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Research article

Bus timetable optimization model in response to the diverse and uncertain requirements of passengers for travel comfort

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Abstract: Most existing public transit systems have a fixed dispatching and service mode, which cannot effectively allocate resources from the perspective of the interests of all participants, resulting in resource waste and dissatisfaction. Low passenger satisfaction leads to a considerable loss of bus passengers and further reduces the income of bus operators. This study develops an optimization model for bus schedules that considers vehicle types and offers two service levels based on heterogeneous passenger demands. In this process, passenger satisfaction, bus company income, and government subsidies are considered. A bilevel model is proposed with a lower-level passenger ride simulation model and an upper-level multiobjective optimization model to maximize the interests of bus companies, passengers, and the government. To verify the effectiveness of the proposed methodology, a real-world case from Guangzhou is presented and analyzed using the nondominated sorting genetic algorithm-II (NSGA-II), and the related Pareto front is obtained. The results show that the proposed bus operation system can effectively increase the benefits for bus companies, passengers, and the government.

Keywords: bus timetable; multiple service levels; pareto front; nondominated sorting genetic algorithm-II

1. Introduction

Public transportation can be a practical and efficient means of solving traffic-related problems, reducing air pollution, and saving energy. The total number of vehicles can be considerably reduced by operating more buses, as buses have much higher resource utilization than private cars. The scheduling of buses is an essential aspect of public transportation operations [1]. However, there are two drawbacks to traditional bus scheduling systems. First, the time interval between the departures of two buses is usually fixed and regulated by experienced engineers [2]. In fact, there are seasonal, weekly, and day-of-the-week fluctuations in transit demand according to the location, time of day, and direction of travel. Second, only one type of bus is typically used by bus companies. However, a bus company needs to provide a comfortable travel experience to passengers instead of simply carrying them from point A to point B [3]. Due to the fluctuating nature of passenger flows and the diversity of passenger needs, traditional fixed bus timetables and single bus services are insufficient to meet the requirements of the public.

There are several solutions to these problems. Mobile phones and electronic payments have enabled the creation of flexible transit systems based on the internet, and such systems are widely used by public transportation companies [4]. Using emerging big data and Internet of Things (IoT) technologies, we can design timetables based on passenger flows [5]. Expecting passengers to adapt to a fixed timetable instead of altering the schedule to meet passenger requests [6] is a crucial cause of unreliable service. One good solution is to solve the multiple-vehicle-type vehicle scheduling problem (MVT-VSP) [7].

There are three key participants in public transportation operation services [8]:

- **Public transport passengers** desire a transportation system that is convenient, economical, and comfortable.
- **Bus companies** provide transportation services and primarily focus on minimizing operating costs and maximizing profits under the market supervision of the government.
- **The government** formulates policies that are intended to ensure an essential quality of life for its citizens, and it creates transportation patterns that satisfy relevant needs and ensure the stable operation of the transportation system.

This paper develops an optimization model for bus schedules that considers vehicle types and offers two service levels based on passenger flow situations. This optimization model considers the respective interests of bus companies, passengers, and the government. Finding a balance between these competing goals is the most difficult aspect of transportation design. The subject considered here pertains to optimizing public transportation systems, which is part of a broad field of study.

1.1. Literature review

In recent decades, research on bus schedule optimization problems has been conducted. Many studies have addressed multitype bus scheduling problems, where types can be classified into large and small buses based on capacity [7]. Many researchers have explored the impact of different vehicle capacities on bus systems. Some scholars have studied the relationship between bus capacity and operating costs. For example, Hassold and Ceder [6] investigated the effect of introducing different bus sizes into a bus system on operating costs. Some scholars have studied the problem of dispatching multitype combinations with multiple capacities and compared it with a single-capacity-model dispatching scheme. For example, Tang et al. [9] developed three dispatching optimization models that considered the coexistence of large and small buses, large buses only and small buses only. The results of their study showed that compared with the single-vehicle-type dispatching approach, the multivehicle dispatching approach can reduce passenger travel time and overall bus operating costs, alleviate the problems caused the large differences in congestion between bus vehicles and poor on-time arrival rates, improve bus service levels and reduce the waste of bus resources. Some scholars have considered various vehicle types with different capacities and have combined them with specific situations to determine the optimal solution [10,11]. With technological development, scholars have considered the problem of variable-capacity self-driving vehicle dispatching. For example, Dai et al. [12] studied the bus operation environment with mixed human-driven buses and self-driving buses in mixed traffic, where the mixed human-driven buses have a fixed passenger capacity, while the self-driving buses can change their capacity by combining or separating multiple small buses.

A key focus and difficulty of current research on multimodel bus scheduling is the scheduling problem of two-two combinations or all considerations of full buses, interval buses and large stop express buses on conventional bus routes [13]. Due to the uneven spatial and temporal distribution of passenger trips [14], the traditional single bus dispatching method may result in crowded vehicles in high-traffic sections and large unused spaces in low-traffic sections, which may lead to a less attractive bus service for passengers. Bus systems can be equipped with multiple vehicle operating modes to address this problem. Moreover, route-variable multivehicle dispatching can reduce travel time [15,16] and improve vehicle utilization [17]. Some scholars have studied the impact of route-variable multivehicle dispatching on bus operating costs [18,19] and environmental pollution [20].

Electric buses have the advantages of low energy costs [21–23] and carbon emissions [24] compared to traditional fuel buses. The share of new energy buses in public transportation is increasing, but it remains difficult to completely electrify a bus system in a short period of time. Thus, the mixed operation of fuel buses and electric buses will continue to be common. In the problem of scheduling multimodel buses with different power sources, some scholars have studied the optimal mixing ratio of the two types of buses. For example, Liu et al. [25] showed that the optimal ratio of electric buses and conventional buses changes with changes in passenger flow. Each fuel bus may be replaced by two electric buses because electric and fuel buses transport the same number of people. Several researchers [26,27] have analyzed the effect of multivehicle bus scheduling with hybrid sources on the cost of bus systems.

Some scholars have considered the problem of multivehicle bus scheduling in real-world situations. For example, Ceder [28] stated that in actual public transport operations, multiple vehicle types are often used, e.g., small buses, articulated buses, and double-decker buses may be simultaneously used in addition to regular buses with different levels of comfort and seating capacity. When acquiring buses, a bus company must consider the required types of buses. To develop a cost-effective framework for determining the level of vehicle comfort, the relationship between the geographic characteristics of journeys (urban, intercity, etc.) and the type of bus needed for the journey was examined. Baldoquin and Rengifo-Campo [29] solved the operational scheduling problem for an urban public transportation system by developing an optimization model. The system contained four different bus companies and three bus types: complementary, articulated, and standard. Yao et al. [30] constructed a mathematical optimization model and successfully applied the method to a real transportation network in a city. The optimization results gave the optimal number of electric buses and associated schedules for two real vehicle types and the number of charging facilities that must be equipped.

The bus departure scheduling optimization problem is a complex multiobjective optimization problem, and the objectives in the problem are often in conflict with one another. Table 1 lists the optimization objectives involved in the multiobjective bus dispatching optimization problem and the solution methods used by scholars.

Table 1. Comparison of related studies.

