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INTERNAL TORQUES OF HUMAN UPPER EXTREMITY DURING ITS OPTIMAL MOTION IN VERTICAL PLANE

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Abstract

In this paper a number of optimal control problems for the motion of the human upper extremity (HUE), for different types of working tasks, are considered. The performance index used in these problems is the integral over the duration of the working task of the sum of the square of the controlling stimuli acting at the joints of the human arm. Under some conditions this performance index can be used for evaluation of the muscles' energy expenditure during human movements. A HUE is simulated by a plane multibody system of rigid masses. This system comprises the three elements with mass and rotatory inertia model the upper arm, the forearm and the hand. The controlled motions of the mechanical system are described in terms of joint angles and Cartesian coordinates of the shoulder joint, through the application of Lagrange's equations. The main aim of the study is an investigation of the interaction between the gravity forces and the internal torques acting at the joints during goal-directed extremal motions of the HUE. The analysis of the internal torques, energetic and viscoelastic characteristics of the shoulder, the elbow and the wrist joints for the exstremal controlled motions of the human arm under the external load acting on the hand has been done.

Introduction

The HUE is a complex indeterminate biomechanical system containing a number of joints connecting the shoulder elements of the upper arm, the lower arm and the wrist. Various models of the HUE have been developed over the last decade [1-6]. These models were used mainly to study the kinematics of the HUE or the static effects of muscle actions and joint forces but not the goal-directed extreme motion of the human arm.

Presently, limited information is available about how the HUE performs dynamically in achieving its extremal (extreme) motion, e.g. time-optimal motion, energy-optimal motion, etc. What type of torque and force distribution is needed for these extreme motions? Which are the basic principles of the stiffness phenomenon of the human arm during performance of the typical working tasks? What kind of interaction between the gravity forces and the internal controlling stimuli during extremal goal-directed motion of the HUE is needed?

This paper is concerned with an investigation within the plane mechanical model of the HUE's controlled motion between fixed boundary conditions and given constraints on the phase coordinates. A computational method for mathematical modelling of the optimal control laws that govern the reaching motion of the HUE is presented. This method is based on a mixed Fourier and spline approximation of the variable functions and inverse dynamics approach [7-10]. The method proposed makes it possible to satisfy the boundary conditions and some constraints on the phase coordinates automatically. Using the proposed method a number of optimal control problems of the HUE have been solved. The performance index used in these problems is the integral over the duration of the working task of the sum of the square of controlling stimuli acting at the joints of the human arm. Under some conditions [11] this performance index can be used for evaluation of the muscles' energy expenditure during human movements. We have also estimated the torques, energetic and viscoelastic characteristics of the shoulder, the elbow and the wrist joints for the optimal controlled motion of the human arm under the external load acting on the hand.

The Mathematical Model

A human arm is simulated by a plane multibody system of rigid masses. This system comprises the three elements with mass and rotatory inertia model the upper arm (link **SE**), the forearm (link **EW**) and the hand (link **WH**), (see Fig.1). In addition to the weights of the links the external forces acting on the HUE include the interaction forces between the upper arm and the human body, which are represented by the principal vector of the reaction forces **R(t)** and the principal moment **p(t)**. There is a load acting on the hand, which is represented by the principal vector of the load forces **F(t)** and principal moment **μ(t)**.

It is assumed that the control torques $q(t), u(t)$ act at the elbow **E** and the wrist **W** joints, respectively. These torques are treated as the internal stimuli.

Let OXYZ be a fixed rectangular Cartesian coordinate system. It's assumed that the HUE moves in a sagittal plane OXY. The following notations are involved: x,y are the Cartesian coordinates of the shoulder joint **S**; α,β,γ are angles that specify the position of the human arm links; m_i,l_i,r_i,J_i are the mass, the length, the distance to the centre of mass and the moment of inertia with respect to the centre of mass of the upper arm (i=1), the forearm (i=2) and the hand (i=3), respectively; $R_x(t),R_y(t)$ and $F_x(t),F_y(t)$ are the horizontal and vertical components of the principal vectors of the reaction forces in the shoulder joint and of the load forces acting on the hand at the point **H**, respectively; g is the acceleration due to gravity.

