Thesis for the degree of Doctor of Philosophy in Applied Acoustics

Sound Field Design for Transducer Array-Based Acoustic Levitation

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Cover:
Visualization of a soundfield with a levitation trap to the left and a quiet zone to the right, superposed ontop of a photograph of two polystyrene balls levitating above a transducer array.

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Abstract

Acoustic levitation is a technique where sound waves are used to hold an object at a designated position, suspending it against other external forces and keeping it stable in the desired position. Transducer arrays are an arrangement of many loudspeaker elements, tuned such that the array creates a sound field that is too complicated for the individual elements to produce on their own. Sound field design refers to the process of prescribing some target criteria for a sound field, describing these criteria mathematically, and apply some method to produce a sound field with these criteria fulfilled.

This thesis is about combining these three concepts. The necessary criteria for levitation to take place are described, using radiation pressure from sound waves in air as the physical mechanism by which the levitation forces are produced. Ultrasonic transducer arrays are modeled using analytical descriptions for the wave propagation, as well as for the predictions of the radiation forces on spherical objects of various sizes. The sound fields required to successfully levitate objects are obtained by numerically optimizing the magnitudes and phases of the elements in the mono-frequency transducer arrays. This is achieved by deriving design criteria from intuitive considerations of the conditions needed for levitation, quantifying these criteria as a single valued cost function which is minimized with a Quasi-Newton method.

The thesis is focused on two main aspects: how to define a suitable cost function for a single levitation trap, and how to levitate multiple objects via mutual quiet zones. The design criteria for a trap are described using a vector field approach, representing properties of the force field with invariant quantities evaluated at the desired levitation position. These quantifiers are scaled by the characteristic quantities of the system and transformed to a satisficing cost function, which avoids over-optimization by reducing the prioritization of a particular criterion when closer to fulfilled. Multiple objects are levitated by superposing sound fields with mutual quiet zones, i.e. each sound field has a trap for one object and quiet zones where all the other objects will be.

Keywords: Acoustic Levitation, Transducer Arrays, Sound Field Design, Numerical Optimization, Nonlinear Acoustics

Thesis

This thesis consists of an extended summary and the following appended papers:

Paper A Carl Andersson and Jens Ahrens. "A Method for Simultaneous Creation of an Acoustic Trap and a Quiet Zone". In: *2018 IEEE 10th Sensor Array and Multichannel Signal Processing Workshop (SAM)*. IEEE, July 8–11, 2018, pp. 622–626. ISBN: 978-1-5386-4752-3. DOI: 10.1109/SAM.2018.8448949

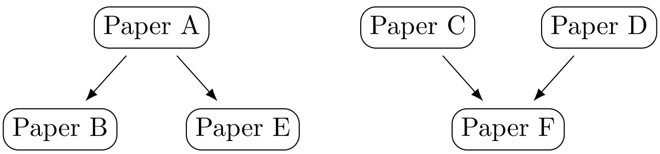
Paper B Carl Andersson and Jens Ahrens. "Minimum Trap Separation for Acoustical Levitation Using Phased Ultrasonic Transducer Arrays". In: *Proceedings of the 23rd International Congress on Acoustics*. Aachen, Germany, Sept. 9, 2019, pp. 1117–1123. ISBN: 978-3-939296-15-7. DOI: 10.18154/RWTH-CONV-239242

Paper C Carl Andersson and Jens Ahrens. "Acoustic Levitation from Superposition of Spherical Harmonics Expansions of Elementary Sources: Analysis of Dependency on Wavenumber and Order". In: *2019 IEEE International Ultrasonics Symposium (IUS)*. Glasgow, United Kingdom: IEEE, Oct. 6–9, 2019, pp. 920–923. DOI: 10.1109/ULTSYM.2019.8926167

Paper D Carl Andersson and Jens Ahrens. "Reducing Spiraling in Transducer Array Based Acoustic Levitation". In: *2020 IEEE International Ultrasonics Symposium (IUS)*. Las Vegas, NV, USA: IEEE, Sept. 7, 2020, pp. 1–4. ISBN: 978-1-72815-448-0. DOI: 10.1109/IUS46767.2020.9251489

Paper E Carl Andersson and Jens Ahrens. "Creation of Large Quiet Zones in the Presence of Acoustical Levitation Traps". In: *2021 IEEE International Ultrasonics Symposium (IUS)*. Sept. 2021, pp. 1–4. DOI: 10.1109/IUS52206.2021.9593601

Paper F Carl Andersson. "Acoustic Levitation of Multi-Wavelength Spherical Bodies Using Transducer Arrays of Non-Specialized Geometries". In: *Journal of the Acoustical Society of America* (Submitted for publication)



The papers are listed in chronological order. The diagram above shows how the papers are related to each other, and is meant to assist the reader in choosing a suitable reading order. Papers A, B and E cover aspects of the quiet zone method, while Papers C, D and F cover aspects of levitation trap creation. A reader not experienced in acoustic levitation should consider starting with Paper D followed by Papers A and B, and the remainder in any order. An expert reader with a background in acoustic radiation force modeling and/or spherical harmonic expansions of sound fields will likely grasp the essence of the methods from Papers E and F without reading the other papers first.

Contents

1. Introduction

One of the earliest works relating to acoustic levitation was done by Rayleigh in 1902, treating the radiation pressure that waves can create over longer periods of time [1]. The earliest demonstration of acoustic levitation in air is likely Bücks and Müller in 1933, where small liquid droplets were levitated in a basic standing wave resonator consisting of a single transducer and a reflector [2]. While the theoretical treatment of radiation force developed during the 20th century, with important contributions by King in 1934 [3], Westervelt in 1951 [4], and Gor'kov in 1962 [5], the experimental apparatus remained essentially unchanged. The first use of ultrasonic transducer arrays for acoustic levitation is in 2014, when Ochiai, Hoshi, and Rekimoto used opposing phased arrays to create movable levitation traps using standing waves [6]. In 2015, Marzo et al. presented a method where the phases of the elements in a transducer array of arbitrary geometry are numerically optimized to create a levitation trap at a designated position in space [7]. In this thesis, the idea of numerically optimizing the input signals to a transducer array of fixed but arbitrary geometry for the purposes of acoustic levitation is explored further.

1.1 Goal and Scope

The core goal in the work presented here was to explore, understand, and improve methods to design sound fields suitable for acoustic levitation. The main focus was to develop a coherent design framework, which allows for expressing the requirements of levitation in various contexts within the scope of acoustic levitation from transducer arrays in air. The choice to work with numerical optimization of the driving signals to the transducer arrays by minimizing a cost function was primarily due to the relative ease of which different contexts can be explored by modifying the cost function. Cost functions also have the advantage that it is usually possible to formulate the cost based on intuitive considerations of some physical properties, which naturally creates a framework where each component is interpretable, facilitating the understanding of the design methods.

A secondary goal was to model the transducer arrays without undertaking a characterization of the radiation properties of each individual element in the array. This was implemented throughout the work by modeling the sound field as a superposition of fields created by a "typical transducer". The radiation model for the typical transducer is obtained from sound field measurements of just a few individual transducers, using the average radiation properties as the radiation model for all the transducers. One advantage with this approach is that the same radiation model can be used repeatedly in different contexts, as long as the same or sufficiently similar hardware is used. This approach has been used successfully by many authors, with a variety of array geometries and trap creation methods [7–14]. The main alternative is to measure the sound field generated by each of the transducers in the array around the target levitation position, and use this information to predict the total sound field from the array [15]. The rationale behind restricting to the "typical transducer" approach is that if the methods developed are so sensitive that they require absolute knowledge of the physical apparatus, they are quite difficult to use in practice.