Ref.	Objective functions			Solution method
	Bus companies	Passengers	Others	
Rojas et al. [31]	Number of passengers served	Vehicle operating costs	-	Constraint method to turn an objective into a constraint so that multiple objectives become a single objective; Pareto
Li and Lu [32]	-	Operational efficiency; service level	-	Weighting method to turn multiple objectives into a single objective
Li et al. [33]	Waiting time	Bus capacity	-	Modified genetic algorithm (GA) and particle swarm optimization (PSO)
Baldoquin and Rengifo-Campo [29]	-	Total vehicle miles to empty; deviation from ideal kilometers	-	Gurobi solver
Yang and Liu [34]	Waiting time	-	Vehicle energy consumption	Two-stage algorithm
Tang et al. [2]	Waiting time	Vehicle operation time	-	Improved GA for nondominated sorting; Pareto
Liu and Wang [35]	Travel costs; waiting time	Operating costs	-	Weighting method to make multiple objectives into a single objective; modified adaptive large neighborhood search with nearest vehicle dispatch (NVD) algorithm (MALNSN)
Our study	Operating cost	Passenger satisfaction, cost of passenger rides and passenger waiting time cost	Government subsidies	Modified nondominated sorting genetic algorithm-II (NSGA-II); Pareto

In Table 1, a common method is to combine multiple objectives in a weighted manner to form a single objective and then solve the problem via a single-objective optimization method. Furthermore, the problem of assigning appropriate weights to each objective is not easy to solve [36]. Therefore, some researchers have started to use heuristic algorithms to solve multiobjective problems, commonly known as genetic algorithms (GAs), tabu search algorithms, simulated annealing algorithms and particle swarm algorithms. Heuristic algorithms with global search capability can successfully balance the quality of solutions and computation time in complex optimization situations.

Most existing multiobjective bus scheduling problems consider the benefits on the passenger side and bus company side, and some studies consider optimization objectives such as energy consumption and environmental pollution. However, few studies have considered the optimization objectives with respect to the control role played by the government in the public transportation system. As indicated by the transportation development process, the government has played an important role in the development history of all modes of transportation because successful operation of the public transportation system is crucial to the economy of a country and the life of its people. When there is a lack of government control and a bus company is fully owned by individuals, the bus company is likely to provide bus services mainly on routes with a high passenger volume for its own benefit, and it often will not pay attention to bus services that are crucial in terms of social benefits but that may not generate much profit, such as bus services in low-income areas and routes with low passenger volumes. Therefore, the role of the government in public transportation operations cannot be ignored.

In studies on multivehicle bus scheduling, researchers focus on multivehicle bus scheduling with different capacities, variable routes, different power sources and practical situations. Unfortunately, less consideration has been given to the differences in passengers' individual needs for bus types. For example, some passengers may be willing to pay a higher fare to ride a bus that provides a higher level of service, while others may feel that the basic-level bus service provided by the existing situations is sufficient to satisfy their needs. Therefore, this study focuses on the differences in individual passenger needs and proposes a multivehicle bus dispatching scheme that considers differentiated services.

The vehicle scheduling problem of bus lines is complex and often involves multiple objectives. To generate Pareto solution sets and avoid an unreasonable assignment of objective weights, this study uses the nondominated sorting genetic algorithm-II (NSGA-II) to solve the established multiobjective optimization model and yield the corresponding Pareto front. Few studies on multivehicle bus scheduling consider the control role played by the government in the public transportation system when considering the optimization objectives. This study considers the role of the government in the public transportation system to ensure social welfare to make the study more comprehensive and practical.

1.2. Contributions

This paper aims to develop a timetable optimization model that offers two levels of service based on different vehicle types to increase the appeal of public bus services while considering a tripartite game involving bus companies, passengers, and the government. In contrast to previous studies, this paper contributes in the following ways.

First, this paper considers vehicle types offering two service levels: high-level and basic-level service buses. A high-level service bus provides high comfort and a good sanitary environment, ensuring that every passenger has a seat. A basic-level service bus provides only basic-level services and meets only basic-level transportation needs; additionally, passenger comfort is not considered. Providing buses with two service levels can better meet the diversified needs of passengers, thus increasing the attractiveness of public transport services.

Second, the goals in this paper, including maximizing the total revenue of bus companies, minimizing the total expenditure of passengers, and maximizing the total social welfare of the government, consider the three conflicting parties. This approach can comprehensively maximize the social benefits of the whole transportation system. The bus timetable optimization model is more comprehensive than the models of previous studies, as all three stakeholders are considered.

Third, a bilevel optimization model is proposed to optimize the departure time and the service level provided by buses based on the passenger flow in a circular bus line with a single depot, representing a new idea for bus-related research.

This paper provides insight into the optimization of bus schedules, analyzes the service level in the construction of bus schedules, and proposes a numerical method for designing the ideal operating plan for public transportation systems. An overview of the rest of the article is provided in this section. A brief overview of some fundamental notions and presumptions is provided in Section 2. The model and method for multiobjective optimization of timetables are presented in Section 3. Section 4 illustrates this issue through a case study. Finally, Section 5 concludes with the findings of this study and suggestions for future research.

2. System description

In this section, the operation process of a bus system with two service levels and the basic assumptions are introduced. For the convenience of readers, we present the vital parameters and variables in Table 2.

(a) (b)

(c) (d)

Figure 1. The interiors of two bus types. (a) High-level service buses with wide seats; (b) high-level service buses with a comfortable environment; (c) high-level service buses with entertainment; (d) basic-level service buses.

Table 2. Notation.

Sets	
$\mathcal{I}$	Set of bus stations, $\mathcal{I} \coloneqq \{1,2,\dots,I\}$
$\mathcal{N}$	Set of buses, $\mathcal{N} \coloneqq \{1,2,\dots,N\}$
$\mathcal{U}$	Set of passengers, $\mathcal{U} \coloneqq \{1,2,\dots,U\}$
$\mathcal{S}$	Set of bus types, $\mathcal{S} \coloneqq \{0,1\}$
$\mathcal{S}_u$	Set of bus types that passenger u can accept, $\mathcal{S}_u \subseteq \mathcal{S}, \mathcal{S}_u \neq \emptyset$
$\mathcal{T}$	Set of time periods $\mathcal{T} = [0, \mathrm{T}]$ discretely expressed as $\{1,2,\dots,T\}$
$\mathcal{M}$	Set of circles served, $\mathcal{M} = \{1,2,\dots,M\}$
Parameters	
i	Bus station index, $i \in \mathcal{I}$
n	Bus index, $n \in \mathcal{N}$
u	Passenger index, $u \in \mathcal{U}$, with travel demand $\{i_u^+, i_u^-, t_u, s_u\}$
s	Bus type index, $s \in \mathcal{S}$
i_u^+	Boarding station of passenger u, $i_u^+ \in \mathcal{I}$
i_u^-	Alighting station of passenger u, $i_u^- \in \mathcal{I}$
t_u	Departure time of passenger u
m	Number of circles served
C_1	Unit operating cost of a basic-level service bus
C_2	Unit operating cost of a high-level service bus
C_s	Subsidy for a basic-level service bus
C_3	Ticket fee for a passenger on a basic-level service bus
C_4	Ticket fee for a passenger on a high-level service bus
C_μ	Satisfaction score income
C_γ	Passenger time cost
$d_{i,j}$	Distance from station i to station j
D	Distance for the whole route circle
v	Average speed
β	Boarding and alighting time
P	Vehicle capacity for a high-level service bus
p_{nim}	Passenger load for bus n departing from station i in circle m
Decision variables	
x_{nt}	Binary variable $\{0,1\}$, $x_{nt} = 1$ when bus n departs at the end of time t; otherwise, $x_{nt} = 0$, where $n \in \mathcal{N}$ and $t \in \mathcal{T}$
s_n	Binary variable $\{0,1\}$, $s_n = 1$ when bus n is a high-level bus; otherwise, $s_n = 0$, where $n \in \mathcal{N}$
t_{nim}	Departure time of bus n at station i in service circle m, where $n \in \mathcal{N}, i \in \mathcal{I},$ and $m \in \mathcal{M}$
y_{num}	Binary variable $\{0,1\}$; $y_{num} = 1$ when passenger u boards bus n in service circle m, where $n \in \mathcal{N}, u \in \mathcal{U},$ and $m \in \mathcal{M}$