The motion of the mechanical system can be described in terms of joint angles α,β,γ and Cartesian coordinates x,y through the application of the Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial z^{\bullet}}\right)-\frac{\partial T}{\partial z}=G_z-\frac{\partial \Pi}{\partial z}, \tag{1}$$

where $z=(x,y,\alpha,\beta,\gamma)$ is a vector of the generalised coordinates, $z^{\bullet}$ is a vector of generalised velocities, T and Π are the kinetic and potential energies of the system, respectively, and G_z is a vector of the generalised forces. In this paper the following dimensionless variables and parameters will be used:

$$x'=x/l, y'=y/l, t'=t/T_0$$

$$p'=p/Mgl,\ \ q'=q/Mgl,\ \ u'=u/Mgl$$

$$\mu'=\mu/Mgl,\ \ F_x'=F_x/Mg,\ \ F_y'=F_y/Mg$$

$$g'=gT_0^2/l,\ \ R_x'=R_x/Mg,\ \ R_y'=R_y/Mg$$

where $l = l_1 + l_2 + l_3, M = m_1 + m_2 + m_3, T_0$ is the period of natural oscillation of the straight human arm.

The Statement of the Optimal Control Problem

The human arm's working tasks can be modelled in different ways. We shall model the working tasks of the HUE by specifying the initial and the final phase states of the human arm, and the external load acting on the hand, i.e. we have the following conditions:

$$z(0) = z_0, z(\tau) = z_\tau, z^\bullet(0) = z_0^\bullet, z^\bullet(\tau) = z_\tau^\bullet, \tag{2}$$

$$F_x(t) = f_x(t), F_y(t) = f_y(t), \mu(t) = \mu_0(t), t \in [0, \tau], \tag{3}$$

where vectors $z_0, z_0^\bullet, z_\tau, z_\tau^\bullet$ and functions $f_x(t), f_y(t), \mu_0(t)$ are given in advance; τ is a dimensionless time, which is equal to the ratio of the duration of the working task to the period of the natural oscillation of the straight human arm.

As follows from analysis of the experimental data [12] the angular motions of the upper arm, the forearm and the hand have to satisfy on $t \in [0, \tau]$ the following inequalities:

$$-\pi / 3 \le \alpha(t) \le \pi, \quad -\pi / 20 \le \beta(t) - \alpha(t) \le 5\pi / 6$$
$$-7\pi / 18 \le \gamma(t) - \beta(t) \le 4\pi / 9 \tag{4}$$

The possibility of the human body's musculoskeletal system is bounded. Hence the controlled stimuli acting at the joints of the HUE have to be bounded.

We shall assume that the next inequalities are valid:

$$|p(t)| \le p_0, |q(t)| \le q_0, |u(t)| \le u_0$$
$$|R_x(t)| \le R_{x0}, |R_y(t)| \le R_{y0}, t \in [0, \tau]. \tag{5}$$

Consider the following optimal control problem.

Problem A. The human arm's working task is given, i.e. the boundary conditions (2) and the external load acting on the hand (3) are specified. It's required to determine the controlled process of the HUE, i.e. the pare of vector-functions $\{z(t), U(t)\}$, where $U(t) = \{R_x, R_y, p, q, u\}$, which satisfy the equations of motions (1), the boundary conditions (2), the given constraints on the phase coordinates (4), the given restrictions on the controlling stimuli (5) and which minimises the functional

$$\Phi = \int_0^\tau \left[p^2(t) + q^2(t) + u^2(t)\right]dt \qquad (6)$$

Under some conditions [11] the functional (6) can be used for evaluation of the muscles' energy expenditure during human movements. For this reason the minimal value Φ_* of functional (6) we shall call the energy expenditure during optimal controlled motion of the HUE. However, it is not strictly the energy of the system.

The Inverse Dynamics and Mixed Fourier-Spline Based Approach

The necessary conditions of optimality in **Problem A** can be obtained by well-known methods of optimal control theory, e.g. using the Pontryagin's maximum principle [13]. As usual the numerical solution of the optimal control problems for multidimensional non-linear systems based on the necessary conditions of optimality is very complicated.

