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THE IMPORTANCE OF PHASE DYNAMICS IN GENERATION OF COHERENT STRUCTURES

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1. INTRODUCTION

In magnetically confined plasmas (MCP), the transport of heat and particles is determined by collisional and anomalous processes caused by turbulence. A collective effort has been put into modelling the turbulent transport in plasmas using various drift wave (DW) models. However, it is evident that large-scale phenomena have a significant impact on overall transport. Heat transport can be mediated by coherent structures such as streamers and blobs through the formation of avalanche-like events that are intermittent in nature, i.e., localized in time but of large amplitude. Furthermore, at the same time, there are structures such as zonal flows (ZF) and GAMs that are non-linearly generated and mitigate turbulent transport by shearing turbulent eddies. A common denominator for these large-scale structures is the synchronization of smaller scale modes or events to a coherent structure, where phases align in a localized region of space and time. Interestingly, phase synchronization is prevalent in many other fields, such as biological clocks, physiological organisms, and chemical reactors. The dynamical evolution of amplitude and phases have been investigated through simplified equations derived from the Hasegawa - Wakatani (HW) system, where effects of synchronization are studied. Theoretical studies often deal with the amplitudes of the fluctuating quantities and assume that the phases are randomly distributed according to the random phase approximation (RPA) and thus disregard the dynamics of the phases [1,2]. In this approximation, dynamical amplitudes have a slow variation compared to the rapid change of the phases, which are distributed uniformly over a 2π interval [3]. There have been a few general approaches to the randomness in turbulence: the RPA, the diagrammatic method by Wyld and the cumulant expansions, with the aim of systematically characterizing intermittent behavior. Unless a specific case is studied, the diagrammatic method has a drawback since there is no consistent small expansion parameter and no normalization procedure available. Moreover, the intuitive picture of the RPA approach is tempting and is thus widely adopted in turbulence theory. The underlying assumption of randomness in the RPA for the phases of Fourier modes in nonlinearly interacting waves cannot be justified since the phases as well as the amplitudes evolve due to non-linear interactions that act on the same time scales for both. Thus, the phases cannot be randomized faster than the amplitudes, see further discussion in Refs. [4,5]. Understanding the generation of coherent structures and the effects of these structures on transport and turbulence is therefore of crucial importance. In regard to plasma dynamics, simplified models are of interest, assuming an expansion of the state in amplitude and phase, i.e., $\phi \sim \phi_0 \exp(i\theta)$, the basic dynamical equations yield one dynamical equation for the amplitude and one for the phase for each field in the model. In previous papers, models using the passive advected scalar [6] and the Burgers equation [1] where it was found that under certain conditions, the RPA assumption can be invalidated using a phase dependent force and the locking of phases may increase the energy transfer to other modes. The assumption of a fully stochastic phase state of the turbulence is more relevant for high values of scale separation with the energy spectrum following a $k^{7/2}$ decay rate. The dynamic of the three-body interactions between the phases in the non-linear Burgers' turbulence shows that the phases lock intermittently. This is due to the k dependence of the coupling strength in the non-linear term which reduces strongly for high-k range due to the dampening effect of the dissipation which does not allow locking of the phases of the small scales. For lower scale dependence the asynchronized and synchronized phases differ significantly, and one could expect the formation of coherent modulations in the latter case. Moreover, the HW have been studied [7] and the work on the predator-prey model of DW – ZF dynamics, it is observed that synchronization may be transferred between the two populations [8].

In this work, we investigate the role of phase dynamics for turbulent fluctuations in a set of direct numerical simulation (DNS) of homogeneous Taylor-Green driven turbulence, simple 2D rotating turbulence flow. The model is the forced incompressible magnetohydrodynamic (MHD) equations. It should be noted that in the study of coupled oscillators describing chemical reactors, the Kuramoto model has been established, and it has been shown that synchronization occurs when a certain threshold is exceeded. In this case the system is strongly forced to generate a vortex and where the phase locking between close neighbours can be quantified.