A further restriction was to only work with direct optimization of the input signals to the transducer elements, keeping the array geometry fixed during the optimization. This choice promotes the use of "generalist arrays" which are not specifically designed for a certain task. They have the advantage that they can rapidly be reconfigured for different tasks by changing the driving signals. The alternative is to optimize the positioning of the transducer elements, possibly in tandem with the driving signals, to create "specialist arrays" for a given levitation task. While an interesting possibility to create purpose-built systems, this will have more limited use in practice since a new physical array has to be constructed for every targeted scenario.

Finally, it was decided to primarily investigate the type of hardware that is readily available today. For airborne ultrasonic transducer arrays, the most commonly used transducers operate at 40 kHz and have a physical housing that measures 9.8 mm in diameter. The arrays typically consist of between a few dozen transducers in the smallest arrays and a few thousand transducers in the largest arrays, and usually have simple regular placement of the transducers [9, 16]. Of note is that this hardware is inherently spatially undersampled, since the transducer housing is about 1.14 wavelengths across; alias-free spatial sampling would require an element spacing below approximately 4.3 mm. Similar to how a low temporal sample rate cannot reproduce high frequency signals, spatial undersampling reduces the highest spatial frequency in certain directions for a designed sound field [17, Chapter 4]. This essentially restricts how fast the sound field can change over space, but there is no simple relation between the degree of undersampling and the performance in the design.

1.2 Related Works and Applications

This section presents some other works that are related to this thesis, compare different approaches or uses of acoustic levitation and ultrasonic transducer arrays.

A field of research which is closely related to acoustic levitation is ultrasonic mid-air haptic feedback [18]. The type of hardware used to produce the sound fields required for said haptic feedback can in many cases also be used for acoustic levitation. This means that some of the modeling approaches are very similar, and in certain cases there is overlap in the sound field design methods as well. A consequence of using similar hardware is that it is possible to create mid-air haptics and acoustic levitation at the same time, by designing the sound field to include both effects together.

If the interest is shifted from generalist systems to specialized systems, the array geometry can be tuned instead of tuning the driving signals. This was for example used by Marzo in 2016 to create handheld levitation systems [19], and by Marzo, Caleap, and Drinkwater in 2018 for levitation of larger objects using a bowl-shaped array [20]. Another development is to use multiple frequencies simultaneously, which enlarges the design space by creating individual fields. This was used by Puranen et al. in 2019 to orient an ellipsoidal object by creating two traps at 39.6 kHz and 40.4 kHz respectively [15]. It can also be used to create a stiffer trap while maintaining the trap size by the use of harmonics to the base trap frequency, as show by Courtney in 2021 [21].

One noteworthy alternative to designing the sound field by tuning the driving signals of the transducers is to use a physical phase lens, which delays the output from the transducers by a tunable amount. This was used by Melde et al. in 2016 to create patterns of small particles in water [22]. A similar technique was used for airborne applications by Norasikin et al. in 2018 to create a self-bending sound beam, capable of levitating a small object behind an occluding object on the array [23]. The latter also combined the static lens with a dynamic phased array, which enabled some limited steering of the pattern created by the lens. Similar combinations of a static physical lens and a dynamic phased array were also explored for applications in water by Cox et al. in 2019 [24]. One advantage with the physical lens is that it can have higher spatial resolution than the transducers that are available today. The disadvantage is that the lens is fixed and cannot be changed during the operation of the system, which prevents major modifications to the sound field without interruption. It should however be noted that the spatial resolution advantage of the physical lens could potentially be removed in the future, if smaller transducers are developed such that the spacing between them is lower than one half wavelength. Another advantage with the physical lens is that it does not require complicated driving electronics for each element, which will likely offer lower manufacturing costs compared to an equivalent transducer-based system.

The effects of spatial undersampling in the array was investigated for acoustic levitation by Marzo and Drinkwater in 2019 [25]. Their results show that increasing the spatial sampling to at least two transducers per wavelength increases e.g. the trap stiffness, but even denser sampling did not increase the stiffness further. Morales et al. also investigated the spatial undersampling for similar hardware in 2021, but for general amplitude pattern generation [26]. In their experiment, the quality of the amplitude pattern was improved by denser sampling up to four transducers per wavelength. Both these experiments were done with constant physical extent of the array and introducing more transducer elements, with the side effect of increasing the degrees of freedom in the design methods used. The alternative is to keep the number of elements constant, which would decrease the extent of the array and change the spatial truncation effects. None of these two options for investigating the spatial aliasing can do so without also modifying other parameters of the system.

A very promising application area for acoustic levitation is within human computer interaction. Fushimi et al. created a system in 2019, capable of moving a small particle at high speeds along specified paths [11]. This was used as a volumetric display by illuminating the particle to render small colored shapes mid-air, visible to the human eye due to persistence of vision. A larger system was created by Plasencia et al. in 2020, moving and illuminating multiple particles individually which enabled rendering of larger shapes [27]. A different application presented by Freeman et al. in 2019 is to use the levitating object together with stationary physical elements to combine the resolution of the stationary elements and the dynamic capabilities of the levitating object [28].

Other applications include manipulation of liquids for chemical processing [29], biomedical manipulation of particles inside the human body [30], sample containment in e.g. protein crystallography [31], or additive manufacturing [32].

2. Summary of Papers

This part of the thesis gives an overview of the appended papers, explaining the high level goals and the relations between the proposed methods and results from the papers.

The core idea used throughout the work presented in this thesis is that of using numerical optimization of a cost function that represents the design criteria for the sound field. While this idea was first proposed by Marzo et al. [7] in order to create a levitation trap from a single sided array, the cost functions used in this work are modified from the original method. One of the main modifications is to formulate the cost functions based on design criteria that intuitively describe the intended tasks for the overall system. Deriving the design criteria from properties of desired tasks, e.g. forces on a levitating object, instead of the means of performing the tasks, e.g. pressure points in the sound field, creates a framework that is easy to understand and in turn easier to extend. The main benefit of this approach throughout the work has been that ideas first developed for small objects could be reused for larger objects, e.g. by using an appropriate force model for the large object.

As a fortunate by-product, this approach naturally creates design criteria and cost functions that are agnostic to the means of generating the forces, as well as the type of optimization that is used to obtain a good solution. In principle this formulation allows for a mix of forces used to levitate the object, e.g. combining magnetic and acoustic forces in some clever way. While no work was done to develop new optimization algorithms, some effort was spent on understanding how to balance a cost function in the context of acoustic levitation. This is needed in order for the optimization to have an even balance of the various criteria that the cost function is composed of.

The sections in the remainder of this part of the thesis discuss how the design criteria were chosen in various ways to perform different tasks, and some of the analysis that was done on the performance and limitations of the system.

2.1 Stable Levitation Traps

In order for an object to levitate at a designated point in space, two conditions must apply. Firstly, the object should not move away from the designated position if already situated there. Secondly, if the object is not at the designated position, the object should be moved towards the target position. Both these conditions are intuitive and simple to state without requiring any further explanation. Using a very basic understanding of the physics of forces and motion, this naturally translates to the following two design criteria:

1. There should be **no net force** acting on the object when situated stationary at the designated position.

2. If the object is not at the designated position, we desire **converging forces** that push the object toward the designated position.