Central in the proposed approach for solving **Problem A** is the idea that any optimal control problem can be converted into a standard non-linear programming problem by parameterizing each of the variable functions [14-16]. We shall assume that every generalised coordinate of the HUE z_i is a variable function which could be represented as follows [7,8,10]:

$$z_i(t) = P_i(t) + Q_i(t), \qquad i = 1,2,3,4,5$$

$$P_i(t) = \sum_{j=0}^{5} p_{ij} t^j, \qquad (7)$$

$$Q_i(t) = \sum_{k=1}^{N_i} \left(a_{ik} \cos\frac{2\pi k}{\tau} + b_{ik} \sin\frac{2\pi k}{\tau}\right),$$

where N_i, (i=1,2,3,4,5) are given positive integers.

Taking into account the boundary conditions (2) the coefficients of the polynomials (7) can be determined in analytical form [8,9].

Suppose that the law of motion of the HUE, i.e. the vector-valued function $z(t)$ is given by formula (7). Using the equations (1) the inverse dynamics' problem can be solved. All above mentioned makes it possible to convert the **Problem A** into the parameter optimization problem:

$$\Phi = F(C) \Rightarrow \min, \qquad f(C) \leq 0 \tag{8}$$

Here the functions F, f are determined by means of (1)-(7), $C = (\ddot{z}_{i0}, \ddot{z}_{i\tau}, a_{ik}, b_{ik}; k = 1,\dots,N_i; i = 1,2,3,4,5)$ is a vector of the variable parameters, $\ddot{z}_{i0} = \ddot{z}_i(0)$, $\ddot{z}_{i\tau} = \ddot{z}_i(\tau)$, i.e. $\ddot{z}_{i0}, \ddot{z}_{i\tau}$ are the generalised accelerations at the initial t=0 and the final time $t \in [0, \tau]$, respectively.

In order to solve the parameter optimization problem (8) a computational algorithm based on Rosenbrok's method [17] has been devised.

The Numerical Simulation and Analysis

Let us describe some solutions of the **Problem A** for the considered model of the HUE. In the model the following values of the parameters for the HUE links have been used: m_1=2.17kg, m_2=1.26kg, m_3=0.53kg, l_1=0.32m, l_2=0.247m, l_3=0.184m, r_1=0.147m, r_2=0.105m, r_3=0.079m, J_1=0.014kgm^2, J_2=0.007kgm^2, J_3=0.0006kgm^2. These values of the parameters for the HUE correspond to a human body with total mass M=70kg and height of 1.7m [18].

The boundary conditions were specified by formula (2) with

$$\begin{aligned} &\alpha(0) = \beta(0) = \gamma(0) = x(0) = 0, \qquad y(0) = 1 \\ &\alpha(\tau) = \beta(\tau) = \gamma(\tau) = \pi/2, \qquad x(\tau) = 0, \qquad y(\tau) = 1 \\ &\ddot{z}_0 = \ddot{z}_\tau = \dot{z}_0 = \dot{z}_\tau = 0 \end{aligned} \tag{9}$$

The external load acting on the hand was given over the time $t \in [0, \tau]$ in the following way: $F_x(t) = 0, F_y(t) = -AMg, \mu(t) = 0$, where $0 \leq A \leq 2$ is an input parameter.

Example 1.

In this example the optimal lifting of the mass to a given height by straight HUE is investigated. We have the following statement of the optimal control problem.

It's required to determine the controlling torques at the HUE joints which transfer the human arm with given mass at its hand from initial equilibrium vertical down state to final horizontal state subject to given constraints

$$\begin{array}{lll} \alpha(t)=\beta(t)=\gamma(t), & x(t)=0, \quad y(t)=1 & \\ -\pi/3 \le \alpha(t) \le \pi, & 0.1T_0 \le \tau \le 4T_o, & t\in[0,\tau] \end{array} \tag{10}$$

with minimisation of the functional (6). In formulae (10) T_0 is the period of the natural oscillation of the straight human arm.

In Fig.2 the phase portraits of the obtained optimal motions of the straight human arm are shown for different values of the parameter A, which is equal to the ratio of the weight of the external load to the total weight of the HUE (solid curve corresponds to A=0.5, dashed curve - A=1, asterisk solid curve - A=1.5, asterisk dashed curve - A=2). It can be seen from Fig.2 that all obtained optimal motions of the straight arm comprise the reverse motion of the system.