2. THE KURAMOTO MODEL AND SYNCHRONIZATION

Random processes and related anomalous diffusion phenomena have been observed in a wide variety of complex systems such as semiconductors, glassy materials, nano-pores, biological cells, and epidemic spreading. The problem of finding a proper kinetic description for such complex systems is a challenge. The pedagogical applications of simplified models such as the Kuramoto model [9, 10] of random oscillators are particularly helpful in understanding dynamics in many-body interacting systems. It has been used previous work as a paradigm to understand Burgers' turbulence. In particular we are interested to show the dominant impact of singular events with high amplitude on the the long term collective behaviour, and to illustrate the limitations of the Gaussian assumptions in these non-linearly coupled systems. We hope to start a wider discussion on the features that can be expected in the field of plasma physics. The dynamics of the phases of the oscillators are described by coupled first order differential equations of the Kuramoto form 1 :

$$\dot{\theta}_j(t) = \omega_j + \frac{K}{N}\sum_{i=1}^{N} sin(\theta_i - \theta_j), \quad (j = 1, ..., N), \tag{1}$$

where $\theta(t)_j$ is the phase of the jth oscillator with $\dot{\theta}_j(t)$ being its time derivative. Here ω_j is the natural frequency of the oscillator which is often assumed to be distributed according to a Gaussian distribution $f(\omega) = exp(-\omega^2/2)/\sqrt{2}\pi$. K is the strength of the interactions between oscillators ith and jth. Moreover, consider the sum of complex numbers of the form i.e. the average of the complex phases:

$$Z = \frac{1}{N}\sum_{i=1}^{N} e^{i\theta_i}. \tag{2}$$

The amplitude of the complex number Z in (2) increases with how close the phases are to each other. The absolut value of 2 is then a measure of the synchronicity in the system where the local order parameter is then defined as $\lambda = |Z|$. In order to test the phase coherence hypothesis in a reduced plasma model a test case of using direct numerical simulations (DNS) of incompressible magnetohydrodynamics (MHD) is used, see Refs [11-12]. The numerical results are downloadable using Ref. [11] and thus the primary work in the paper is the data analysis of this numerically generated data set. The basic equations solved

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\nabla p \vec{j} \times \vec{b} + \nu\nabla^2\vec{u} + \vec{F} \tag{3}$$

$$\frac{\partial b}{\partial t} = \nabla \times (\vec{u} \times \vec{b}) + \eta\nabla^2\vec{b} \tag{4}$$

$$\nabla \cdot \vec{u} = 0 = \nabla \cdot \vec{b} \tag{5}$$

Here $\vec{u}$ is the velocity field in the incompressible Navier-Stokes equation, b is the magnetic field in the induction equation and p is the pressure and $\vec{F}$ is an external forcing taken to that of the Taylor-Green flow defined by:

$$\vec{F} = f_0(\sin(k_f x)\cos(k_f y)\cos(k_f x)\hat{x} - \cos(k_f x)\sin(k_f y)\cos(k_f z)\hat{y}), \tag{6}$$

where $f_0 = 0.25$ and $k_f = 2.0$. The Prandtl number is proportional to 1 and $\eta = \nu = 1.1 \times 10^{-4}$.

3. RESULTS

In this section we compute the local order parameter, defined by λ_j, for $j = 1$ it calculates the synchronization including only the first neighbours and for $j = 2$ also including the two closest neighbouring modes. There is filter applied in a few cases to the signal, such that modes with $\lambda < 0.1$ are artificially reduced to zero before taking the inverse fast Fourier transform back to real space. The simulations are performed with a periodic boundary in $\hat{z}$ thus the figures below are one cross section in the x-y plane. Also to simplify only one slice in time is analyzed. In Fig. (1) and (2) the phase coherence of the absolute value of the magnetic field are shown respectively. The

