CHALMERS
UNIVERSITY OF TECHNOLOGY

Application of the dimensionless Aggressiveness number in abrasive processes

Downloaded from: https://research.chalmers.se, 2026-04-02 19:57 UTC

Citation for the original published paper (version of record):
Badger, J., Drazumeric, R., Krajnik, P. (2021). Application of the dimensionless Aggressiveness number in abrasive processes. Procedia CIRP, 102: 361-368.
http://dx.doi.org/10.1016/j.procir.2021.09.062

N.B. When citing this work, cite the original published paper.

research.chalmers.se offers the possibility of retrieving research publications produced at Chalmers University of Technology. It covers all kind of research output: articles, dissertations, conference papers, reports etc. since 2004. research.chalmers.se is administrated and maintained by Chalmers Library

(article starts on next page)

Available online at www.sciencedirect.com

ScienceDirect

Procedia CIRP 102 (2021) 361–368

www.elsevier.com/locate/procedia

18th CIRP Conference on Modeling of Machining Operations

Application of the dimensionless Aggressiveness number in abrasive processes

Jeffrey Badger[a,d]*, Radovan Dražumerič[b,d], Peter Krajnik[c,d]

[a]*The Grinding Doc, Texas*
[b]*University of Ljubljana, Faculty of Mechanical Engineering, Ljubljana, Slovenia*
[c]*Chalmers University of Technology, Department of Industrial and Materials Science, Gothenburg, Sweden*
[d]*The International Grinding Institute, USA*

* Corresponding author. Tel.: +1 512-934-1857. *E-mail address:* JB@TheGrindingDoc.com

Abstract

The chip thickness is often used to characterize abrasive processes, particularly grinding. Unfortunately, because of the seemingly random nature of the geometrically undefined cutting points and difficulty in estimating the cutting-point density, chip thickness is notoriously difficult to quantify. Recently, the dimensionless Aggressiveness number has gained popularity because it circumvents the need to quantify the wheel topography and is applicable to any geometry in abrasive contact. This paper shows how the concept of dimensionless Aggressiveness number applies to the most common abrasive geometries and how it can be used to achieve practical results in a variety of applications.

© 2021 The Authors. Published by Elsevier B.V.
This is an open access article under the CC BY-NC-ND license (https://creativecommons.org/licenses/by-nc-nd/4.0)
Peer-review under responsibility of the scientific committee of the 18th CIRP Conference on Modeling of Machining Operation.

Keywords: Abrasive; CBN; Diamond; Dressing; Geometry; Grinding; Kinematic; Modeling; Optimization; Specific energy.

1. Background

Dimensionless parameters have been used for centuries to quantify fundamental physical relationships. Perhaps the most well-known example in engineering is the Reynold's number. It is defined as the ratio of inertial forces to viscous forces in a fluid and is useful for quantifying whether fluid flow is laminar or turbulent. Although it is perhaps the most well-known concept taught in all fluid-mechanics courses today, it did not gain immediate widespread popularity. The concept was introduced in 1851 by British physicist George Gabriel Stokes (1819-1903), but it did not obtain widespread use. Over thirty years later, in 1883, British mathematician and pioneer in fluid mechanics Osborne Reynolds (1842-1912) began to use it. However, it took another twenty-five years, in 1908, before the concept began to gain widespread popularity, when German physicist Arnold Sommerfeld (1868-1951) gave it a name: the Reynold's number.

In a similar vein, another dimensionless number, this time in grinding, was introduced over 100 years ago and used sporadically by various grinding researchers. However, it did not gain popularity until it was recently given a name.

George Alden (1843-1926) may have been the first to use a dimensionless number in quantifying chip thickness, in 1914, with the term $(v/V)\cdot\sin(A+B)$, where v is the workpiece surface velocity, V is the wheel surface velocity, and $\sin(A+B)$ is the unit normal vector into the wheel in cylindrical grinding [1]. Sixty-two years later, Shaw [2], when quantifying the cutting-point density as a function of a parameter proportional to the kinematic depth into the wheel, adopted a dimensionless parameter he referred to as $(v/V)\cdot(d/D)^{0.5}\cdot 10^6$. Nearly twenty years later the same dimensionless parameter was used again, in a 1993 article in Japanese by Inasaki, Toyoma and Shiratori [3], who termed it the "Non dimensional average sectional area of chips", $a_m/w^2 = V_w/V_g\cdot\sqrt{(h/D)}$, in a plot of specific energy when grinding cermets. In addition, Malkin [4] defined a similar, more fundamental term based on first principles – the ratio of the radial component of the feedrate to the wheel speed – but never applied it.

2212-8271 © 2021 The Authors. Published by Elsevier B.V.
This is an open access article under the CC BY-NC-ND license (https://creativecommons.org/licenses/by-nc-nd/4.0)
Peer-review under responsibility of the scientific committee of the 18th CIRP Conference on Modeling of Machining Operation.
10.1016/j.procir.2021.09.062

Fig. 1. Pioneers in dimensionless parameters. In fluid mechanics: Stokes, Sommerfeld and Reynolds; in grinding: Alden, Shaw, Malkin and Inasaki.

It was not until 2008, at a CIRP Conference in Dublin [5], that the concept was given name – the *Aggressiveness number* – and began to gain popularity, 104 years after Alden used it.

Here it was defined as the machine parameters (usually feedrate, depth of cut and wheel speed) within the chip-thickness equation that can be varied by the operator in plunge-grinding operations.

Twelve years later the dimensionless *Aggressiveness number* was finally defined in terms of first principles [6], as the ratio of the velocity normal component to the velocity tangential component in two contacting abrasive surfaces.

In production-grinding operations with basic geometries, such as surface grinding, experienced machine operators often have an intuitive understanding of the concept of Aggressiveness. When increasing material removal rates, they will increase wheel speed, knowing that if they do not the "grinding action will be more aggressive", resulting in a loss of wheel form or a bad surface finish. In other words, they unwittingly keep the aggressiveness of the operation constant.

