the nonlinear conversion efficiency of soliton microcombs," *Nat. Photonics* **17**, 992–999 (2023).
74. Z. Ye, P. Zhao, K. Twayana, *et al.*, "Overcoming the quantum limit of optical amplification in monolithic waveguides," *Sci. Adv.* **7**, eabi8150 (2021).
75. J. Riemensberger, N. Kuznetsov, J. Liu, *et al.*, "A photonic integrated continuous-travelling-wave parametric amplifier," *Nature* **612**, 56–61 (2022).
76. Y. Geng, W. Cui, Q. Wen, *et al.*, "Microcavity-based narrowband parametric amplifier for carrier recovery in optical coherent self-homodyne detection," *Opt. Lett.* **44**, 3490–3493 (2019).
77. F. Lei, Y. Sun, Ó. B. Helgason, *et al.*, "Raw data for article: self-injection-locked optical parametric oscillator based on microcombs," Zenodo: Version 1, 2024, <https://doi.org/10.5281/zenodo.10724831>