Consider a bus system with two bus types, i.e., high-level service buses and basic-level service buses. Let s_n denote the type of bus service, $s_n = 1$ when bus n is a high-level bus; otherwise, $s_n = 0$. A high-level service bus can provide a comfortable environment with in-car entertainment, as shown in Figure 1(a)–(c), in which a soft sofa, sufficient personal space, and proper temperature are carefully furnished or emphasized. A basic-level service bus, as shown in Figure 1(d), provides only basic-level services that satisfy only basic-level transportation travel demands.

Consider a closed bus line circle with bus stations $i \in \mathcal{I}$, as shown in Figure 2. All bus stations are numbered in counterclockwise order, as indicated in Figure 2. Let binary variable x_{nt} denote whether bus station $i \in \mathcal{I}$ is served at time interval $t \in \mathcal{T}$. Both high-level and basic-level service buses start from departure bus station ($i = 1$) and head to destination bus station ($i = I$) under the service scheduling instruction from the centralized control center. Only when a bus starts to provide its service are all the service details, e.g., departure time x_{nt}, service type s_n, and scheduled arrival time t_{nim}, confirmed.

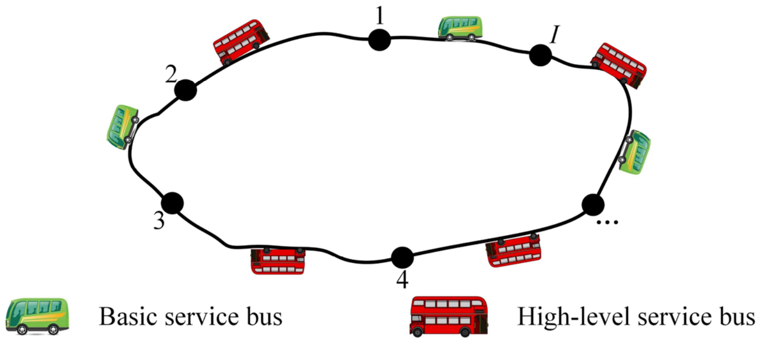

Figure 2. Circle route.

Consider a set of passengers $u \in \mathcal{U}$ and a set of bus types $s \in \mathcal{S}$. Let $\mathcal{S}_u$ denote the bus types that passenger u can accept, $\mathcal{S}_u \subseteq \mathcal{S}$, $\mathcal{S}_u \neq \emptyset$. Each passenger has his/her heterogeneous travel demand origin and destination, departure time, and bus type that he/she is willing to board, denoted as $\{i_u^+, i_u^-, t_u, s_u\}$. To start travel, a passenger is supposed to provide his/her travel demand, including the origin and destination, departure time, and bus type that he/she is willing to board (acceptable bus service levels) in advance. Meanwhile, a sufficient fee is deposited into the passenger's personal account with an agreement on fee deduction. Then, all these heterogeneous demands are integrated into the user app via a mobile phone or platform in advance. Once all these travel demands are input into the centralized control center, an optimal bus operating scheduling is generated based on the proposed method. Here, let binary variable y_{num} denote whether passenger u boards bus n in service circle m.

In this paper, the interests of three participants, i.e., a bus service provider, passengers, and the government, are weighted. The transit system is represented by a triangle relationship and contains three games. The government is the macrocontroller of these relationships. It guarantees basic-level transportation services to passengers and subcontracts bus operations to bus companies, which provide transportation services to passengers. The government hopes to reach optimal social welfare, paying fewer subsidies to bus companies and achieving greater passenger satisfaction. The role of the bus company is vital. It obtains money from both the government and passengers. The more normal buses that the bus company operates, the more subsidies that the company receives from the government, but the less bus fare income that it receives from passengers. Bus departure times and the order of departure of the two types of buses significantly affect the total income of the bus company. The role of passengers is straightforward. They hope to pay less money and receive better service. Passenger satisfaction feedback is closely related to personal preferences and money spent relative to the service level experienced. These participants are interrelated and influence each other, as shown in Figure 3.

- The game between the government and the bus company can be described as a principal-agent game. In this game, the government acts as a principal, and the bus company acts as an agent. The government entrusts and subsidizes the bus company to provide basic-level public transportation services. These two game participants have conflicting and incompatible goals. The government tries to maximize social benefits by providing moderate subsidies, while the bus company focuses on its total revenue.
- The game between the bus company and passengers can be described as a leader-follower game. In this game, passengers provide their willingness to pay in advance and reach a postgame agreement after a leader-follower game. In this game process, the bus company wants to gain more fare revenue, while passengers want to pay less for a basic-level service or pay slightly more for a higher-level service.
- Similar to the relationship between the bus company and the passengers, the relationship between the government and passengers is a leader-follower game. In this game, the government tries to reach maximum passenger satisfaction with the transit system, which highly depends on the service over the fee that all passengers obtain from the transit service provider.

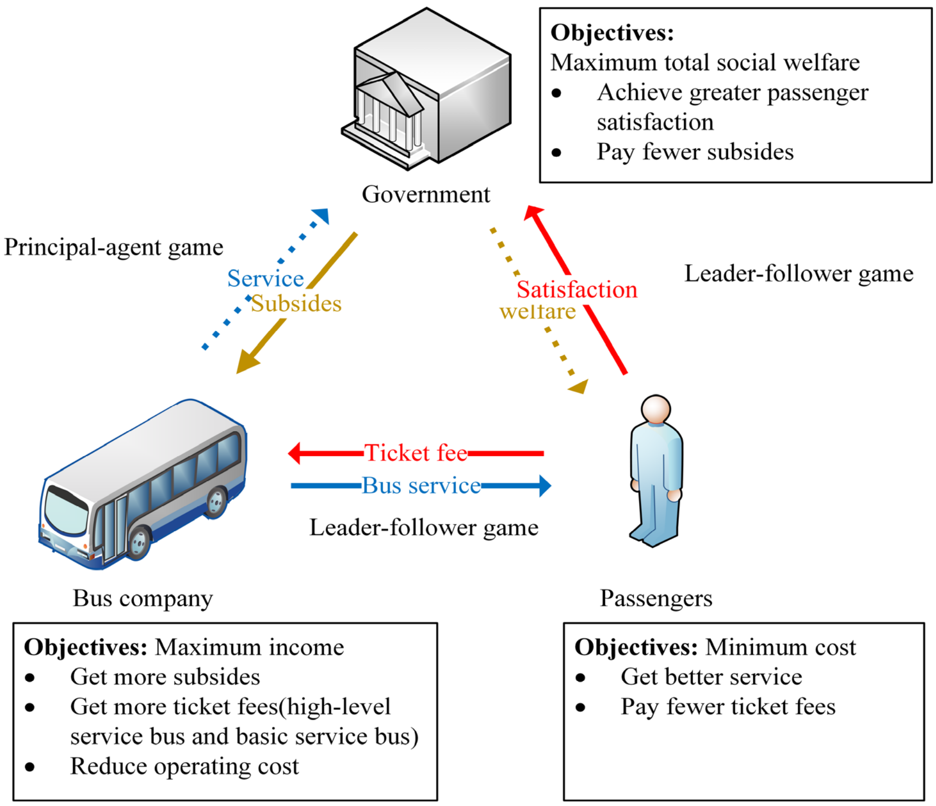

Figure 3. Game process.