In fact, the concept of constant Aggressiveness is not new. Between 1513 and 1517, Leonardo Da Vinci [7] designed a grinding machine for polishing telescopic mirrors (at that time made of bronze), in a sketch titled "Design for a Machine for Grinding Convex Lenses". The focus of the design was on achieving a proportional increase in lens velocity for an increase in wheel velocity.

Between 2008 and 2020, with the Aggressiveness number now defined, the concept allowed for numerous advances in many applications, both in complex geometries and simple, everyday changes made by machine operators.

This paper explores the development of Aggressiveness number and gives examples of its application in production.

2. Introduction

Chip thickness plays an important role in machining. In turning, drilling and milling, parameters that yield a larger chip thickness typically produce greater cutting efficiency, larger forces on the cutting edge and rougher surface finishes. In metal-cutting operations, the respective calculations are rather straightforward as the geometry of the cutting edges and the distance between them are generally known.

In grinding, chip formation also occurs, albeit on a much smaller scale. Here the chip thickness also plays an important role. Larger calculated chip thicknesses in grinding typically produce a greater proportion of cutting over rubbing and plowing, lower specific energies, larger forces on the grits, greater wheel wear and a rougher surface finish [4].

Unfortunately, the calculation for chip thickness in grinding is not as straightforward as in machining. A typical grinding wheel may contain two million active grits (d_s=400, b_s=100, C=16 grits/mm^2, $N_g=\pi\cdot d_s\cdot b_s\cdot C=2\cdot 10^6$). Each individual grit has a unique geometry, with the distance between cutting points following a seemingly random distribution. To complicate matters, dulling of the grits alters the cutting-point geometry, and grit fracture and grit pullout alter the cutting-point density. To make matters even worse, the calculation of chip thickness is a recursive function – as the value of cutting-point density used in the chip-thickness calculation itself depends on the depth of penetration, i.e., the chip thickness [2,10,11]. Finally, the well-known equation for chip thickness applies only to one specific type of abrasive interaction – grinding – and to one specific geometry – plunge grinding. Different equations are required for face, cup-wheel and crankpin grinding, etc.

Efforts to quantify the cutting-point density, C, have proven difficult [8] as the distribution of cutting edges depends on numerous, often transient, parameters such as grain size, dressing parameters and depth of measurement into the wheel. In addition, different measurement methods have yielded vastly different results, and even the same method performed by different researchers has yielded different results [2,10,11,12]. In addition, the "shape factor" used in the chip-thickness calculation depends on the grit geometry, and different researchers have used different values [11].

This paper is therefore concerned with the practical application of a more fundamental approach to capture the contact geometry and kinematics of an abrasive process: the dimensionless Aggressiveness number. It circumvents the problems of quantifying the wheel topography and is not restricted to any specific geometry or type of abrasive contact.

The fundamental parameter is defined, and then specific examples are given where it has been successfully applied in production operations. It is then compared to the popular parameters of specific material removal rate, equivalent chip thickness and speed ratio. Finally, recommendations are given for its practical use in common grinding geometries.

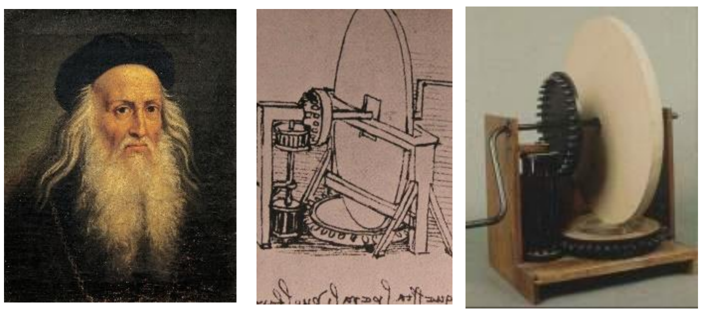


Fig. 2. DaVinci, and his design for grinding of lenses [7].

Nomenclature

a_e	depth of cut, mm
Aggr	Aggressiveness number
$Aggr^*$	point aggressiveness
$Aggr'$	line aggressiveness
b_s	wheel width
C	cutting point density, points/mm^2
CF	cross-feed, in spiral grinding, mm
d_{eq}	equivalent wheel diameter, mm
d_g	grit diameter, mm
d_s	wheel diameter
e	specific energy, J/mm^3
h_m	maximum chip thickness, µm
ℓ_c	contact length, mm
N_g	number of grits
q	speed ratio
r	chip shape factor
R_a	surface roughness, µm
v_s	wheel speed, m/s
v_w	feedrate, in mm/s, also called the table speed

3. Evolution of basic models in grinding

Over the years, several parameters have been developed to quantify the intensity of an abrasive interaction. The most basic may be the specific material removal rate, *Q'*, defined in terms of plunge grinding as the depth of cut, a_e, and the feedrate, v_w, according to $Q'=v_w \cdot a_e$ [4]. In general, grinding operations with higher specific material removal rates will be "more aggressive", with the individual cutting points penetrating deeper into the workpiece.

Unfortunately, in terms of quantifying the chip thickness, the specific material removal rate suffers from not taking into account the wheel speed. In 1974, Snoeys and Peters [12] proposed the equivalent chip thickness, h_{eq}, defined as $h_{eq}=Q'/v_s=a_e \cdot v_w/v_s$. It gained widespread popularity in academia. In addition to this, the speed ratio, *q*, is often used, particularly in industry. It is defined as [13] $q=v_s/v_w$, with lower values recommended for roughing, higher for finishing.

However, all of these fail to take into account arc length, ℓ_c, such that a cylindrical outer-diameter operation and a creep-feed operation could have the same values *Q'*, h_{eq} and *q*, but behave vastly differently because of arc lengths that differ by over an order of magnitude.