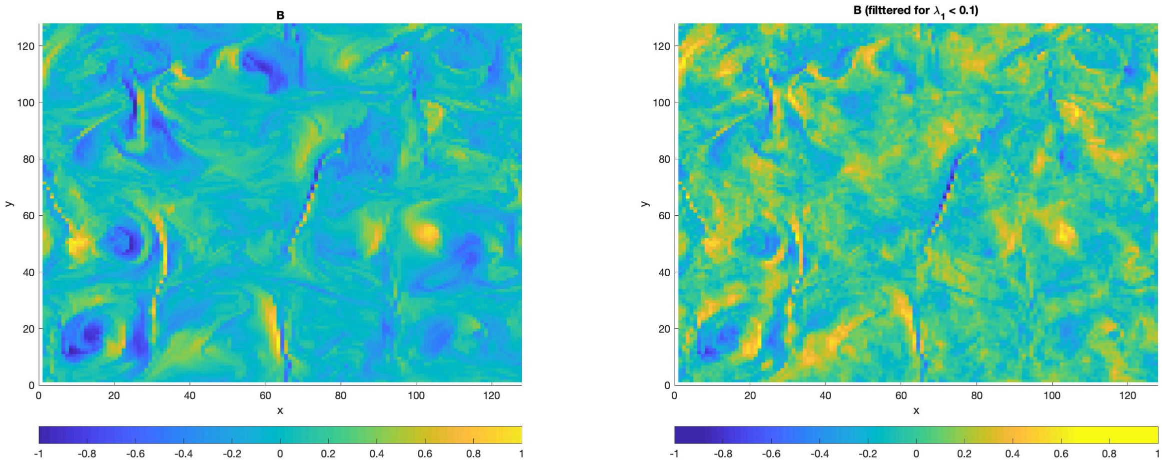

FIG. 1. B field (original) and B field filtered for λ_1 (first neighbors) < 0.1.

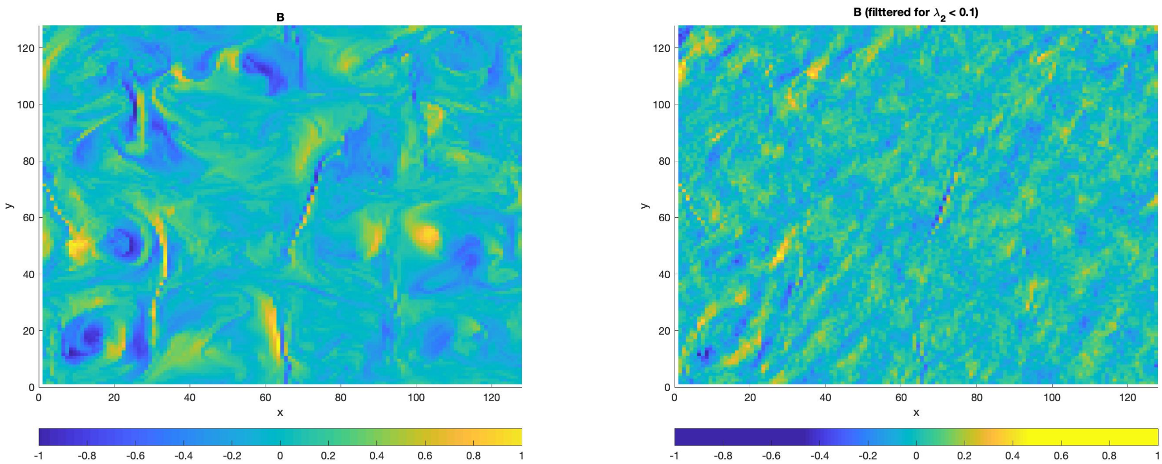

FIG. 2. B field (original) and B field filtered for λ_2 (second neighbors) < 0.1.

original signal and a filtered version of the signal is shown where the filter articially put all small coherence values to zero. The phase coherence is plotted in the x-y plane? In Fig. (3) and (4) the phase coherence of the absolute value of the velocity are shown respectively in the x-y plane.

4. SUMMARY

In this work, we have introduced an analysis of a DNS incompressible MHD simulation where the phase coherence is investigated. The phase coupling is assumed to follow the well-established Kuramoto paradigm that has been shown to represent systems displaying self-organisation well. The model is intended to isolate the importance of the collective phase a-synchronisation/synchronisation states on the time evolution of the velocity and magnetic fields. Note that the full picture is varied with phase locking and breaking. This has been observed in other non-linear systems such as in the Burgers turbulence [1] and in the HW model [7]. In both these systems generation of large scale structures is possible however in many cases this is impeded by phase breaking due to the non-linear