These are the fundamental design criteria for a levitation trap, which were partly used in e.g. Paper B as the design criteria for the traps.

If the levitation is done in air on Earth, the first criterion means that the acoustic radiation force needs to balance the gravitational force. If done in an environment where the object is naturally buoyant, e.g. in space or with a floating particle in water, this criterion means that the acoustic radiation force needs to be different. It could also happen that the acoustic radiation force should balance other external forces that act on the object e.g. electrostatic forces or wind drag.

A converging force field is closely related to a negative divergence of the force, which can be seen as the sum of axial stiffnesses in the trap. This means that the second design criterion is very similar to maximizing the stiffness in the levitation trap, which was part of the original approach by Marzo et al. [7]. If the trap stiffnesses are summed with equal weights the two formulations are identical, while a weighted sum of the axial trap stiffnesses will depend on the chosen coordinate system. In some cases this is useful to prioritize the stiffness in a particular direction, while in other cases it can lock the trap orientation to the chosen coordinate system thus complicating the design process.

The criteria can be further extended by considering the structure of the force field in the nearby region of the trap, and how this force field interacts with the levitated object over time. In particular it is important that an object situated in a trap does not continuously receive kinetic energy, since this increasing energy will eventually exceed the potential well that the trap constitutes. This can be prevented by a third criterion, preventing the build up of kinetic energy for the object in the trap:

3. In the vicinity of the trap, we need a **conservative force field.**

Since any conservative vector field is curl-free, this criterion is included in the optimization by minimizing the magnitude of the curl of the force field at the center of the trap. This formulation is explored in Paper D, where the curl reduction is shown to increase the stability of designed levitation traps. An example of a force field with an obvious curl is shown in Figure 1, together with the curl-reduced version obtained with the proposed method. The effect that force curl has on the stability of levitation traps can also explain why the so called "vortex trap" is not stable for as large objects as the so called "twin trap", as first noted by Marzo et al. [7].

2.2 Large Object Levitation

The design criteria for acoustic levitation are typically expressed with quantities derived from the interaction between the sound field and the levitated object. This interaction

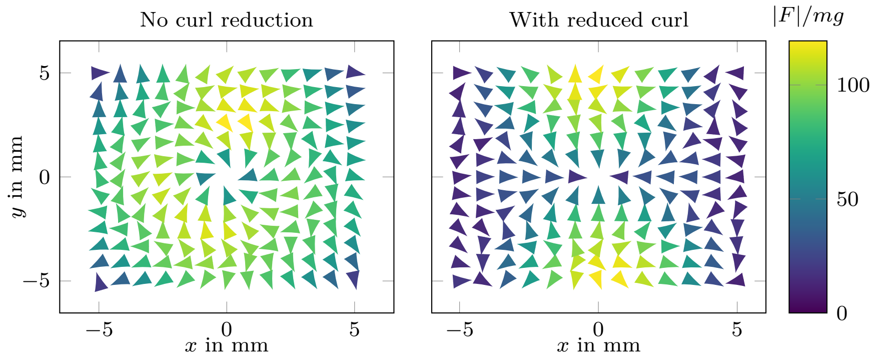


Figure 1: The structure of force fields obtained from optimization without regards to force curl (left) and including curl reduction (right). For additional examples, see Paper D where the method was proposed or Paper F where it was applied for larger objects.

takes place at the surface of the object, and for small objects the sound field cannot vary much over this surface. As a consequence, the information required to express the design criteria is contained in the sound field at the desired levitation position. However, for larger objects the interaction surface is sufficiently large that the sound field can vary significantly over the surface. Therefore, the design criteria require information about the sound field over a larger region of space. This comes with two complications, one abstract and one concrete. The abstract complication is that the sound field is implicitly designed in the entire volume occupied by the levitated object, instead of locally at a single point. Together with the fact that lifting a larger object typically require more force, the task of levitating a large object is inherently more difficult. The concrete complication is that the interaction between the sound field and the object is mathematically more difficult to handle. In the context of airborne acoustic levitation, this was first solved by Inoue et al. using the boundary element method to numerically find the sound field scattered from the target object, and to calculate the radiation force using a discretized integration over the interaction surface [16]. To avoid computationally intensive numerical methods, we restrict the levitated objects to be spherical, which enable the use of standard analytical solutions for the scattering problem and the calculation of the radiation force [33–40].

In theory this mathematical formulation requires an infinite series, which naturally is not feasible in practice. The influence of the truncation of this series was investigated in Paper C, explicitly for transducer array-based computations of the acoustic radiation force on spherical objects of various sizes. The main conclusion is that the series converges nicely with a finite number of terms, and an indicative relation between the size of the object and the number of terms needed for convergence is presented. The design criteria of a converging conservative force field was expressed for large objects using this analytical formulation in Paper F, and a sound field was designed where a sphere of 20 mm diameter successfully levitated 60 mm above a planar 16x16 element array, see Figure 2.

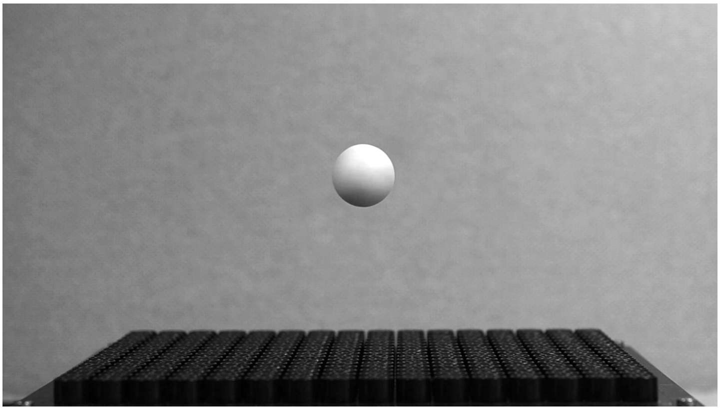


Figure 2: Photograph of a spherical object with a diameter of 20 mm levitating 60 mm above a transducer array, originally from Paper F.

2.3 Modelling Error Analysis

When using a modeling tool in practice, it is good to know to what extent the predictions are accurate and how different factors influence the model. In Paper F the sensitivity of predictions of the radiation force on objects of various sizes was investigated for a few representative array geometries.

The transducer arrays used for acoustic levitation are often built with relatively inexpensive transducers due to the quantity required. Such transducers have some manufacturing tolerance, which give rise to "mismatch" between the transducer elements, e.g. variations of the magnitude or phase of the emitted sound wave [9, Supplementary materials, section 3]. The influence of element mismatch in magnitude and phase was investigated by varying the complex driving amplitudes that are the input to the model, and analyzing how the magnitude and direction of the radiation force on the target object changes in the region around a levitation trap. This analysis shows that the influence of mismatch decreases slightly with increased size of the levitated object, and that generally the phase-mismatch has a stronger influence than the magnitude-mismatch. The random variations used for the elements in the simulated arrays were chosen to be similar to those that can be found in some transducers available on the market, ranging from high quality name brand transducers to low cost options. The general trend is that the mismatch in high quality transducers is small enough to barely influence the predicted force ($< 1\,\%$ error in magnitude, $< 1^\circ$ error in direction), but the mismatch found in low cost options cause typical errors up to $25\,\%$ in force magnitude and 5° in force direction.