One parameter, common in textbooks, does take arc length into account. It is the maximum undeformed chip thickness, h_m, typically given in terms of plunge grinding as [4]:

$$h_m = \sqrt{\frac{4}{C \cdot r}\left(\frac{v_w}{v_s}\right)\sqrt{\frac{a_e}{d_{eq}}}} \; . \qquad (1)$$

where *C* is the cutting point density, *r* is the chip shape factor, and d_{eq} is the equivalent wheel diameter for cylindrical grinding, or simply the wheel diameter for surface grinding. Unlike the other equations, this equation takes into account all the required geometric parameters necessary to quantify the depth of penetration of an individual grit into the workpiece. However, it suffers from several issues: a) the choice of cutting-point density and chip-shape factor, as described above; b) its restriction to grinding; and c) its restriction to the geometry of plunge grinding assuming a trochoidal trajectory of the abrasive.

3.1. Aggressiveness term first coined

In 2008, in an effort to circumvent these issues, Badger [5] proposed using only the parameters that can be readily altered by the machine operator, namely the depth of cut, feedrate, wheel speed and (to a lesser extent) the wheel diameter. He termed it the Aggressiveness number, *Aggr*, calculated by:

$$Aggr = 10^6 \frac{v_w}{v_s}\sqrt{\frac{a_e}{d_s}} \; . \qquad (2)$$

The constant of 10^6 was used to provide more graspable numbers (*Aggr* = 21.4 as opposed to *Aggr* = 0.00000214).

This simple concept enabled engineers and machine operators to vary several variables at once and see whether the Aggressiveness number increased or decreased. In addition, once the characteristic curve of G-ratio or specific-energy vs. aggressiveness was determined for a given process, it could be used to predict wheel wear and specific energy (and therefore temperatures) for any combination of parameters.

This parameter was adopted by numerous researchers: in camshafts [14], crankshafts [15], nickel alloys [16], face-grinding [17], and truing [18], while also gaining popularity in industry [13].

However, the parameter was still limited to trochoidal-path plunge operations and only to grinding operations. It did not apply to other geometries and other abrasive geometries.

3.2. Grand unified theory based on first principles

In 2020, these disparate theories were unified into one Theory of Aggressiveness [6]. The theory used first principles to take into account the instantaneous geometry and kinematics at any point of contact between any two contacting surfaces in abrasive contact.

The point aggressiveness, $Aggr^*$, was defined as:

$$Aggr^* = v_{w,N} \,/\, v_s \qquad (3)$$

where $v_{w,N}$ is the component of velocity acting normal to the point of contact and v_s is the component of velocity acting tangential to the point of contact. In most grinding operations, v_w is the workpiece velocity and v_s is the wheel velocity. In addition, the line aggressiveness, *Aggr'*, was defined as the average aggressiveness along a line of contact as a point on surface 1 passes through surface 2.

The term that has been used in all previous publications and which is used here is the Aggressiveness number, *Aggr*, which is defined as the average point Aggressiveness in a given abrasive contact area.

In practical use, *Aggr* is often multiplied by the constant 10^6 for the reasons described above, and the user must be aware of this. In most cases, however, whether the constant was used or not is obvious as operations using the 10^6 constant typically produce *Aggr* numbers in the range of 5-200 for grinding and in the thousands for dressing.

In operations with a trochoidal path (such as surface and cylindrical grinding), the point aggressiveness increases from 0 to its maximum value (for up-grinding) or maximum to zero (for down-grinding). Here, the maximum point aggressiveness is double the Aggressiveness number, or $Aggr^*_{max} = 2 \cdot Aggr$. In operations with a linear path (such as face grinding, cup-wheel grinding and cutting-off grinding), the aggressiveness is constant throughout the cut, such that $Aggr^{*}_{max} = Aggr$.

3.3. Spiral grinding – from fundamental equation

An example of a rather complicated geometry that can be analyzed using the first principles of Aggressiveness is spiral grinding, used in diamond-grinding of ceramics for the semiconductor industry. Fig. 3 shows a top view and side view of a vertical-spindle spiral grinding operation with a fixed (axial) depth of cut, a_e. The wheel traces a spiral path around the workpiece, with a traverse cross-feed lead, *CF*, at a workpiece surface velocity v_w.

Here the standard equations of chip thickness would break down. However, we can apply the fundamental definition of Aggressiveness – the normal component of feedrate into the abrasive surface divided by the wheel speed, averaged over the contact area. This is done by using two unit-normal vectors, *V1* and *V2*. The first, *V1*, is normal to the taper surface on the cup-wheel; the second, *V2*, is normal to the surface-curvature of the wheel. Then, the Aggressiveness number is simply the average point aggressiveness ($Aggr^{*} = v_w/v_s \cdot V1_N \cdot V2_R$) in the contact area on the wheel. This shows how Aggressiveness theory can be applied to any complicated abrasive geometry using first principles. This particular operation is given in the Appendix.

3.4. Calculating grit penetration depth from Aggressiveness

In spite of its ambiguity, there are situations where an end-user prefers a concrete value for chip thickness (often referred to as grit penetration depth). It can be more intuitive to say, "My grit penetration depth is 1.4 microns", particularly to those who come from a machining background, than to say, "My Aggressiveness is 21.4." Therefore, the basic conversion from Aggressiveness number is given in the Appendix. Again, the constants *C* and *r* must be chosen, and we leave this up to the end-user. However, some suggested values are given. While these values can be disputed, all values can be disputed, and so some standardized value must be chosen. Moreover, the *relative* values of h_m are typically more important than the *absolute* values of h_m.

Now that this fundamental parameter is established, we can apply it to various grinding geometries and compare it to other parameters (such as equivalent chip thickness and speed ratio). In addition, we are no longer restricted to plunge grinding. We can incorporate other geometries, such as face grinding, using the basic equations given in the Appendix.

4. Application of Aggressiveness number

4.1. Specific energy in surface grinding of WC

Zelwer and Malkin [19] performed surface grinding tests on tungsten-carbide-cobalt using a resin-bonded diamond wheel. The results, plotted in Fig. 4 (a), show that specific energy decreased with increasing depth of cut and feedrate, represented by five distinct curves. If we needed to know the specific energy at, say, a_e=5 µm and v_w=20 mm/s, we would have to "eyeball" an imaginary curve between the blue and black curves, along with "eyeballing" the shape of that curve for values of a_e<5 µm.