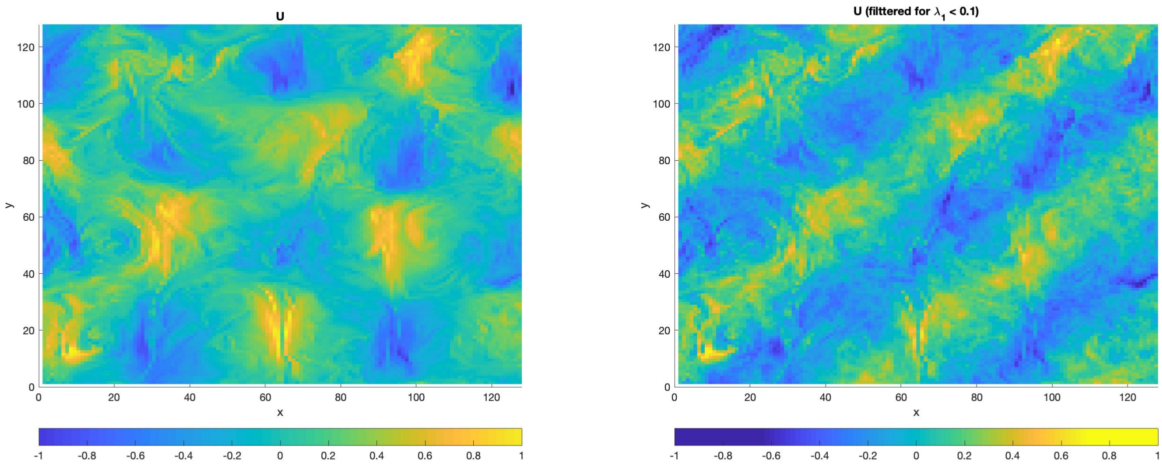

FIG. 3. U field (original) and U field filtered for λ_1 *(first neighbors)* < 0.1.

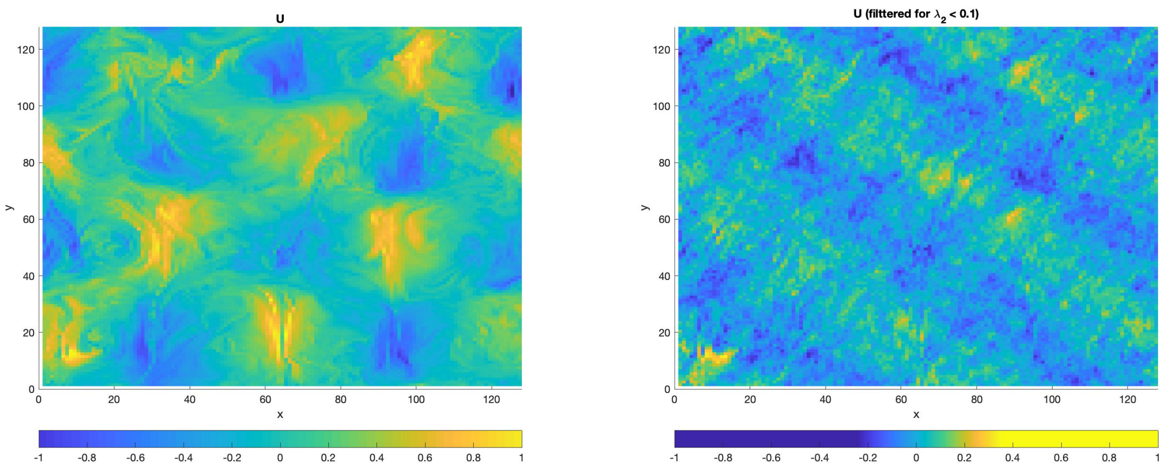

FIG. 4. U field (original) and U field filtered for λ_2 *(second neighbors)* < 0.1.

interaction. It was noted that due to particular form of the systematic dependency of the frequencies to the wave-numbers through the dispersion relation, this system does not seem to tend toward synchronization. It is possible that boundary conditions come into play non-trivially stop formation of synchronized large scale structures in some cases.

The results are shown as discrete time slices in the z-periodic simulation box. We find that phases are organized in band like structures diagonally across the simulation domain where synchronization extends beyond local areas. This seems to hold even when the local order parameter is extended beyond the immediate nearest neighbour interaction indicating that indeed synchronization in this particular forced system is present. This is, moreover, similar to what was found in the forced Burgers equation where banded synchronized structures could be generated by forcing the non-linear system.

This is a first attempt to investigate phase locking in 3D MHD turbulence where no unique order parameter is defined. This opens up synchronization of other system where large scale modes are present.

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