We formulated an optimization model to simulate this game process. This paper considers two decision variables: the time at which each bus departs from the first station and the level of service it provides. The problem is formulated as a bilevel optimization problem. The lower-level problem occurs when, given the departure conditions determined based upon the above objectives, passengers select the bus nearest to their planned arrival time that meets their riding requirements and does not exceed the vehicle's upper capacity limit. In the upper-level problem, the goal is to realize the multiobjective collaborative optimization of the interests of the bus company, passengers, and the government. By solving this three-level model, we can achieve a balance of interests and obtain the operation strategies of bus companies.

For convenience of formulation, the following assumptions are proposed to simplify the modeling process.

Assumption 1. All passengers are rational travelers and will board the first qualified bus based on their demand requests. Moreover, they do not take more than one circle.

Assumption 2. The bus line is a continuously serviced circular or round trip, and all buses share the same average speed.

3. Methodology

3.1. Step 1: The lower-level problem

The lower-level problem can be described as a problem where a passenger chooses to take the most qualified bus that meets his/her departure time needs after the bus company places buses with a certain service level into operation at defined times based on the departure strategy provided by the system.

3.1.1. Objective function

Equation (1) can be used to calculate the departure time after a bus arrives at the platform and waits for passengers to board and alight. The departure time is equal to the prior departure time plus the driving time on the road segment and the platform waiting time. We assume that the time necessary for each bus to wait at each station for people to board and alight is a constant.

$$t_{nim} = \sum_{t\in\mathcal{T}} x_{nt} t + \frac{d_{1i}+D(m-1)}{v} + \beta(i-1+Im-I) \quad i \in \mathcal{I}, n \in \mathcal{N}, m \in \mathcal{M} \tag{1}$$

The objective function formulated in (2) aims to minimize the waiting time for each passenger. Before taking the bus, each passenger provides their travel information, including the boarding station, alighting station, departure time and acceptable bus service level; that is, each passenger has a request label $\{i_u^+, i_u^-, t_u, s_u\}$. If and only if bus n and the corresponding number of running circles m are the optimal solutions to (2) will passengers choose to take the bus, i.e., $y_{num} = 1$.

Constraint (3) indicates that the departure time of the bus should be later than that of passengers, which means that passengers cannot board the bus earlier than their departure time. In constraint (4), S_u denotes the acceptable bus service level of passenger u. For each passenger, three possible option sets exist, i.e., only basic-level service buses $S_u = \{0\}$, both basic-level service buses and high-level service buses $S_u = \{0,1\}$, and only high-level service buses $S_u = \{1\}$. Passengers board a bus based on their acceptable set.

$$\min_{r\in\mathcal{N},k\in\mathcal{M}} \{t_{ri_u^+k} - t_u\} \tag{2}$$

s.t.

$$t_{ri_u^+k} \geq t_u \tag{3}$$

$$s_r \in \mathcal{S}_u \tag{4}$$

3.1.2. The lower-level constraints

A high-level service bus ensures that each passenger has a seat, but the space in the bus is limited. Hence, to ensure a comfortable distance between passengers, this bus type has a capacity limit, i.e., the passenger load p_{nim} of bus n departing from station i in circle m is less than or equal to the bus capacity limit P, as shown in Eq (5).

$$p_{nim}s_n \leq P \qquad i \in \mathcal{I}, n \in \mathcal{N}, m \in \mathcal{M} \tag{5}$$

$$p_{n0m} = p_{nI(m-1)} \qquad i \in \mathcal{I}, n \in \mathcal{N}, m \in \mathcal{M} \tag{6}$$

$$p_{nim} = p_{n(i-1)m} + \sum_{u\in\{u|i_u^+=i\}} y_{num} - \sum_{u\in\{u|i_u^-=i\&i_u^->i_u^+\}} y_{num} - \sum_{u\in\{u|i_u^-=i\&i_u^-<i_u^+\}} y_{nu(m-1)} \qquad i \in \mathcal{I}, n \in \mathcal{N}, m \in \mathcal{M} \tag{7}$$

Since buses continuously drive in one direction on a closed circle line, Eq (6) indicates that the passenger load p_{n0m} of bus n departing from "station 0" in circle m is equal to the passenger load $p_{nI(m-1)}$ of bus n departing from station I in circle $(m-1)$. Equation (7) calculates the number of passengers on bus n running in circle m after leaving platform i. The first term in Eq (7) represents the number of passengers on bus n running in circle m after leaving platform $(i-1)$. The second term in Eq (7) represents the number of passengers who board at platform i and choose to take bus n running in circle m. The third term indicates the number of passengers who board at a platform before i, decide to take bus n running in circle m and alight at platform i. The fourth term represents the number of passengers who board at a platform after i, choose to take bus n running in circle $(m-1)$, and alight at platform i when bus n runs in circle m.

3.2. Step 2: The upper-level problem

The upper-level problem can be described as calculating the incomes of the bus company, passengers and the government based on the results of the lower-level problem, as shown in Eqs (9)–(11), and exploring the synergistic maximization of the interests of the three participants, as shown in Eq (8).

$$\max_{x_{nt}, s_n} F \coloneqq [F_1, F_2, F_3] \tag{8}$$

$$F_1 = \sum_{u\in\mathcal{U},n\in\mathcal{N},m\in\mathcal{M}} C_3 y_{num}(1-s_n) + \sum_{n\in\mathcal{N}}(C_s - C_1)(1-s_n) + \sum_{u\in\mathcal{U},n\in\mathcal{N},m\in\mathcal{M}} C_4 y_{num}\, s_n - \sum_{n\in\mathcal{N}} C_2 s_n \tag{9}$$

$$F_2 = \sum_{i\in\mathcal{I},n\in\mathcal{N},m\in\mathcal{M}} C_\mu \min\{P, p_{nim}\}(1-s_n) - \sum_{u\in\mathcal{U},n\in\mathcal{N},m\in\mathcal{M}} C_3 y_{num}(1-s_n) - \sum_{u\in\mathcal{U},n\in\mathcal{N},m\in\mathcal{M}} C_4 y_{num}\, s_n - \sum_{u\in\mathcal{U}} C_r\left(\sum_{u\in\mathcal{U},n\in\mathcal{N},m\in\mathcal{M}} y_{num}t_{nim} - t_u\right) \tag{10}$$

$$F_3 = \sum_{i\in\mathcal{I},n\in\mathcal{N},m\in\mathcal{M}} \min\{P, p_{nim}\}(1-s_n)C_{\mu} - \sum_{n\in\mathcal{N}} C_s(1-s_n) \tag{11}$$

Equation (9) calculates the income of the bus company and consists of four terms: the total fare paid by passengers who take a basic-level service bus, the difference between the government's subsidy for the operation of basic-level service buses and the operating cost of basic-level service buses, the total fare paid by passengers who take a high-level service bus, and the cost of operating high-level service buses.