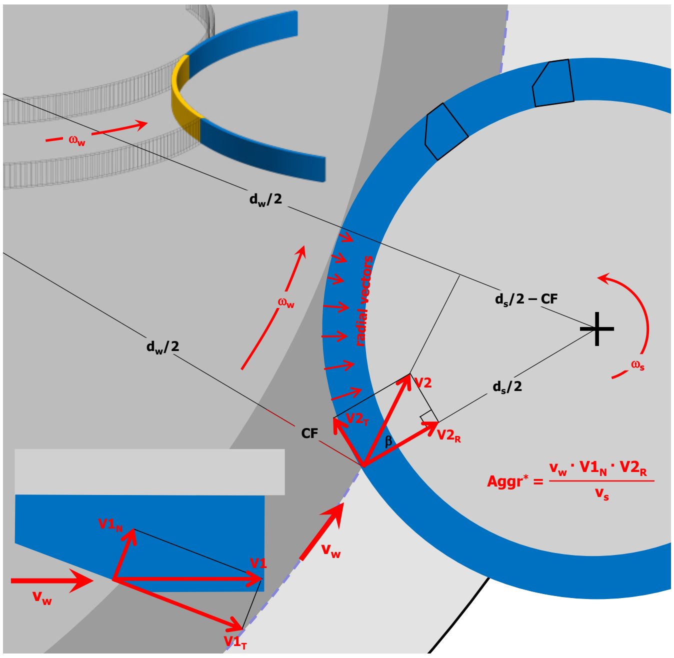


Fig. 3. Unit-normal vectors used to calculate *Aggr* in spiral grinding.

However, if we put these values into the equation for *Aggr*, we obtain a curve that lines up much better, (Fig 2.b).

To determine the specific energy, we simply calculate the Aggressiveness number at these values (a_e=5 µm, v_w=20 mm/s; *Aggr*=3.97) and determine the associated specific energy (*e*=168.4 J/mm^3), giving us a much more accurate estimate.

4.2. Surface roughness in cylindrical grinding of steel

As grinding operations become more aggressive, surface roughness (R_a) values increase [4]. This is shown in Fig. 5, from the results of Opitz and Gühring [20]. Here we can compare the various parameters of (a) *Q'*; (b) speed ratio; (c) equivalent chip thickness; and (d) Aggressiveness number. The Aggressiveness number, with an R^2 value of 0.99, is clearly the most accurate, even remarkable, considering the large spread typically associated with measuring surface roughness.

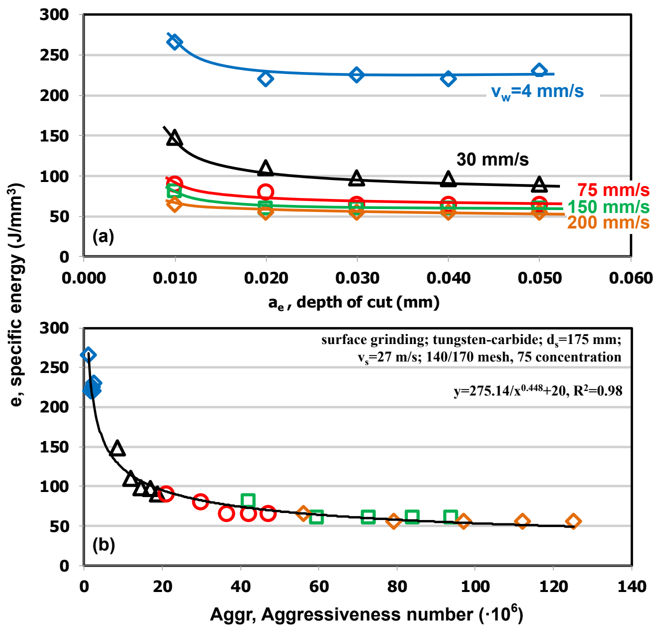


Fig. 4. Surface grinding of tungsten-carbide [19].

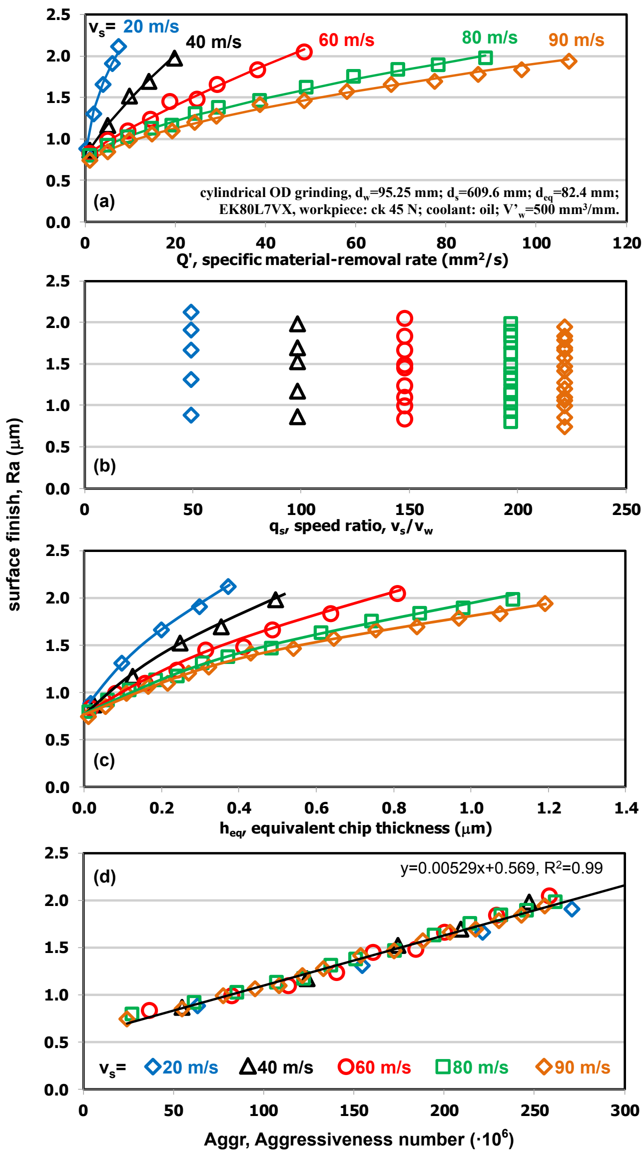


Fig. 5. R_a vs. various parameters, from [20].