Fig.3 shows the phase portraits for the optimal lifting of the load by straight arm in the case when the parameter A=1 and the duration of the control process τ is given in advance. These phase pictures correspond to different values of the dimensionless time τ (solid curve corresponds to τ=0.25, dashed curve - τ=0.5, asterisk solid curve - τ=1, asterisk dashed curve - τ=1.25). The analysis of the phase portraits depicted in Fig.3 shows that for different values of the parameter τ there are different types of the optimal motion of the human arm for the same working task. If the duration of the controlled process is less than a quarter of the period of the natural oscillation of the arm then the obtained optimal motion of the arm does not comprise the reverse motion (solid curve in Fig.3). If the time of the controlled process is more than a quarter of the period of the natural oscillation then the optimal motion of the arm is an oscillatory motion with a number of reverse motions.

Fig.4 shows the graphic dependence of the minimal dimensionless "energy cost" Φ_* with respect to the duration τ of the controlled process for the obtained optimal motions of the straight human arm (solid curve). It can be seen from Fig.4 that the energy expenditure reaches minimum when the time of the controlled process is equal to the period of the natural oscillation of the straight human arm.

Example 2.

In this example the optimal lifting of the mass is investigated under the assumption that the elbow angle of the human arm can be varied during the working task.

The position of the hand is defined by the polar coordinates (r,ψ) (See Fig.5). We have the following statement of the optimal control problem.

It's required to determine the controlling torques at the joints of the HUE which transfer the human arm with given mass at its hand from the initial for a given joint and a given subject [19-21]. Such approach is a cause [22] of a great variety of the stiffness coefficients of the elbow joint obtained by the researchers: from 0.59Nm/rad [21] to 260Nm/rad [19]. A disagreement between experimental results has led to a different approach to the stiffness concept [19-23].

In this paper the following approximate procedure has been used to estimate the stiffness characteristics of the HUE during its extremal motion.

Let $p_*(t), q_*(t), u_*(t)$ be the control torques for the optimal motion of the HUE at the shoulder, the elbow and the wrist joints, respectively. Consider the following problem.

It's required to determine the vector-parameters ξ_η, which minimise the functional

$$E = \int_0^{\tau} \left\{ [p_*(t) - w(t,\xi_p)]^2 + [q_*(t) - w(t,\xi_q)]^2 + [u_*(t) - w(t,\xi_u)]^2 \right\} dt, \tag{11}$$

where

$$w(t,\xi_\eta) = \begin{cases} C_{0\eta} + C_{1\eta}\alpha_*(t) + K_{1\eta}\dot{\alpha}_*(t), t \in [0, t_{1\eta}] \\ C_{2\eta}\alpha_*(t) + K_{2\eta}\dot{\alpha}_*(t), t \in [t_{1\eta}, t_{2\eta}] \\ C_{3\eta}\alpha_*(t) + K_{3\eta}\dot{\alpha}_*(t), t \in [t_{2\eta}, \tau] \end{cases}$$
$$\xi_\eta = \{C_{0\eta}, C_{j\eta}, K_{j\eta}, t_{1\eta}, t_{2\eta}; j = 1,2,3\}, 0 \le t_{1\eta} \le t_{2\eta} \le \tau \tag{12}$$
$$\eta = (p,q,u).$$

The vector-parameters ξ_η^* that minimise the functional (11) will determine the viscoelastic characteristics of the HUE joints corresponding to the controlled torques $p_*(t), q_*(t), u_*(t)$ and optimal motion $\dot{\alpha}_*(t)$.

The graphic dependencies of the viscoelastic approximation of the controlled torques for the obtained optimal controlled motion of the straight HUE at the wrist, the elbow and the shoulder joints are depicted in Figures 8,9,10, respectively. In these pictures the solid curves are the optimal torques for the $A = 1, \tau = 1$; dashed curves are the solutions of the approximation problem (11)-(12).

The analysis of the graphic dependencies in Fig. 8-10 shows that the obtained optimal control torques of the HUE can be constructed with sufficient accuracy by the linear viscoelastic controllers.

These controllers are specified by formulae (12) and the following data of the dimensionless variable parameters.