The directivity of the transducers in the array influence the radiation force on the target object. In the formulation of the radiation force on large objects, the theoretical framework requires that the sound radiation from the transducers is described using a multipole model. This has consequences both for the practical modeling of the sound radiation and for the computational requirements of the system. The modeling is problematic mainly since the process to obtain a multipole decomposition of a source radiation is difficult to perform accurately in practice, either using analytical considerations or fitting to measured data. The computational load is increased due to the multipole translation theorems used to calculate the sound field decomposition required to compute the radiation force. An alternative approach is a far-field approximation, in which the sources are modeled as linearly decreasing spherical waves with some directivity. The parameters for such models are much easier to obtain from measurements or from standard analytical source models, compared to a full multipole decomposition.

However, this approach will only approximate the required sound field decomposition when calculating the radiation force. The influence of this approximation, as well as the influence of the transducer directivity in general, was investigated in Paper F for planar arrays of sizes up to 32x32 elements, levitation positions between 3 cm and 8 cm away from the array, and objects of diameters up to 5 cm. The results indicate that the use of a proper multipole expansion is more relevant when the target object is larger than one or two wavelengths. If the more convenient far-field approximation is used for large objects the typical errors in force magnitude and direction can be up towards 20 % and 10°. While producing notable errors, the far-field approximation still predicts forces that are much more accurate than if the directivity is not accounted for at all. Replacing the actual transducer directivity with an omnidirectional characteristic, forces are predicted with typical errors in magnitude of up to a factor 20 and typical errors in direction of up to 40°, for objects of most sizes. This large error is likely due to the high angle between some the of the transducer normals and the object position relative to the transducer.

2.4 Levitation of Multiple Objects

In certain applications it is desired to levitate multiple objects at once using the same transducer array. If the objects are far enough apart, this can be done by dividing the array in local sub-arrays and create a single trap from each part. It can also be done by superposing the complex driving amplitudes obtained by designing a sound field for each levitation trap individually, thus superposing the sound fields and generating multiple traps at once. However, if the target objects are close enough in space, the levitation traps will interfere with each other due to the superposition of sound fields of the same frequency, regardless of whether the individual fields are created by sub-arrays or by driving signal superposition. One solution to this comes in the form of mutual quiet zones. Multiple fields are superposed, each responsible for levitation of a single object and with quiet zones where all the other traps will be created by the other fields. This prevents the interference of the sound fields locally where the traps are, such that the trap created by one field is not destroyed by the simultaneous creation of the other traps.

One way to achieve such mutual quiet zones is described in Paper A, where the sound field design criteria was extended to also include the suppression of the sound field at secondary positions. This is then applied twice to design one sound field with a trap for the first target object and a quiet zone for the second target object, then again to design a sound field with a trap for the second target object and a quiet zone for the first target object. The complex driving amplitudes obtained by those two optimizations are then superposed to find the required driving amplitudes to create both traps simultaneously using the entire transducer array. In the same paper, this method was successfully used to demonstrate the levitation of multiple small objects in relatively close proximity.

Since a levitation trap is created by strong gradients in the sound pressure, it requires high pressure amplitudes close to the desired trap. This is in contrast to the condition of a quiet zone nearby to the trap, raising the question of how close the trap and the quiet zone can be. Paper B describes an investigation on the required spacing of the two levitating objects when using the method of mutual quiet zones, and in what manner the method breaks down when the objects are positioned too close. The general conclusion is that the target positions should be at least one wavelength apart for the method to have optimal performance. When the target positions are too close for two individual traps to form where intended, the final result depends on the relative phase between the two superposed fields. It is possible to find a relative phase such that two traps form, but at a larger separation than intended. Otherwise, a single trap forms between the two target positions, effectively merging the two traps to one larger but weaker trap. This effect is likely due to the fact that a mono-frequency sound field cannot create a strong pressure amplitude too close to a pressure null, and is thus a physical limitation that cannot be circumvented with other mono-frequency approaches. Using a higher frequency of course decreases the absolute separation between the objects but it also means that the objects are larger in comparison to the wavelength, introducing the same difficulties as levitating a larger object with the same frequency.

When used for simultaneous levitation of multiple objects, each quiet zone needs to be large enough to cover the region of space where the structure of the sound field is important for the levitation trap. The original approach in Paper A was formulated for small levitating objects and small quiet zones. However, if the goal is to levitate larger objects, the size of the quiet zone needs to extend to cover at least the size of the levitated object. This can be achieved by modifying the design criterion to consider a larger region of space, which is described in Paper E. The sound field in the target quiet zone was decomposed in spherical harmonics at the center of the region. The suppression of those decomposition coefficients is used as the quiet zone criterion, at the same time as criteria for a levitation trap outside the quiet zone. See Figure 3 for a visualization of a field with a quiet zone and a trap. By including more coefficients in the cost function, a larger region of space is considered for the suppression. Simulations show that if the suppression of the coefficients is properly balanced, the size of the quiet zone increases with increasing number of coefficients. One such sound field was measured to show the existence of a region where the sound field is suppressed, with the same qualitative performance as the simulations. A small spherical object was shown to levitate in a trap generated in the same sound field as the quiet zone, verifying experimentally the simultaneous creation of a large quiet zone and a simple levitation trap. Of note is that this experimental result is obtained without measuring the sound field generated by each of the transducers, relying on the measured far-field directivity of a single transducer in the middle of the array to model all the transducers.

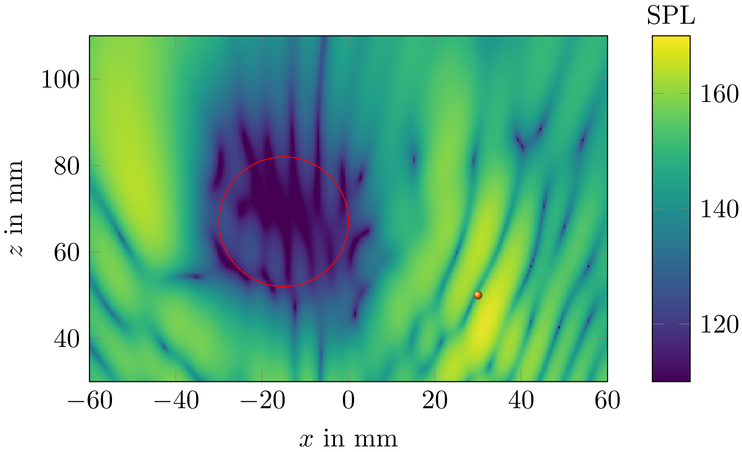


Figure 3: Simulated sound field with a large quiet zone and a levitation trap. The quiet zone is indicated with the 15 mm red circle, while the target levitation position is indicated by a small orange ball. This is an extension of the sound field visualized in Paper E.

2.5 Cost Functions

One important step in the sound field design is to assemble a suitable cost function from the prescribed levitation criteria. If the components in a cost function are not balanced to properly represent all the criteria, the minimization of the cost function might not necessarily mean that the criteria are optimally fulfilled. In most works, the cost function is formed by adding quantities that represent different criteria, scaled by some prioritization weights. These weights are typically found manually by running the optimization, analyzing the resulting sound field by some means other than the cost function, then change the prioritization weights to compensate any flaws in the sound field. This process is done iteratively and manually until some suitable weights are found for a specific case. Unfortunately the weights found suitable for one context do not necessarily transfer to other contexts, so that the design process requires extensive manual tuning and experience.