Equation (10) calculates the passenger income value and consists of four terms: the satisfaction score of passengers who choose to take a basic-level service bus, the fare paid by passengers taking basic-level service buses, the fare paid by passengers taking high-level service buses, and the waiting time cost of passengers.

Equation (11) is the formula for calculating the income value of the government. The government income value is equal to the satisfaction score income of passengers who choose to take a basic-level service bus minus the subsidy paid by the government to the bus company to operate basic-level service buses.

The lower optimization model has an analytic solution that can be transformed into a single-layer optimization model by substituting the analytic solution into the upper optimization model. The two-layer architecture is used to maintain the scalability of the model and can be applied to more situations by transforming the lower-layer model into other user behavior models.

3.3. Solving techniques

3.3.1. Problem complexity

The bilevel programming problem is a nonconvex optimization problem. Even the simplest linear bilevel programming problem has been proven to be NP-hard [37,38]. The difficulty in bilevel optimization lies in the nested problem structure; that is, the upper and lower problems are affected by each other's decision variables. Clearly, double-layer optimization problems are much more difficult than ordinary single-layer mathematical optimization problems.

The study of the theoretical number of alternatives provides an idea of the overall size of the problem. It is assumed that the total time associated with the departure strategy set by the system is T minutes, the total number of buses is N, and the service levels provided by buses are S. The total number of departure strategies generated will be $(T \times S)^N$. Ignoring the characteristics of the buses, the total number of departure strategies generated is $(\sum_{i=0}^{N} C_N^i \sum_{i=0}^{N-1} C_T^{N-i} C_{N-1}^i)$. For example, given that the total time of the departure strategy is 60 minutes, with 9 buses and 2 service levels, in this case, 120^9 (hundreds of millions) departure strategies will be created. For an understanding of the significance of this problem scale, assume that every possible departure strategy is executed within 0.001 seconds. Based on these assumptions, it would take approximately 164 million years to execute them all.

In theory, this bilevel optimization model cannot obtain the optimal global solution. Therefore, it is solved with a heuristic algorithm.

3.3.2. Algorithm

To solve the model, an NSGA-II algorithm is selected as an elite strategy. The algorithm is implemented with gamultiobj, a programming package in MATLAB. The specific steps are as follows.

- **Step 1**: Randomly assign the passenger request labels $\{i_u^+, i_u^-, t_u, s_u\}$.
- **Step 2**: Initialization. Randomly assign the initial solutions for departure strategy x_{nt} and s_n.
- **Step 3**: Solve the lower-level problem. For a given x_{nt} and s_n, solve the lower model to obtain bus departure times t_{nim}, passenger boarding information y_{num} and bus passenger loads p_{nim}.
- **Step 4**: Solve the upper-level problem. Substitute t_{nim}, y_{num} and p_{nim} into the upper model to obtain the income values of the bus company, passengers and the government: F_1, F_2 and F_3.
- **Step 5**: Form the Pareto front. Use the results of steps 0-3 to establish the fitness function of the NSGA-II algorithm. Then, use the programming package gamultiobj in MATLAB to form the Pareto front.

The gamultiobj algorithm creates a Pareto front using a controlled elite GA (a variation of NSGA-II [39]). The algorithmic steps are as follows: 1) In the same manner as the GA algorithm, the first step of the gamultiobj algorithm is to create the initial population for the algorithm. 2) Iteration: a. Select the parents using the binary tournament selection method (binary tournament); b. Create children by crossing, mutating and crossing the selected parents; c. Calculate the objective function value of the children and their feasibility; d. Create an extended population matrix by combining the current population and children; e. Calculate the rank (rank) and crowding distance (distance between individuals) for each individual in the extended population; f. Retain the appropriate number of individuals from each rank (rank) when pruning the extended population. 3) Stopping: stop the iteration when the stopping condition, such as the maximum number of generations or a time limit, is reached.

As part of the GA's binary tournament selection technique, two individuals are removed from the population simultaneously (put-back sampling) and compared for merit. The superior individual is then selected to join the offspring population. This operation is repeated until the size of the new population reaches the size of the original population.

The vital difference between the gamultiobj algorithm and the traditional NSGA-II algorithm is that in step f of the iterative process, the NSGA-II algorithm retains only the appropriate number of individuals of better rank to form the offspring. In contrast, the gamultiobj algorithm retains the appropriate number of individuals in each rank, including a better or worse rank of the target value, to form the offspring to increase the diversity of the population.

In the MATLAB gamultiobj solver, ParetoFraction is the proportion of individuals set to remain on the first Pareto front when the solver selects individuals from a higher front. This option is a scalar between 0 and 1. When the number of individuals in the other ranks in step f of the iteration is insufficient, the proportion of individuals in the first rank will exceed that proportion appropriately.

4. Numerical example

4.1. Test case description

The algorithm described in the previous section is used in this test case. Assume that a closed circle route is approximately 14 km long and has ten bus stations. The distance between stations is shown in Table 3. The departure station is station 1, and the other stations are numbered in a counterclockwise circle.

Table 3. Distance between stations (unit: m).

1→2	2→3	3→4	4→5	5→6
1379	708	1408	1614	1256
6→7	7→8	8→9	9→10	10→1
1593	506	1582	2090	1866

The bus company is equipped with nine buses. There are two types of buses: high-level service buses and basic-level service buses. The driving speeds of the two types of buses are the same. After receiving a departure order, a bus starts from the departure station and continues to drive one way around the closed-circle route.

The run time of the case simulation is 60 minutes, and all travel information for 750 passengers, including the boarding station, alighting station, departure time, and acceptable bus service level information, is randomly generated.

The parameter values and sources used in the case are shown in Table 4.

Table 4. Default parameter settings.

Parameter	Value	Data source
C_1	3.69 yuan/km	People's Government of Guangdong Province (http://www.gd.gov.cn/zwgk/lsgb/content/post_153718.html)
C_2	4.52 yuan/km	People's Government of Guangdong Province (http://www.gd.gov.cn/zwgk/lsgb/content/post_153718.html)
C_s	1 yuan/km	Guangzhou Municipal Transportation Bureau (http://jtj.gz.gov.cn/gkmlpt/content/5/5494/post_5494004.html#371)
C_3	2 yuan/passenger	People's Government of Guangdong Province (http://www.gd.gov.cn/zwgk/lsgb/content/post_153718.html)
C_4	5 yuan/passenger	People's Government of Guangdong Province (http://www.gd.gov.cn/zwgk/lsgb/content/post_153718.html)
C_μ	0.51 yuan/passenger·station	Guangzhou Statistics Bureau (http://112.94.72.17/portal/queryInfo/statisticsYearbook/index)
C_γ	0.14 yuan/min	Guangzhou Statistics Bureau (http://112.94.72.17/portal/queryInfo/statisticsYearbook/index)
v	31.85 km/h	Department of Transport of Guangdong Province (http://www.gzjt.gov.cn/gzjt/jtzt_sjkf_jtysyb/201903/)
β	1.5 min	Department of Transport of Guangdong Province (http://www.gzjt.gov.cn/gzjt/jtzt_sjkf_jtysyb/201903/)
P	38 passengers	Yutong Bus (https://www.yutong.com/thtml/product7281007000.thtml?order=hits)

4.2. Implementation process

The NSGA-II algorithm is a multiobjective optimization algorithm that can be effectively applied in public transportation [23–26]. We use the MATLAB toolkit gamultiobj to solve this problem. The algorithm was executed on a personal computer equipped with an ADM7 4800H processor and 16 GB DDR4 RAM.