For the wrist joint's controller:

$$C_{0u}^{*} = 0.0068, C_{1u}^{*} = 0.1856, K_{1u}^{*} = 0.0045$$
$$C_{2u}^{*} = 0.1832, K_{2u}^{*} = 0.0020, C_{3u}^{*} = 0.1382 \qquad (13)$$
$$K_{3u}^{*} = 0.0039, t_{1u}^{*} = 0.39, t_{2u}^{*} = 0.86$$

For the elbow joint's controller:

$$C_{0q}^{*} = 0.0398, C_{1q}^{*} = 0.258, K_{1q}^{*} = 0.0249$$
$$C_{2q}^{*} = 0.272, K_{2q}^{*} = 0.0103, C_{3q}^{*} = 0.2756 \qquad (14)$$
$$K_{3q}^{*} = 0.0000, t_{1q}^{*} = 0.55, t_{2q}^{*} = 0.91$$

For the shoulder joint's controller:

$$C_{0p}^{*} = 0.1140, C_{1p}^{*} = 0.0000, K_{1p}^{*} = 0.0892$$
$$C_{2p}^{*} = 0.0000, K_{2p}^{*} = 0.0393, C_{3p}^{*} = 0.4984 \qquad (15)$$
$$K_{3p}^{*} = 0.0000, t_{1p}^{*} = 0.56, t_{2p}^{*} = 0.99$$

The analysis of the obtained numerical data (13)-(15) shows that the stiffness phenomenon of the joints of the human arm for the considered optimal controlled motion of HUE can not be described by the constant stiffness and damping coefficients. The stiffness curves have three characteristic parts corresponded to the time intervals: $t \in [0, t_{1\eta}], t \in [t_{1\eta}, t_{2\eta}], t \in [t_{2\eta}, \tau]$. This is confirmed by the experimental results [22].

Discussions and Conclusions

In this paper the analysis of the goal-directed controlled motions of a human arm is based on the solution of the optimal control problem for a plane three-link mechanical system. The performance index is the integral over a duration of the working task of the sum of the square of controlling stimuli acting at the joints of the human arm.

To solve the non-linear optimal control problem under given boundary conditions, restrictions on the phase coordinates and on the controlling stimuli, a parameter optimization approach based on the special Fourier and spline approximation of the variable functions has been proposed. Using this approach a number of optimal control problems for the HUE have been solved.

The analysis of the obtained numerical results shows that there is a strong interaction between the gravity forces and the internal controlling stimuli during the goal-directed motions of the HUE. This obvious statement is illustrated by a number of new results from the mathematical modelling of the extremal controlled motions of the human arm lifting a load in a vertical plane. For instance, as follows from Fig.4 (solid curve) the duration of the interaction between the gravity forces and the internal stimuli has a great influence on the energetic characteristics of the extremal motions of the straight HUE. The energy expenditure reaches a minimum when the duration of the controlled process is equal to the period of the natural oscillation of the straight human arm. We shall call this phenomenon "energetic resonance" of the straight human arm in the gravity field.

The obtained results of the mathematical modelling of the extremal motions of the considered mechanical system have demonstrated the important influence of the internal torque acting at the elbow joint on the quantitative and qualitative characteristics of the goal-directed motion of the HUE. The incorporation into the HUE of an additional degree of freedom in the elbow joint and the harmony of the interaction between internal stimuli acting at the shoulder, the elbow, the wrist joints and gravity forces makes it possible to obtain the kinematic advantage (disappear of the reverse motions of the hand, See Fig.6,7) and to reduce the energy expenditure of the goal-directed extreme motion of the human arm (See Fig.4, dashed curve).

In order to investigate the stiffness phenomenon of the HUE a special approximate procedure is proposed. This procedure made its possible to design the linear viscoelastic controllers of the shoulder, the elbow and the wrist joints that govern the HUE during the extremal lifting of the load at the hand in a vertical plane. The analysis of the designed controllers (formulae (12)-(15)) shows that within the framework of our assumptions the stiffness coefficients of the wrist and of the elbow controllers are of the same order. The damping coefficients of the elbow's controller are an order of magnitude larger than those of the wrist's controller. The damping coefficients of the shoulder's controller are approximately 3-4 times greater than those of the elbow's controller.