4.3. Specific energy in diamond dressing

The fundamental concept of aggressiveness applies to any abrasive contact. While diamond dressing of grinding wheels typically does not fall in the category of grinding, it can be considered fundamentally a grinding process, with the diamond dresser acting as the abrasive and the grinding wheel acting as the workpiece. Therefore, we can apply the concept of dimensionless aggressiveness to rotary dressing.

Experiments were performed using a large CNC machine for orbital grinding of crankshafts [6]. A vitrified CBN wheel was used (b151-grain, wheel diameter 700 mm, wheel width 46 mm). Dressing was performed using a diamond disc with the following parameters: a) two values of the speed ratio: q=+0.43 and q=+0.86, achieved by changing the dresser speed; b) two values of effective dressing depth: 2 μm and 6 μm; and c) three values of the collision number: 2.3, 5.4 and 13.5, achieved by changing the traverse velocity.

The results are shown in Fig. 6. When plotted against *Q'* (a), there are two distinct curves depending on the speed ratio. In contrast, when plotted against equivalent chip thickness (b), the correlation improves, although there are still two distinct curves. However, when plotted against *Aggr* (c), the dependency on speed ratio disappears. This is because the speed ratio is incorporated into the fundamental equation.

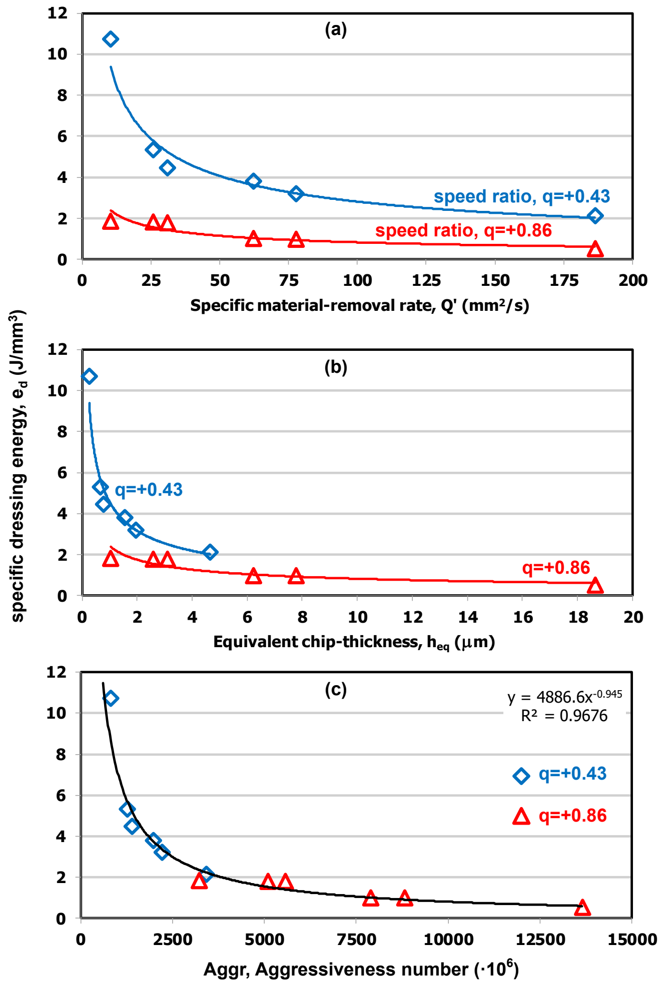


Fig. 6. Specific energy in diamond dressing [6].

4.4. Specific energy in face grinding

The commonly used equation for chip thickness is only valid for trochoidal-path plunge-grinding operations. It does not apply to linear-path face grinding. However, the fundamental definition of aggressiveness can be used. An example is given in Fig. 7, which shows the measured specific energy vs. feedrate and Aggressiveness number for data taken from diamond-grinding of PCBN inserts [21]. The figure shows that, for a given wheel speed, specific energy decreases with increasing feedrate before reaching a near-steady-state value. However, if the wheel speed is changed, very different specific energies are measured.

In contrast, the same data plotted vs. *Aggr* gives a strong correlation for all feedrates and wheel speeds.

4.5. Inner-diameter outer-diameter grinding

One grinding operations that suffers greatly when arc length is not taken into consideration (as is the case with *Q'*, h_{eq} and *q*) is cylindrical grinding – when considering inner-diameter vs. outer-diameter grinding.

In cylindrical outer-diameter grinding, the convex-on-convex contact creates a shorter arc length, whereas in inner-diameter grinding the concave-within-concave contact creates a longer arc length. This is illustrated in Fig. 6, taken from Lindsay [22], when cylindrical-plunge grinding hardened 52100 steel. Two grinding operations with the same values of *Q'* exhibit a significant shift in specific energy owing to the different arc lengths (and equivalent diameters, which differ by a factor of 4.75). However, when plotted against Aggressiveness number, this shift disappears.

This means that once the characteristic curve of specific energy (or G-ratio or R_a) is established, it can be applied to any geometry, be it inner-diameter, outer-diameter or shoulder grinding (given in the Appendix) – and still be able to predict grinding behavior. In fact, this was successfully done in crankshaft grinding, which exhibits enormous differences in arc length between the crankpin, radius and sidewall [15].