4.3. Parameter adjustment

We try to obtain the exact Pareto front surface by solving a small case using the enumeration method. Then, we select the appropriate gamultiobj algorithm parameters based on the exact solution.

In the small case, the system runs for 15 minutes with three vehicles and two service levels: high-level and basic-level service buses. All other parameters were taken to be identical to those of the large case. In total, 5440 departure options were involved in the small case. Using the enumeration method, 164 Pareto optimal solutions can be identified.

We performed sensitivity analyses of the population size parameters, optimal individual coefficient, crossover ratio, and maximum evolutionary generation. Three replicate code runs were performed for the value of each parameter, and the mean values are presented in Table 5 and Tables 7–9.

In Table 5, we consider six different population sizes. The initial settings of the other parameters of the solver are shown in Table 6. When the population size approaches 500, the proportion of real Pareto optimum solutions increases with the number of outcomes. The ratios corresponding to population sizes of 500 and 600 are similar, and we chose 500 as the population size parameter considering the influence of the program running time.

Table 5. Average boundary values of the Pareto front regarding population size.

Population size	Used times(s)	Total number of results	Number of results on the Pareto front (deduplication)	Proportions
100	111.74	35	15	0.4190
200	219.87	70	37	0.5238
300	324.42	105	59	0.5651
400	432.75	140	88	0.6286
500	529.56	175	116	0.6629
600	634.71	210	140	0.6667

Table 6. Gamultiobj solver initial settings.

Parameter	Value
Populationsize	-
ParetoFraction	0.35
CrossoverFraction	0.80
Generations	200
stallGenLimit	100
TolFun	1e-100
gaplotPareto	@gaplotPareto

In Table 7, we set four different optimal individual coefficients. The table indicates that when the individual factor is 0.35, the number of trustworthy Pareto optimal solutions obtained is the largest proportion of the total number of results. Thus, 0.35 is chosen as the optimal individual factor.

Table 7. Average boundary values of the Pareto front regarding the optimal individual factor.

Pareto Fraction	Used times(s)	Total number of results	Number of results on the Pareto front (deduplication)	Proportions
0.25	544.31	77	77	0.6187
0.35	529.56	116	116	0.6629
0.45	532.04	143	143	0.6341
0.55	544.77	157	157	0.5697

In Table 8, we consider seven different crossover ratios. Two peaks appear at approximately the ratio of the number of proper Pareto optimal solutions to the total results obtained when the crossover ratio is 0.75 and 0.90. Since the latter is larger than the former and has a shorter running time, the crossover ratio is chosen to be 0.90.

Table 8. Average boundary values of the Pareto front regarding the crossover ratio.

Crossover Fraction	Used times(s)	Total number of results	Number of results on the Pareto front (deduplication)	Proportions
0.65	550.48	175	108	0.6190
0.7	547.82	175	113	0.6438
0.75	552.68	175	119	0.6800
0.8	557.31	175	114	0.6514
0.85	550.15	175	115	0.6571
0.9	545.83	175	122	0.6952
0.95	535.90	175	120	0.6876

In Table 9, we set four different maximum evolutionary generations. The table shows two peak points at approximately the ratio of the proper Pareto optimal solutions to the total results obtained when the maximum evolutionary generation is 200 and 400. Although the ratio at 200 is slightly smaller than that at 400, the system running time at 200 is almost half that at 400. Therefore, 200 is chosen as the maximum evolutionary generation.

Table 9. Average boundary values of the Pareto front regarding the maximum number of evolutionary generations.

generations	Used times(s)	Total number of results	Number of results on the Pareto front (deduplication)	Proportions
100	278.45	175	112	0.6419
200	553.43	175	118	0.6724
300	826.29	175	115	0.6590
400	1088.43	175	120	0.6857

The final parameters from the parametric sensitivity analysis are shown in Table 10.

Table 10. Gamultiobj solver settings

Parameter	Value
Populationsize	500
ParetoFraction	0.35
CrossoverFraction	0.90
Generations	200
stallGenLimit	100
TolFun	1e-100
gaplotPareto	@gaplotPareto

4.4. Results

Based on the given optimal parameter settings, we ran the gamultiobj function to solve the large case. Because the algorithm involves a random component, it performed five independent runs to reduce the effect of randomness on the results. Figure 4 shows the five different Pareto fronts obtained from the five independent runs. The different Pareto fronts contain 823 nondominant situations.

The results of the five different runs are integrated into a new Pareto front, which contains only the nondominated solutions with respect to the combined results, as shown in Figure 5(a). The integrated Pareto front contains, in total, 399 situations. The Pareto front is not a plane but a curved surface. Moreover, three different projections of nondominated solutions are plotted in Figure 5(b)–(d). The projection in Figure 5(b) is classified into two curves. Among them, the solutions located on the outer projection curves are better for passengers and bus company than the inner ones. However, from the government's perspective, the solutions on the inner projection curve are slightly better than those on the outer projection curve. Table 11 demonstrates that the boundary values are similar for each run, which indicates that the performance of the multiobjective optimization algorithm is stable.

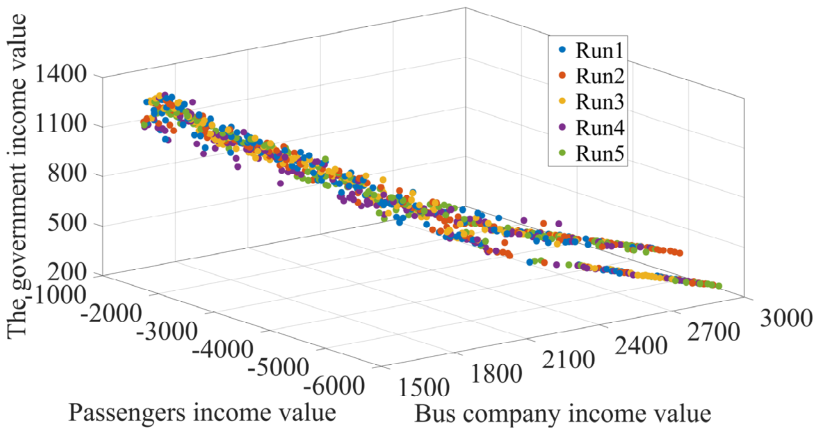

Figure 4. Pareto fronts for different runs.

Figure 5 shows that an increase in bus company revenue is negatively correlated with growth in the value of passenger revenue. The income of the bus company is also negatively correlated with the income of the government. The income of passengers is positively correlated with the income of the government, which coincides with the game relationship between them and verifies the validity of the model.