In Paper F a method for automatic balancing of the cost function was proposed, based on applying a specific transform to the various components of the cost function. In essence, the method decreases the weights for a specific component of the cost function when its value is small, i.e. the corresponding criterion is (partly) fulfilled. The approach still requires some slight manual tuning but the process is not as sensitive as the one previously used, and enabled the same weights to be used across a variety of test cases with successful results. In the paper, this is applied to the levitation of large objects, but the same idea has also been used successfully in unpublished experiments to create large quiet zones, and to create traps for small objects in scenarios other than the ones described in the paper.

3. Levitation System Considerations

This part of the thesis contains some considerations on the levitation system as a whole. Most of the content presented here is unpublished, and is presented in a less formal way.

3.1 Trap Strength and Stability

There are three main descriptions used to quantify the trap strength: the divergence of the force field, the axial stiffnesses, and the eigenvalues of the force gradient matrix. Conceptually, they all represent similar ideas. The eigenvalues sum to the same as the axial stiffnesses, which in Cartesian coordinates is the same as the negative force divergence. If the force field in the trap is described with a linear approximation

$$\vec{F}(\vec{r}) \approx \nabla\vec{F}(\vec{r}_0) \cdot (\vec{r} - \vec{r}_0)$$

the force gradient matrix $\nabla\vec{F}(\vec{r}_0)$ is the matrix of partial derivatives of the force components w.r.t. the coordinates in the position vector $\vec{r}$, evaluated at the trap center $\vec{r}_0$ which we without loss of generality take as the origin of the coordinate system. On the diagonal of this matrix we find the trap stiffness in the coordinate axial directions, and the eigenvalues of this matrix is the stiffness along the trap principal axes independent of the orientation of the coordinate axes.

The choice of using the sum of these eigenvalues as a quantifier for the trap is often motivated by the time evolution of dynamical systems. A first order linear dynamical model

$$\dot{\vec{x}} = A \cdot \vec{x}$$

is stable if the eigenvalues λ_i of the system matrix A have strictly negative real parts. This can easily be understood by writing the homogeneous solution as the time-varying superposition of the corresponding eigenvectors $\vec{\xi}_i$,

$$\vec{x}(t) = \sum_i a_i \vec{\xi}_i \mathrm{e}^{\lambda_i t}$$

which is easily confirmed to be a solution. If all the eigenvalues have negative real parts, the overall solution will decrease exponentially over time. However, this is not directly applicable to the force gradient matrix. From Newton's third law, we have a second order system with

$$m\ddot{\vec{r}} = \nabla\vec{F}\cdot\vec{r}$$

where we can again write the solution in terms of the eigenvalues and eigenvectors of the matrix $\nabla\vec{F}/m$,

$$\vec{r}(t) = \sum_i a_i\vec{\xi}_i \mathrm{e}^{\pm\sqrt{\lambda_i}t}$$

but the eigenvalues now appear with a square root in the exponential. This means that the sign of the real part of the eigenvalues of the force gradient matrix has no direct relation to the system stability, only the real part of the square roots of the eigenvalues matter.

This can be further understood by taking a simple two dimensional case and aligning the trap principal axes with the coordinate system. Quantifying the trap by some axial stiffness parameter s and some force-coupling parameter ζ, we can write

$$\nabla\vec{F} = \begin{pmatrix} -s & \zeta \\ -\zeta & -s \end{pmatrix}.$$

This parametrization is chosen since s is directly related to the force divergence while ζ is related to the force curl. If we disregard the force curl by setting $\zeta = 0$, we immediately find the eigenvalues $\lambda_{1,2} = -s/m$, producing the complex time constants $\tau = \sqrt{\lambda_{1,2}} = \pm\mathrm{i}\sqrt{s/m}$ i.e. a harmonic mass spring system. However, reintroducing a non-zero curl changes the eigenvalues to $\lambda_{1,2} = (-s\pm\mathrm{i}\zeta)/m$, producing four time constants $\tau = \pm\mathrm{i}\sqrt{(s\mp\mathrm{i}\zeta)/m}$. Considering the location of these in the complex plane, writing the eigenvalues on polar form as $\lambda_{1,2} = |\lambda|\mathrm{e}^{\mathrm{i}(\pi\pm\alpha)}$, we find the four complex time constants as $\tau = \sqrt{|\lambda|}\mathrm{e}^{\mathrm{i}(\pm\pi/2\pm\alpha/2)}$. Choosing different signs for $\pi/2$ and $\alpha/2$ in the exponent places the time constant in the right half plane, i.e. with a positive real part. This shows that a consideration of the eigenvalues of the force gradient matrix must include both the real part and the imaginary part to evaluate the stability of the trap.

The physical mechanism by which a non-zero force curl is kept in check is the damping that comes from the drag forces. To include this in the dynamical model of the system we have to transform the second order system to a coupled first order system for both the position and the velocity, written with block matrices as

$$\begin{pmatrix} \dot{\vec{r}} \\ \dot{\vec{v}} \end{pmatrix} = \begin{pmatrix} 0 & \mathbf{I} \\ \nabla\vec{F}/m & -\gamma\mathbf{I} \end{pmatrix} \begin{pmatrix} \vec{r} \\ \vec{v} \end{pmatrix}$$

where $\gamma \geq 0$ is some linear damping quantifier with appropriate physical units and $\mathbf{I}$ is the unit matrix. With some work, the eigenvalues of this first order dynamical system can be found as

$$\lambda = \frac{-\gamma \pm \sqrt{\gamma^2 - 4s/m \pm 4\mathrm{i}\zeta/m}}{2}$$

where the two sign choices are independent. In cases where the curl ζ is insignificant it is easy to see that the real parts will always be negative. When the curl is significant, the expression under the root is written as

$$\sqrt{(\gamma \pm 2\mathrm{i}\sqrt{s/m})^2 \pm 4\mathrm{i}(\zeta/m - \gamma\sqrt{s/m})}$$

where both sign choices are the same as the one originally under the root sign. If this is seen as a function $f(\delta) = \sqrt{z^2 \pm \mathrm{i}\delta}$ and expanded in δ, without going too deep into complex mathematics we find

$$\sqrt{(\gamma \pm 2\mathrm{i}\sqrt{s/m})^2 \pm 4\mathrm{i}(\zeta/m - \gamma\sqrt{s/m})} \approx \gamma \pm 2\mathrm{i}\sqrt{s/m} \pm \frac{2\mathrm{i}(\zeta/m - \gamma\sqrt{s/m})}{\gamma \pm 2\mathrm{i}\sqrt{s/m}},$$

giving the eigenvalues as

$$\begin{aligned}\lambda_{1,2} &\approx \pm\mathrm{i}\sqrt{s/m} + \frac{\zeta/m - \gamma\sqrt{s/m}}{2\sqrt{s/m} \mp \mathrm{i}\gamma}\\ \lambda_{3,4} &\approx \pm\mathrm{i}\sqrt{s/m} - \frac{\zeta/m + \gamma\sqrt{s/m} \mp \mathrm{i}\gamma^2}{2\sqrt{s/m} \mp \mathrm{i}\gamma}.\end{aligned}$$

Since a typical trap is not very damped the denominator in the fractions will be close to real and positive, which ensures $\Re\{\lambda_{3,4}\} < 0$ unconditionally. However, if the curl quantifier $\zeta/m > \gamma\sqrt{s/m}$, the system is not stable due to the real parts of $\lambda_{1,2}$. Remembering that the converging forces here are quantified with s, and $\sqrt{s/m}$ is the resonance frequency in the system, we can attempt a physical interpretation of this expression. After some initial transient, only the behavior of $\lambda_{1,2}$ remains. At that point, the overall energy in the system is increased by the curl quantifier ζ and decreased by the product of the damping γ and the resonance frequency $\sqrt{s/m}$. For a fixed damping parameter, the stability of the trap can be increased by increasing the stiffness or by decreasing the curl. However, the stability is proportional to $\zeta/\sqrt{s}$ indicating that it is more favorable to decrease the curl.