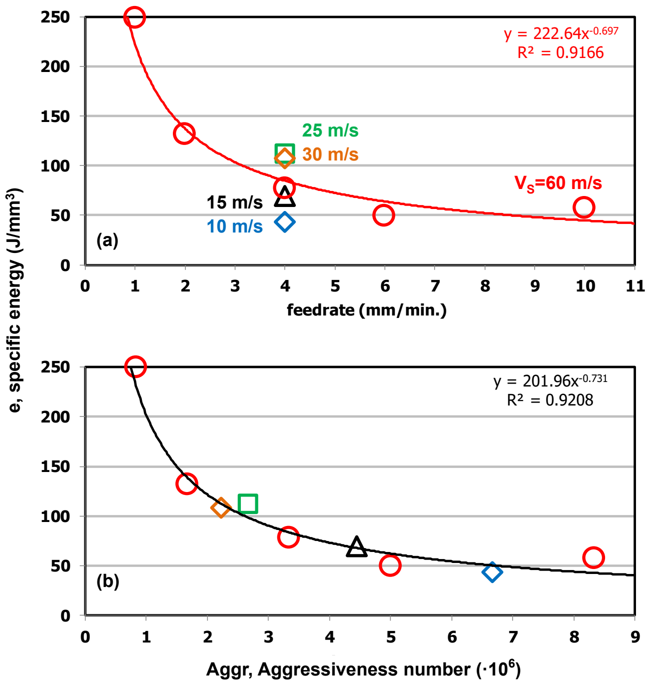


Fig. 7. Specific energy in face-grinding of PCBN [21].

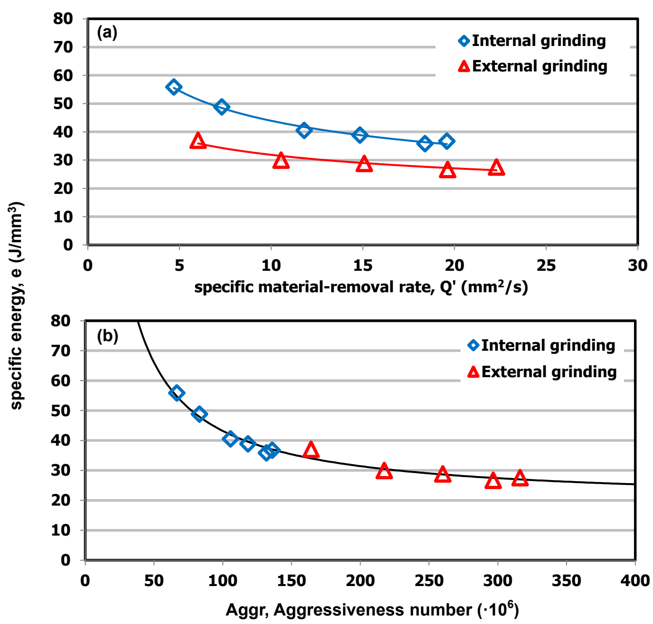


Fig. 8. Specific energy in inner-diameter and outer-diameter grinding [22].

4.6. Wheel wear in cutting-off grinding

There is a direct relationship between wheel wear and Aggressiveness number. Higher values of *Aggr* create larger forces on the grits and a larger chance of grit fracture and bond fracture. Fig. 9 shows the results from Shaw [8] in the form of G-ratio (the volume of material ground away divided by the volume of wheel worn away in the process) for cutting-off grinding of AISI 1020 steel (A24R6B, v_w=3.6-12.7 mm/s, v_s=62.5m/s). Here, the standard equations for *Q'* and h_m are not applicable, whereas the equation for *Aggr* is simple (given in the Appendix). Moreover, once this characteristic curve is developed, wheel wear can be accurately predicted for any combination of feedrate and wheel speed.

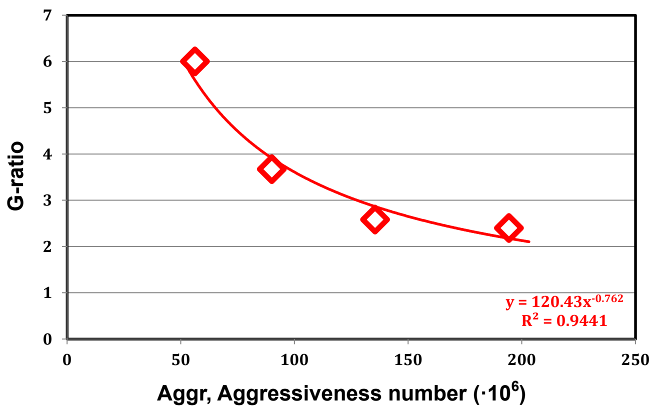


Fig. 9. G-ratio vs. *Aggr*, from [8].

5. Practical considerations

One of the benefits of the Aggressiveness number is that once the characteristic equation of *e-vs.-Aggr* is established for a given process, it can be used to predict temperatures at all points within a complex geometry. (Or, for wheel wear, *G-ratio-vs.-Aggr.*) This was well demonstrated in optimization of camshaft and crankshaft grinding at a major European truck-engine manufacturer [14,15]. Here, the constraints of temperature, aggressiveness, wheel wear and machine limitations (typically acceleration and jerk) are defined. Then, finding the parameters that minimize cycle time is quite straightforward. These are often referred to as "constant temperature" processes.

In addition, keeping the Aggressiveness number constant while varying other parameters has proven useful in production, particularly in everyday use on the shop floor where the goal is often to increase feedrates. Here, all that is needed is the basic formula and a handheld calculator. Often, when attempting to reduce cycle times, operators simply increase the feedrate – without changing other parameters – and suffer from increased wheel wear, edge chipping, surface finish, etc. In The Book of Grinding [23], Badger gives examples of using constant Aggressiveness to increase feedrates. For example, in creep-feed grinding of flutes in tungsten-carbide endmills, simply increasing the feedrate by 25% resulted in edge-chipping in the semi-brittle tungsten-carbide. However, by increasing both the feedrate and the wheel speed by 25% – and thus maintaining constant aggressiveness – large batches were able to run without edge-chipping. The feedrate was then increased again by 50% (with a 50% increase in wheel speed) without problem. In the second example, feedrates in cylindrical-plunge grinding of tungsten-carbide shafts were successfully increased by 300% while maintaining constant aggressiveness (by increasing the wheel speed and decreasing the workpiece RPM according to the fundamental equation). However, simply increasing the feedrate by 100% without other changes (and therefore increasing the aggressiveness) resulted in immediate form breakdown after one part.