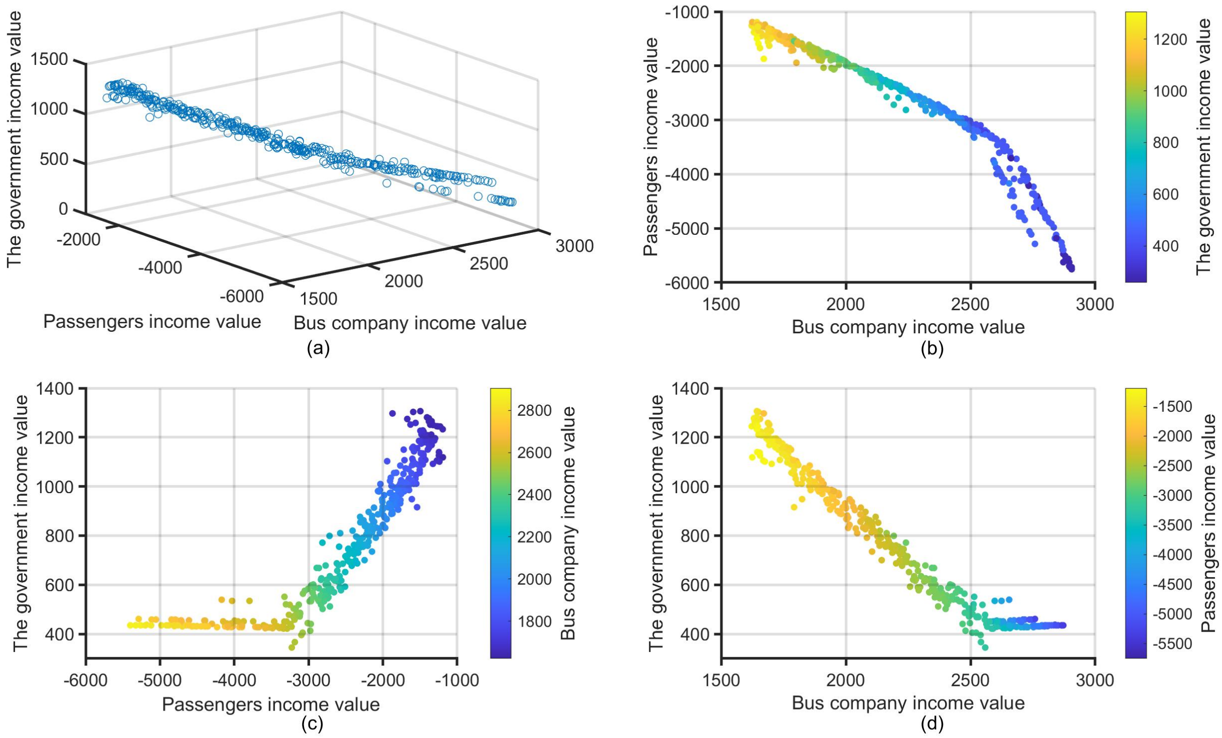

Figure 5. Integrated Pareto front: (a) in three-dimensional view; (b)–(d) in projection views.

Table 11. Pareto optimal solutions.

Runs		1	2	3	4	5
	F_1	2867.49	2914.83	2858.85	2888.27	2917.27
When F_1 is optimal	F_2	-5458.95	-5808.67	-5390.49	-5666.43	-5902.33
	F_3	274.14	274.14	275.39	275.39	277.01
	F_1	1634.97	1624.72	1643.46	1633.11	1619.22
When F_2 is optimal	F_2	-1207.06	-1193.30	-1195.52	-1176.19	-1230.19
	F_3	1110.11	1118.46	1232.69	1084.51	1106.07
	F_1	1649.48	1643.32	1637.84	1643.32	1625.66
When F_3 is optimal	F_2	-1592.16	-1516.60	-1425.07	-1496.69	-1387.78
	F_3	1303.02	1275.02	1296.39	1306.13	1259.21
Used times(s)		2425.68	2414.08	2444.78	2474.86	2361.29

According to the analysis, the bus company and the government are profitable, and the passengers are in a negative state. Thus, when the bus system simultaneously provides two bus service levels, the bus company can achieve gains, and the passengers are in a state of spending money to purchase the service, as demonstrated by the positive government revenue. Furthermore, the passengers are more satisfied with the bus system and willing to spend money to purchase the service.

5. Conclusions

The motivation of this paper is to provide possible solutions to the problems of low comfort, lack of seats, and overcrowding in existing public transportation systems, which could lead to a decline in the public transport volume. This paper provides insight into the optimization of bus schedules, analyzes the service levels in schedules, and proposes a mathematical model for designing operating plans for public transportation systems. This model comprehensively considers three stakeholders in the public transport system, i.e., passengers, bus companies, and the government. The model is a bilevel optimization model in which the lower-level problem is based on passenger flow analysis and considers the passenger travel situation. The upper-level problem involves three objective functions that maximize the income of bus companies, passengers, and the government. A case study illustrates the effectiveness of our model.

To verify the effectiveness of the proposed method, a real-world case from Guangzhou is considered and analyzed using the NSGA-II algorithm to obtain the related Pareto front. The results show that adopting the two-track bus operation mode can increase the income of bus companies and meet the diversified needs of passengers while ensuring positive income for the government.

Due to the limitations of the input data, the case study in this paper is a numerical example. In future work, we will examine the model's sensitivity to changes in input data or algorithm parameters. Furthermore, a network with multiple bus lines will be considered, and various bus service levels and defined service level indicators will be combined with actual scenarios to improve the universality of the model.

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Conflicts of interest

The authors declare that there are no conflicts of interest.

References

1. V. Guihaire, J. K. Hao, Transit network design and scheduling: A global review, *Transp. Res. Part A Policy Pract.*, **42** (2008), 1251–1273. https://doi.org/10.1016/j.tra.2008.03.011
2. J. J. Tang, Y. F. Yang, W. Hao, F. Liu, Y. H. Wang, A data-driven timetable optimization of urban bus line based on multi-objective genetic algorithm, *IEEE Trans. Intell. Transp. Syst.*, **22** (2021), 2417–2429. https://doi.org/10.1109/tits.2020.3025031
3. J. Zhang, L. Tu, F. Zhang, X. J. Yin, J. Sun, H. S. T. Chen, Flexible express bus line planning and operating based on passenger flow analysis, in *IEEE International Conference on Intelligent Transportation Systems-ITSC*, (2018), 2511–2518.
4. M. Y. Pei, P. Q. Lin, R. H. Liu, Y. Y. Ma, Flexible transit routing model considering passengers' willingness to pay, *IET Intel. Transport Syst.*, **13** (2019), 841–850. https://doi.org/10.1049/iet-its.2018.5220