Generalizing this simple example to arbitrary force gradient matrices is not trivial, but by inserting the general quantities we can arrive at some indicative metric for trap stability. The general quantity corresponding to the stiffness parameter s is the negative of the force divergence, i.e. $-\nabla \cdot \vec{F}$. Similarly, the general quantity corresponding to the curl parameter is the magnitude of the curl vector, i.e. $|\nabla \times \vec{F}|$. In a linear approximation, almost all drag models tend to a simple Stokes drag

$$\vec{F}_{\mathrm{drag}} = -6\pi a\mu\vec{v}$$

where a is the radius of the object and μ is the dynamic viscosity of the medium, giving the damping parameter as

$$\gamma = 6\pi a\mu/m.$$

Using this we can restate an indicative metric for trap stability as

$$|\nabla \times \vec{F}|/m \lesssim \frac{6\pi a\mu}{m}\sqrt{-\nabla \cdot \vec{F}/m}$$

or, by squaring and expressing the mass of the spherical object using its density ρ_*

$$|\nabla \times \vec{F}|^2 \lesssim -\frac{27\pi\mu^2}{a\rho_*}\nabla \cdot \vec{F}.$$

One possible use for this result is as a relative importance of the curl magnitude and the force divergence when creating cost functions. It could also potentially work as an approximate check for trap stability, instead of running a full kinetic simulation.

3.2 Size Scaling

One aspect which is important to investigate for an early stage technological system is how the system could scale to perform more challenging tasks. The determining factor for the success of a levitation task is how much force can be generated. If the capabilities of the system need to grow, the main requirement is to produce more force on the levitated objects. Starting with some simple considerations of how various generalized parameters of the system are related to each other: the force F is proportional to the square of the local pressure p, which in turn depends on the number of transducers N, some transducer output measure p_0, and the overall length scale L, as

$$F \propto p^2 \qquad \text{and} \qquad p \propto N p_0 / L.$$

With the acoustic power density w of the transducers, and the spacing Δ between the elements, we can re-express the number of transducers and the transducer output as

$$N \propto L^2/\Delta^2 \qquad \text{and} \qquad p_0 \propto \sqrt{w}\Delta^2,$$

which give us that

$$F \propto p^2 \propto (N p_0/L)^2 \propto ((L^2/\Delta^2)(\sqrt{w}\Delta^2)/L)^2 = (\sqrt{w}L)^2 = W$$

where W is the total acoustic power in the array, and it is assumed that the same length scale is applied to the size of the array and the distance between the array and the target object. This shows that the forces generated are directly proportional to the total acoustic power available to create the forces, all else being equal. Scaling the system to larger sizes will thus be preferable, since there will be more power output available which can create the required forces. Important to note is that this result is obtained by moving the target object further away from the array by the same proportion as the dimensions of the array. This allows us to neglect the influence of the transducer directivity in the analysis, since all angles remain the same. If the object is kept at the same distance from the array, increasing the size of the array will eventually not bring more power to the levitation position, reaching diminishing returns for a particular position.

The other size parameter which is interesting to vary is the size of the levitated object. Unfortunately, the expressions for the radiation force acting on a large spherical object are difficult to analyze w.r.t. scaling of the radius of the object. To investigate this empirically, the force on the object was optimized numerically for different array geometries and a

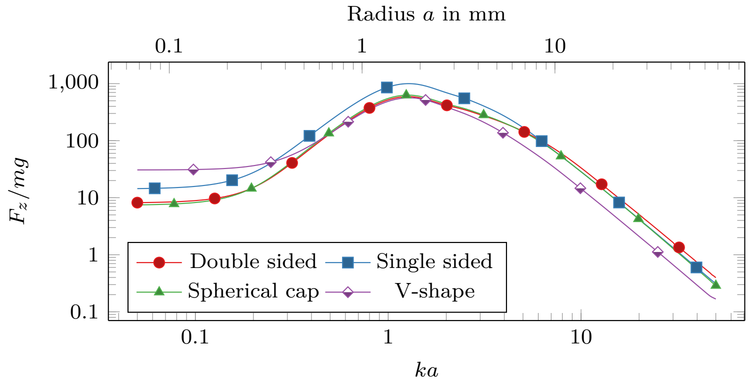


Figure 4: Maximum lift forces obtainable in relation to the gravitational pull for different sizes of the target object in four array geometries. The experiment was also repeated for other variations of these geometries, with qualitatively similar results.

large range of object sizes. The first array in the experiment was a square double sided array with 16x16 elements in each side (512 in total), and a separation of 30 cm between the two sides, targeting an object in the middle between the two sides. The second array used was a rectangular single sided array with 23x22=506 elements, targeting an object 15 cm centrally above the array. The third array is a "spherical cap" with 554 transducers placed in concentric circles, all at 25 cm from the target object. The fourth and last array is two halves with 12x12 elements each (288 in total), forming a v-shape by tilting them 30° from horizontal and targeting an object 12.5 cm above the tip in the v. The goal for the optimization was simply to maximize the lift force on the object, restricted only by the maximum input amplitude to the transducers. While this goal does not create a levitation trap, it avoids the issues of defining a trap cost function and analyzing the results.

The obtained forces are shown in relation to the gravitational pull in Figure 4, for a wide range of object sizes. For the smallest objects the obtained force is constant in relation to the gravitational pull. This is expected from the Gor'kov potential, which scales with the volume of the object. In the range from $ka \approx 0.2$ to $ka \approx 0.8$ the force goes approximately as $F_z/mg \propto (ka)^{2.5}$, while in the range of $ka \gtrsim 6$ we get $F_z/mg \propto (ka)^{-2.75}$. The maximum lift capability is around $ka = 1$ for all the investigated setups.

This decreasing slope indicate that for a given array geometry there is an upper limit on how large objects can be levitated, which is not a surprising result. If the intention is to levitate an object of twice the radius whilst keeping the same level of control over the motion, the array needs to have about seven times the power ($2^{2.75} \approx 6.7$) to keep the same strength for the forces. If the length dimensions of the array are increased together with the distance between the array and the target object (to avoid the influence of the directivity) the array has to increase in size by approximately 160 %, e.g. from a 15x15=225 element array to a 39x39=1521 element array with the same spacing between the transducers.

4. Discussion

The purpose of this work was to explore, understand, and improve methods to design sound fields used for acoustic levitation. Taking acoustic levitation as a whole, there are too many different methods and too many various contexts to cover within a single thesis. For this reason, the scope was restricted to using a cost function-based method to design the sound fields, within the context of general-purpose ultrasonic transducer arrays. While this reduction in scope seems limiting at first, it has allowed for a few key advancements in the field as well as new insight into how a cost function can be extended and interpreted.