From the obtained numerical results it is also possible to conclude that for the considered extremal controlled motions of the HUE the stiffness curves of the designed controllers have three characteristic parts with different stiffness and damping coefficients. This is an example of variability of the stiffness characteristics of the living tissues.

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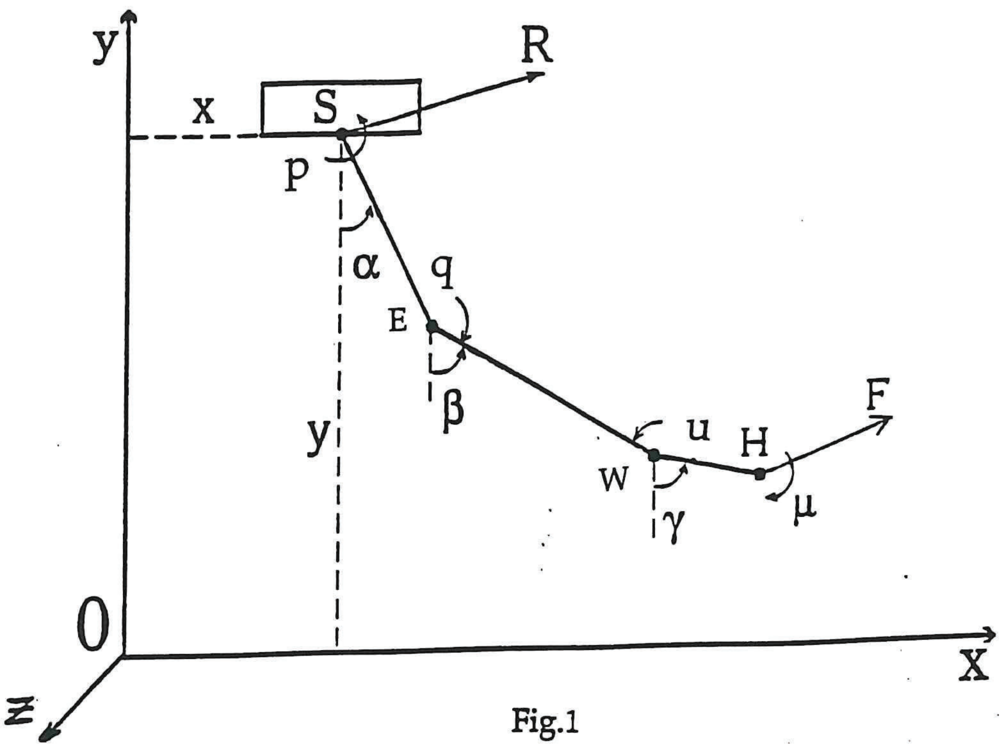
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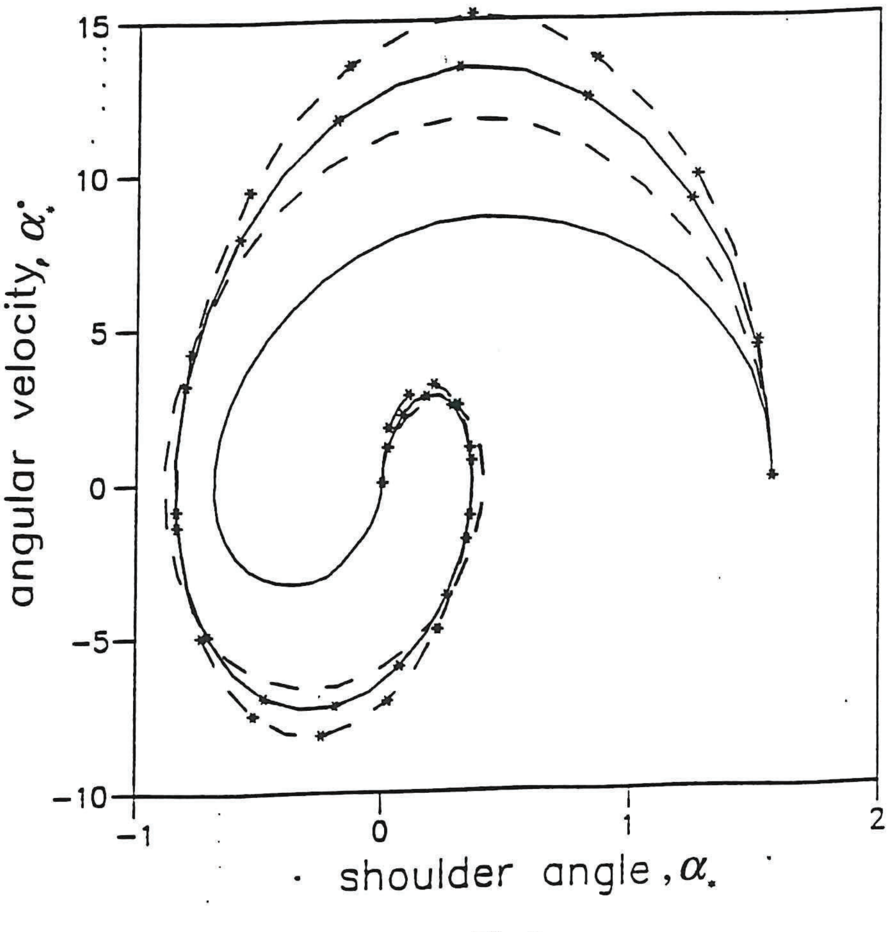
Fig.1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-5
-10
-1
0
1
2
angular velocity, α̇
shoulder angle, α