These practical examples illustrate how machine operators can be empowered to achieve desired goal (less wheel wear, better wheel self-sharpening, lower temperatures, better surface finishes, shorter cycle times) simply by understanding the relationships between aggressiveness and these parameters, even when making multiple changes at once. In operations with more complex calculations (for example, cylindrical grinding), a simple spreadsheet based on the formulas in the Appendix allows the operator to deal with multiple parameters at once (for example, plunge velocity, workpiece RPM, wheel speed, swivel angle, workpiece diameter and workpiece shoulder height) and calculate the Aggressiveness number – and then increase feedrates within the limits of temperature and spindle power while maintaining a constant Aggressiveness number.

6. Conclusions

A fundamental relationship has been established quantifying how aggressively two surfaces in abrasive contact interact. It is called the dimensionless Aggressiveness number. It is proportional to the chip thickness (or grit penetration depth) but avoids the problems associated with quantifying the abrasive-surface topography and shape. It is based on first principles – i.e., it is not geometry-specific – and so can be used on any abrasive operation. It gives a better correlation to specific energy, surface finish, G-ratio, etc., than commonly used parameters such as speed ratio and equivalent chip thickness. It has been demonstrated effective in optimizing complex operations such as crankshaft grinding and camshaft grinding and, on a more basic level, has proven an effective tool for machine operators in making quick calculations on the shop floor when optimizing grinding operations and troubleshooting problems. The basic equation is given, along with a table containing the equations for the most common geometries in grinding (surface, cylindrical, face, cutting-off, cup-wheel, vertical-spindle spiral, etc.).

Acknowledgements

This work was partially funded by the International Grinding Institute, The Grinding Doc, and Scania CV AB.

References

[1] Alden GI. Operation of grinding wheels in machine grinding. *Transactions of ASME* 1914; 36:451–460.

[2] Brecker JN, Shaw MC. Measurement of the effective number of cutting points in the surface of a gringing wheel. *Proceedings of Interntional Conference on Production Engineering Tokyo* 1974; 1:740.

[3] Toyama I, Inasaki I, and Shiratori Hidehisa. High Efficiency Grinding of Cermet (in Japanese). *Japan Society for Mechanical Engineers* 1993; 59/558/2,No. 92-0946:267-272.

[4] Malkin S, Guo C. Grinding Technology: Theory and Applications of Machining with Abrasives, 2nd ed. New York: Industrial Press Inc; 2008.

[5] Badger J. Practical application of aggressiveness and chip thickness in grinding. *CIRP 3rd International Conference on High Performance Cutting (HPC) Dublin* 2008; 599–606.

[6] Dražumerič R, Badger J, Roininen R, Krajnik P. On geometry and kinematics of abrasive processes: The theory of aggressiveness. *International Journal of Machine Tools & Manufacture* 2020; ***154***:1-13.

[7] Da Vinci L. Design for a Machine for Grinding Convex Lenses. 1513-1517. www.astrosurf.com/ gap47/ T400/ Machine/ 1-machines-histoire_eng.htm

[8] Shaw MC. Principles of Abrasive Processing, Oxford: Oxford University Press; 1996.

[9] Badger J, Torrance AA. A comparison of two models to predict the grinding force from wheel surface topography. *International Journal of Machine Tools & Manufacture* 2000; 40:1099–1120.

[10] Brecker JN, Sauer WJ, Andrew C, Shaw MC. *Seventh Annual Report, Grinding Wheel Institute of Grinding Fundamentals*: Carnegie Mellon University 1971; 46.

[11] Rowe WB. Princples of Modern Grinding Technology, 2nd ed. William Andrew; 2009.

[12] Snoeys R, Peters J. The significance of chip thickness in grinding. *CIRP Annals* 1974; 23:227–236.

[13] Graf W. Handbook Cylindrical Grinding. Switzerland, Winterthur Schleiftechnik AG; 2010.

[14] Krajnik P, Drazumeric R, Badger J, Hashimoto F. Cycle optimization in cam-lobe grinding for high productivity. *CIRP Annals* 2014; ***63***(1):333-336.

[15] Drazumeric R, Roininen R, Badger J, Krajnik P. Temperature-based method for determination of feed increments in crankshaft grinding, *J. Mater. Process. Technol* 2018; *259*:228–234.

[16] Curtis D, Krain H, Winder A, Novovic D, Impact of grinding wheel specification on surface integrity and residual stress when grinding Inconel 718. *Proc IMechE Part B: J Engineering Manufacture* 2020; 1-14.

[17] Urgoiti L, Barrenetxea D, Antonio-Sánchez J, Lanzagorta JL. Detailed Thermo-Kinematic Analysis of Face Grinding Operations with Straight Wheels. *MDPI Metals* 2020; 1-13.

[18] Dražumerič R, Badger J, Klement, U, Krajnik P. Truing of diamond wheels–Geometry, kinematics and removal mechanisms. *CIRP Annals* 2018.

[19] Zelwer O, Malkin S. Grinding of WC-Co cemented carbides. *ASME Journal of Engineering for Industry* 1980; 102:133-139.

[20] Opitz H, Gühring K. High speed grinding. *CIRP Annals* 1967; *16*:61–73.

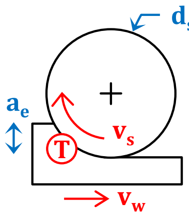
[21] Denkena B, Köhler J, Ventura CEH. Influence of grinding parameters on the quality of high content PCBN cutting inserts. *Journal of Materials Processing Technology* 2014; 214: 276– 284.

[22] Lindsay, R. Principles of Grinding. Norton Company; 1985; Figure 15.