When this work started, acoustic levitation of objects larger than the wavelength had not been done using transducer arrays. Now it has been presented at least twice independently from my research [16, 20], once within my own research (Paper F), and at least once in another work partly based on methods developed in my research [14]. Another key advancement in my research is the "quiet zone" method, where a levitation trap is produced at one position at the same time as a quiet zone at another area. Some additional aspects and uses of this method are discussed in section 4.1. Much of my work has focused on formulating cost functions based on criteria for the force structure in and around the trap. The advantages of this approach is more oriented towards increasing the understanding of the system and opportunities to improve or generalize the methods. The usefulness of this approach is discussed further in section 4.2.

4.1 Quiet Zones

The idea of suppressing the sound field in one area while at the same time performing a task in another area is not radically new, and has been an active research topic within the spatial audio community for about 20 years [41–43]. However, it had not been applied to acoustic levitation before this work. In the context of acoustic levitation, the suppression of the sound field in the secondary area is usually not a goal on its own, but as a means to also perform some other task. This second task is typically fulfilled by creating a second sound field which operates mainly within the quiet zone, and the sound field for this second task might also need a quiet zone to avoid interfering with the first task. The quiet zone method is not inherently tied to creating a levitation trap, but can also be used for simultaneous fulfillment of other tasks that the array is capable of. These tasks can be active tasks, such as mid-air ultrasonic haptic feedback or focus point-based parametric audio, or passive tasks such as avoiding unwanted scattering from some external object present in the sound field.

Applying the quiet zone method for a certain task requires that it can be formulated with a cost function, which is then extended with the quiet zone criterion. The optimization of this extended cost function is ideally started from a set of variables that solves the original task without consideration of the quiet zone. This has two consequences. Firstly, the optimization typically converges in relatively few iterations since the new solution is close to the starting point in the search space. Secondly, the new sound field will share many properties with the sound field for the original task, which is increases the reliability of the approach. Finding the starting point for the new optimization can be done with any method that can solve the original task, allowing the quiet zone method to leverage advancements in the creation of sound fields for the original task.

One typical application of this method is to levitate multiple objects. This is done by creating one sound field for each of the objects, with a trap for the corresponding object and quiet zones for all the other objects. This allows the design of each of the traps to be separated from the others to a greater extent than if they are designed as a whole. Since the design is separated for each of the individual sound fields, they are easier to design than a comparable single field with multiple traps. These individual sound fields are then superposed to form a single field with multiple traps.

In this superposition, the power available for each of the fields is decreased, weakening the individual traps compared to the single trap case. This decrease in power makes it clear how the capabilities of an array relate to the number of tasks and/or the complexity of each task. One disadvantage with the quiet zone approach is that the total sound field might not be the optimal solution, e.g. there might be another field which creates stronger traps at the same positions. This highlights the approach as a general-purpose solution, while some more specialized methods could create more powerful solutions for specific cases.

4.2 Decoupled Design Process

When the cost function is derived from the structure of the force field in the trap, the goal of the design is clear: to achieve a certain force field. On a conceptual level, the structure of a sound field does not matter as long as it delivers a suitable force field. This mindset has further implications for the overall design process, as it decouples the cost function, the force modeling, and the optimization process. Since the end goal is to achieve a certain force field acting on a target object, the cost function has to be described based on considerations of the kinetics of the forces and the object. The modeling of the forces acts as an interface layer between the abstract cost function and the actual physical system used to create the forces. Since the two are fully separated, the same cost function can be used for a variety of physical systems, generalizing within acoustic levitation as well as to optical levitation and magnetic levitation. The third component to the design process is the numerical optimization used to minimize the cost function. This has to be done in different ways depending on what the variables in the system are and what form the desired outcome takes. In this thesis the variables have been the complex valued amplitudes for the transducers in an array, but the same overall process could be used to optimize the placement of the transducers in a specialized array or to determine the locations of focus points close to the target object.

This decoupled approach greatly assisted in some specific studies. When enhancing the trap stability by reducing the force curl, how to calculate the curl or how to reduce it was never an issue. When the levitation was extended to larger objects, no effort was required to find a proper cost function. Including an additional function in the system by the means of a quiet zone mainly required the formulation of the proper criterion. Further extensions or improvements in the future can likewise be approached as targeted efforts within one of these three aspects of the design process.

One disadvantage with this mindset is that it prioritizes understanding over results. Other presented methods show impressive results, either by lowering the computational effort needed for levitation, or by performing more difficult tasks. The disadvantage in some of these methods is that it is not apparent why they work, only that they do. Both of these approaches have an important role within the community of acoustic levitation, introducing new knowledge in a different way.

4.3 Future Work

4.3.1 Trap criteria

The trap description used in most of this work centers around three parts: the net force, the force divergence, and the force curl. While these criteria can create traps that are stable and strong, the produced force fields can still be improved in some ways. It is common that the trap stiffness is not symmetrical, e.g. that the restoring forces are stronger along the y-axis than along the x-axis as in the right hand side of Figure 1. For such a case, it would often be preferable to sacrifice some of the stiffness in y-direction to increase the stiffness in x-direction, in particular if the "weak" direction is so weak that it does not properly restore the object to the intended position. This is not covered by the force divergence, where increasing an already high stiffness and decreasing a low stiffness can make the cost function decrease in value. Since we aim for a cost function independent of the chosen coordinate system, this symmetry criterion should not be based on the stiffness along the coordinate axes, but rather on the eigenvalues of the stiffness matrix. A basic metric for the symmetry of the trap is the variance of these eigenvalues. Including this variance directly in the cost function could be sufficient to create more symmetrical traps. However, increasing the stiffness in one principal direction without decreasing the stiffness in the other directions should still cause a net decrease in the cost function. This means that a balance is needed to make sure that the stiffness will be increased in the weaker directions at the expense of the strong directions, while still allowing a single direction to increase in strength if this does not weaken the other directions.

Another symmetry that is desirable in the force field is the extent of the trap, i.e. the region of space within which the forces actually act to push the object back to the center of the trap. In most cases the forces are symmetrical around the trap center, such that moving the object a specific distance one or the other way along a certain axis will produce the same restoring force. However, it sometimes happens that the produced

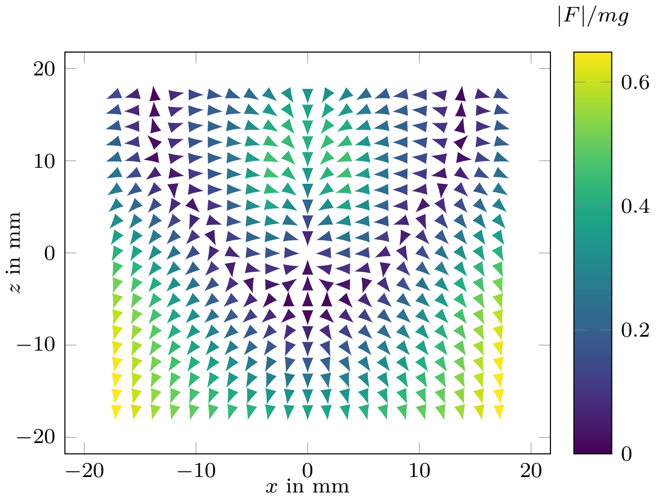


Figure 5: Example of trap where the geometrical center and the kinetic center do not coincide.