5. G. Cheng, C. L. He, Analysis of bus travel characteristics and predictions of elderly passenger flow based on smart card data, *Electron. Res. Arch.*, **30** (2022), 4256–4276. https://doi.org/10.3934/era.2022217
6. S. Hassold, A. Ceder, Public transport vehicle scheduling featuring multiple vehicle types, *Transp. Res. Part B Methodol.*, **67** (2014), 129–143. https://doi.org/10.1016/j.trb.2014.04.009
7. S. Bunte, N. J. P. T. Kliewer, An overview on vehicle scheduling models, *Public Transp.*, **1** (2009), 299–317. https://doi.org/10.1007/s12469-010-0018-5
8. D. Pena, A. Tchernykh, S. Nesmachnow, R. Massobrio, A. Y. Drozdov, S. N. Garichev, Multiobjective vehicle type and size scheduling problem in urban public transport using MOCell, in *2016 International Conference on Engineering and Telecommunication (EnT)*, (2016), 110–113. https://doi.org/10.1109/EnT.2016.30
9. R. Tang, Y. Bie, Q. Song, X. Xiong, Timetable optimization method for single bus line based on multiple vehicle size model, in *CICTP 2019*, (2019), 1882–1894.
10. Y. Bie, R. Tang, L. Wang, Bus scheduling of overlapping routes with multi-vehicle types based on passenger OD data, *IEEE Access*, **8** (2020), 1406–1415. https://doi.org/10.1109/access.2019.2961930
11. X. Dou, Q. Meng, Feeder bus timetable design and vehicle size setting in peak hour demand conditions, *Transp. Res. Rec.*, **2673** (2019), 321–332. https://doi.org/10.1177/0361198119846462
12. Z. Dai, X. C. Liu, X. Chen, X. Ma, Joint optimization of scheduling and capacity for mixed traffic with autonomous and human-driven buses: A dynamic programming approach, *Transp. Res. Part C Emerging Technol.*, **114** (2020), 598–619. https://doi.org/10.1016/j.trc.2020.03.001
13. J. Zhou, Initial boundary value problem for a inhomogeneous pseudo-parabolic equation, *Electron. Res. Arch.*, **28** (2020), 67–90. https://doi.org/10.3934/era.2020005
14. Y. Liu, F. Y. Wu, C. Lyu, S. Li, J. P. Ye, X. B. Qu, Deep dispatching: A deep reinforcement learning approach for vehicle dispatching on online ride-hailing platform, *Transp. Res. Part E Logist. Transp. Rev.*, **161** (2022), 102694. https://doi.org/10.1016/j.tre.2022.102694
15. Y. Bie, R. Tang, Z. Liu, D. Ma, Mixed scheduling strategy for high frequency bus routes with common stops, *IEEE Access*, **8** (2020), 34442–34454. https://doi.org/10.1109/access.2020.2974740
16. Y. Bie, M. Hao, M. Guo, Optimal electric bus scheduling based on the combination of all-stop and short-turning strategies, *Sustainability-Basel*, **13** (2021). https://doi.org/10.3390/su13041827
17. B. Hu, X. Wang, W. Chen, Study on combinational scheduling between inter-zone vehicle and regular vehicle for urban public transit, *J. Wuhan Univ. Technol.*, **36** (2012), 1192–1195.
18. M. Kim, P. Schonfeld, Conventional, flexible, and variable-type bus services, *J. Transp. Eng.*, **138** (2012), 263–273. https://doi.org/10.1061/(asce)te.1943-5436.0000326
19. Z. Zhou, Z. Ye, Y. Xu, Optimizing combined bus service pattern and frequencies with genetic algorithm, in *CICTP 2018: Intelligence, Connectivity, and Mobility*, Reston, VA: American Society of Civil Engineers, (2018), 881–892.
20. M. Y. Jin, S. X. He, S. C. Zhang, Bus combinational scheduling based on improved genetic algorithm and considering exhaust emission, *Transp. Res.*, **7** (2021), 55–65. https://doi.org/10.16503/j.cnki.2095-9931.2021.02.007
21. Y. Liu, L. Wang, Z. Zeng, Y. Bie, Optimal charging plan for electric bus considering time-of-day electricity tariff, *J. Intell. Connected Veh.*, **5** (2022), 123–137. https://doi.org/10.1108/JICV-04-2022-0008

22. J. He, N. Yan, J. Zhang, Y. Yu, T. Wang, Battery electric buses charging schedule optimization considering time-of-use electricity price, *J. Intell. Connected Veh.*, **5** (2022), 138–145.
23. W. Zhang, H. Zhao, M. Xu, Optimal operating strategy of short turning lines for the battery electric bus system, *Commun. Transp. Res.*, **1** (2021), 100023. https://doi.org/10.1016/j.commtr.2021.100023
24. J. Ji, Y. Bie, Z. Zeng, L. Wang, Trip energy consumption estimation for electric buses, *Commun. Transp. Res.*, **2** (2022), 100069. https://doi.org/10.1016/j.commtr.2022.100069
25. Y. Liu, E. Yao, S. Liu, Energy consumption optimization model of multi-type bus operating organization based on time-space network, *Appl. Sci.-Basel*, **9** (2019). https://doi.org/10.3390/app9163352
26. G. J. Zhou, D. F. Xie, X. M. Zhao, C. Lu, Collaborative optimization of vehicle and charging scheduling for a bus fleet mixed with electric and traditional buses, *IEEE Access*, **8** (2020), 8056–8072. https://doi.org/10.1109/access.2020.2964391
27. A. Jahic, M. Plenz, M. Eskander, D. Schulz, Route scheduling for centralized electric bus depots, *IEEE Open J. Intell. Transp. Syst.*, **2** (2021), 149–159. https://doi.org/10.1109/ojits.2021.3096115
28. A. Ceder, Public-transport vehicle scheduling with multi vehicle type, *Transp. Res. Part C Emerging Technol.*, **19** (2011), 485–497. https://doi.org/10.1016/j.trc.2010.07.007
29. M. G. Baldoquin, A. J. Rengifo-Campo, A model for solving vehicle scheduling problems: a case study, *Rev. Fac. Ing. Univ. Antioquia*, **88** (2018), 16–25. https://doi.org/10.17533/udea.redin.n88a03
30. E. Yao, T. Liu, T. Lu, Y. Yang, Optimization of electric vehicle scheduling with multiple vehicle types in public transport, *Sustainable Cities Soc.*, **52** (2020), 101862. https://doi.org/10.1016/j.scs.2019.101862
31. O. J. Ibarra-Rojas, R. Giesen, Y. A. Rios-Solis, An integrated approach for timetabling and vehicle scheduling problems to analyze the trade-off between level of service and operating costs of transit networks, *Transp. Res. Part B Methodol.*, **70** (2014), 35–46. https://doi.org/10.1016/j.trb.2014.08.010
32. L. Ye, L. Danni, Bus scheduling model based on peak hour volume clustering, in *CICTP2015*, (2015), 1065–1080.
33. J. Li, J. Hu, Y. Zhang, Optimal combinations and variable departure intervals for micro bus system, *Tsinghua Sci. Technol.*, **22** (2017), 282–292. https://doi.org/10.23919/TST.2017.7914200
34. X. Yang, L. Liu, A multi-objective bus rapid transit energy saving dispatching optimization considering multiple types of vehicles, *IEEE Access*, **8** (2020), 79459–79471. https://doi.org/10.1109/access.2020.2989334
35. R. Liu, N. Wang, Data-driven bus route optimization algorithm under sudden interruption of public transport, *IEEE Access*, **10** (2022), 5250–5263. https://doi.org/10.1109/access.2022.3140947
36. X. Zuo, C. Chen, W. Tan, M. Zhou, Vehicle scheduling of an urban bus line via an improved multiobjective genetic algorithm, *IEEE Trans. Intell. Transp. Syst.*, **16** (2015), 1030–1041. https://doi.org/10.1109/tits.2014.2352599
37. O. Ben-ayed, C. E. Blair, Computational difficulties of bilevel linear programming, *Oper. Res.*, **38** (1990), 556–560. https://doi.org/10.1287/opre.38.3.556

38. X. Zhao, S. Li, A linear time approximation scheme for scheduling unbounded batch machines with delivery times and inclusive processing set restrictions, *Electron. Res. Arch.*, **30** (2022), 4209–4219. https://doi.org/10.3934/era.2022213
39. K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*, Springer London, 2001.