[23] Badger J. The Book of Grinding 2019 Edition. Texas:2.2676-2.2774.

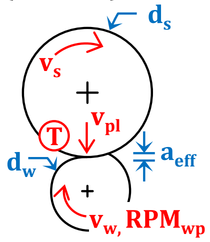
Appendix: Calculations of Aggressiveness number for common abrasive-contact geometries.

1. Surface grinding



$$Aggr = \sqrt{\frac{a_e}{d_s} \cdot \frac{v_w}{v_s}}$$

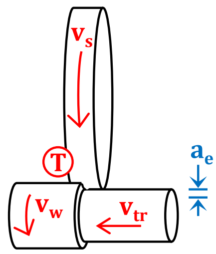
2. Cylindrical plunge (OD & ID)



$$Aggr = \sqrt{\frac{a_{eff}}{d_{eq}} \cdot \frac{v_w}{v_s}}$$

$$a_{eff} = v_{pl}/RPM_{wp}$$

3. Cylindrical traverse, no taper (OD & ID)

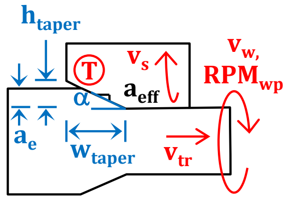


$$Aggr = \sqrt{\frac{a_e}{d_{eq}} \cdot \frac{v_w}{v_s}}$$

In cutting portion of wheel, w_{cut}

$$w_{cut} = v_{tr}/RPM_{wp}$$

4. Peel, with taper, (cylindrical traverse)

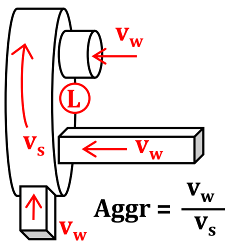


$$Aggr = \sqrt{\frac{a_{eff}}{d_{eq}} \cdot \frac{v_w}{v_s}}$$

$$a_{eff} = \frac{h_{taper}}{w_{taper}} \cdot \frac{v_{tr}}{RPM_{wp}}$$

note: $Aggr \neq f(a_e)$
$a_{eff} \neq f(a_e)$

5. Face grinding

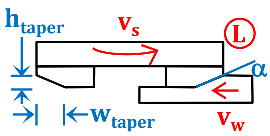


$$Aggr = \frac{v_w}{v_s}$$

Applies for both workpiece rotation and non-rotation when $v_w << v_s$

Note: $Aggr \neq f(d_s, b_w)$

6. Cup wheel, with taper

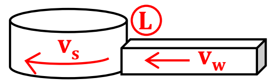


$$Aggr = \frac{v_w}{v_s} \cdot \sin(\alpha)$$

$$= \frac{v_w}{v_s} \cdot \frac{h_{taper}}{\sqrt{h_{taper}^2 + w_{taper}^2}}$$

note:
$Aggr = f(h_{taper}, w_{taper})$
$Aggr \neq f(a_e)$

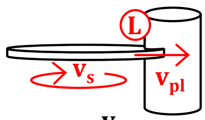
7. Cup wheel, no taper



$$Aggr = \frac{v_w}{v_s}$$

note: $Aggr \neq f(a_e, d_s, b_w)$

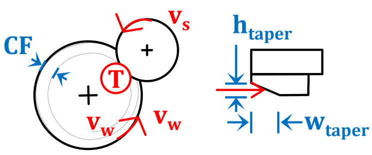
8. Cutting-off



$$Aggr = \frac{v_{pl}}{v_s}$$

Note: $Aggr \neq f(d_s, b_w)$

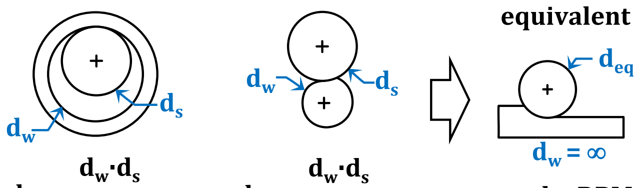
9. Vertical spindle, with taper



$$Aggr = \frac{v_w}{v_s} \cdot \sqrt{\frac{CF}{d_{eq}}\left(1 - \frac{CF}{d_{eq}}\right)} \cdot \frac{h_{taper}}{\sqrt{h_{taper}^2 + w_{taper}^2}}$$

A. Conversions (applies to all cylindrical geometries)

outer-diameter | inner-diameter | surface-grinding equivalent



$$d_{eq} = \frac{d_w \cdot d_s}{d_w + d_s} \qquad d_{eq} = \frac{d_w \cdot d_s}{d_w - d_s} \qquad v_w = \pi \cdot d_w \cdot RPM_{wp}$$

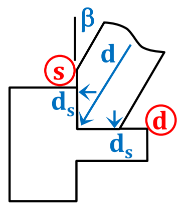
B. Swivel adjustments

at shoulder: (s)

$d_s = d/\sin(\beta)$
$(d_w = \infty)$

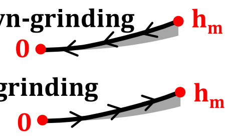
at diameter: (d)

$d_s = d/\cos(\beta)$
$(d_w = d_w)$



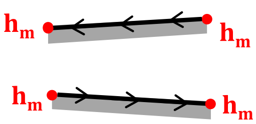
C. Shapes:

(T) Trochoidal path



$$h_m = 1000 \cdot \sqrt{\frac{4}{C \cdot r} \cdot Aggr}$$

(L) Linear path



$$h_m = 1000 \cdot \sqrt{\frac{2}{C \cdot r} \cdot Aggr}$$

h_m = maximum chip thickness, in µm (constant in linear path)
C = cutting point density, in points/mm^2
suggested value: $C = 2/d_g$
d_g = grit diameter, in mm
= FEPA/1000, or = 15.2/Mesh#
r = ratio of chip width to chip height
suggested value: r = 10
Shapes:
Trochoidal path: $Aggr = ½\ Aggr^*_{max}$
Linear path: $Aggr = Aggr^* = Aggr^*_{max}$