force field is has a longer region with restoring forces on one side than on the other. If we describe the extent of the trap by the region of space where the forces are actually restoring the object towards the center, we can define the geometrical center of the trap as the spatial middle point in this region. Similarly, we can define a kinetic center, which is the point where the forces are converging to, i.e. the resting position of the object and the designed target position for the levitation trap. If the restoring forces do not extend the same distance in both directions from the target position, the geometrical center and the kinetic center will not coincide. This can in unfortunate cases place the kinetic center very close to the edge of the converging region of the trap, effectively making the trap very sensitive to external disturbances. In the force field shown in Figure 5 the boundary of the trap is just a few millimeters from the kinetic center when looking towards negative z, but more than 20 mm away in positive z. This is clearly a problem that can cause traps to be very sensitive, even if they fulfill the currently used criteria. Symmetries of a function around a point can be described locally with higher order derivatives. Hence, a potential solution to this issue could lie in higher order spatial derivatives of the force. How to include these derivatives, which derivatives are important, and how to prioritize this criterion is not clear at the moment.

4.3.2 Multi-frequency approaches

As mentioned in the introduction, almost all of the work on acoustic levitation uses a mono-frequency sound field, with multiple frequencies only used recently. There are two main use-cases for multi-frequency levitation: to create independent fields or to obtain higher spatial resolution.

Under linear conditions, the sound field created by sources at one frequency f_1 will not modify the sound field created by sources at a different frequency f_2. This can be used to design two individual sound fields without considering the interactions that otherwise happen during the superposition. However, the radiation force is related to the time average of the square of the total sound pressure, which requires further consideration. If averaged over sufficiently long time, related to $1/|f_1 - f_2|$, the mixing terms from the squaring average to zero. If the two frequencies are far enough apart, such that the required averaging time is much shorter than the kinetic dynamics of the levitated object, the acoustic radiation forces from each of the fields will manifest independently. However, if the two frequencies are close it is more appropriate to interpret the superposition as a single field at an intermediate frequency, amplitude modulated at the difference frequency $|f_1 - f_2|$. This will generate an acoustic radiation force which is also modulated at this difference frequency. If the modulation is on a similar time scale as the kinetic dynamics of the levitated object, it is no longer appropriate to consider the radiation force as a static quantity, which makes the modeling and design much more complicated.

Since the spatial resolution of a sound field is limited by its wavenumber, the spatial control over the force is also limited by the same. This means that in order to notably increase the spatial resolution in the force field, sound fields of much higher frequencies has to be added to the first one. In such cases the radiation force from each of the sound fields will be essentially independent from the others, since the frequencies are well separated. As shown recently, this can be used to create traps with a flatter force curve in large parts of the trap whilst also increasing the stiffness in the central portion of the trap [21]. The same could be done for levitation traps in air with the optimization approaches used for this thesis. Since the forces from the different frequency sound fields are independent, this could be modeled as multiple independent arrays radiating at one frequency each. The predicted force is simply the sum of the forces produced by each of the arrays, and the variables to the optimizer is the ensemble of the complex driving amplitudes to the arrays. Currently the main difficulty is in the hardware aspect, in particular that the widely used array systems all operate at 40 kHz due to the powerful transducers available.

If used with the intention of creating force fields that are more independent from each other, the frequencies can be in the same approximate range if the traps should have similar qualities. This is beneficial since forces created by sound fields at different frequencies interfere less than those created at the same frequency. The force F generated by a single field $p_1 = \hat{p}\cos\omega t$ is proportional to the time average of the squared field, i.e.

$$F \propto \langle(\hat{p}\cos\omega t)^2\rangle = \hat{p}^2/2.$$

Considering a superposition with another sound field of smaller amplitude but at the same frequency, the change in the force is greater than the change in the sound pressure magnitude. Taking e.g. $p_2 = \frac{\hat{p}}{10}\cos\omega t$, we get the force

$$F \propto \langle(p_1 + p_2)^2\rangle = \hat{p}^2\left(\frac{11}{10}\right)^2\langle\cos^2\omega t\rangle = \frac{121}{100}\hat{p}^2/2,$$

i.e. a 10 % relative disturbance in the superposition between p_1 and p_2 created a 21 % change in the force. This change is also highly dependent on the relative phase of the two sound fields, such that the force can either increase or decrease depending on whether the sound fields add constructively or destructively. Considering instead a superposition with a sound field of the same small magnitude but at a different frequency, i.e. $p_2 = \frac{\hat{p}}{10}\cos\omega_2 t$, the situation is different. If the two frequencies are separated enough for the forces to be independent, say a few kHz, the mixing terms will average to zero as mentioned above. This gives the force as

$$F \propto \langle (p_1 + p_2)^2 \rangle = \hat{p}^2 \langle \cos^2 \omega t + \frac{\cos^2 \omega_2 t}{100} + \frac{2}{10} \cos \omega t \cos \omega_2 t \rangle = \frac{101}{100} \hat{p}^2 / 2,$$

i.e. the 10 % disturbance in the pressure fields causes only a 1 % change in the force, which furthermore is not dependent on any phase relation between the two fields. This shows that using different frequencies for individual traps relaxes the conditions under which the superposition of multiple sound fields preserves the performance of the designed traps. A similar idea has also been used previously to create a trap with orientation control by placing two traps at the same position but with different orientation of the traps [15].

4.3.3 Signature-based optimization

The result from an optimization procedure for a levitation trap is the set of complex amplitudes for the transducers in the array. When optimizing for a single levitation trap, many authors have noted that the phase angles of these driving signals have structural patterns in relation to the physical positions of the individual transducers. If these phase angles are compensated for the propagation delay between each of the transducers and the target levitation position, the new compensated pattern is often independent of the target position of the levitated object. This compensated pattern is sometimes called the "signature" of the array, and is specific for each array geometry and sometimes for different types of trap. The usefulness of these signatures is that a similar trap can be placed at a different point in space by adding the corresponding propagation phase delays to the signature. This allows the positioning of a trap to be changed without requiring any further optimization at all. For certain arrays types, these signatures are so reliable that they can be used immediately without any optimization at all, essentially working as an explicit solution for a levitation trap of a certain type.

However, finding and describing these signatures is not always easy. When inspecting a signature obtained from optimization it is often clear that there is a pattern, but it can be difficult to describe it explicitly. One very interesting option would be to skip the optimization of the individual transducers in the first place, and instead optimize this signature directly. This would require some clever parameterization of signatures, such that an optimizer can vary the signature in a systematic way. Calculating the radiation force from a signature could be done as an extension of the array-based model, simply by applying the proper propagation delay for individual transducers chosen to sample the signature. This signature-based optimization further allows the cost function to be applied at many points in space simultaneously, such that the optimizer is searching for a signature that would create traps at any of the chosen points when added with the corresponding propagation delays.

If successful, this framework would be a systematic way to derive phase signatures that are suitable for acoustic levitation in a chosen context. This signature can then reliably be used to rapidly and explicitly create traps at any chosen locations, which allow for much faster movement of the traps and real-time responses to external inputs. Such a signature is also a very good starting point for a quiet zone-based approach to levitate multiple objects, since it is known to create a single trap at the desired position.

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