Fig.2

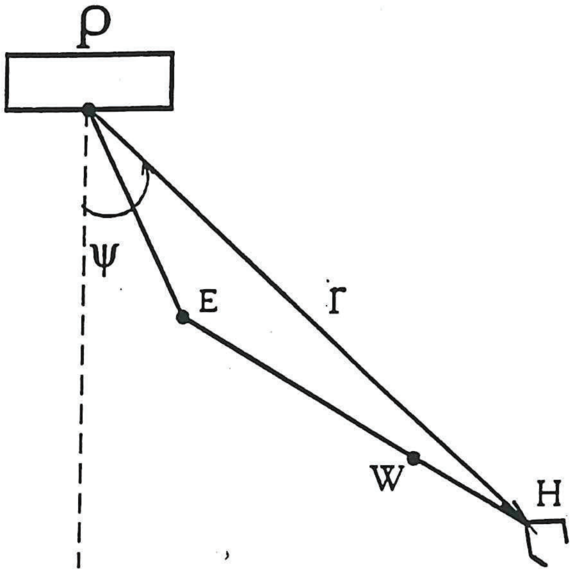

Fig.5

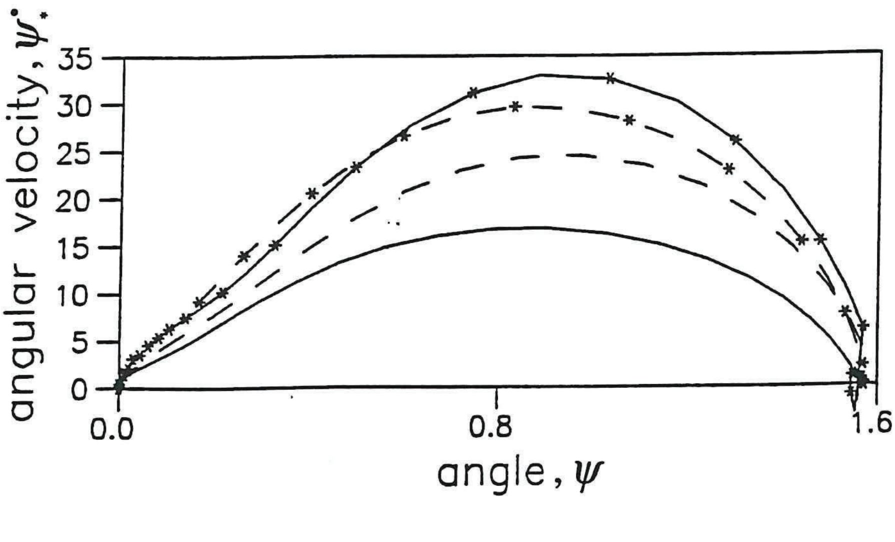

Fig.6

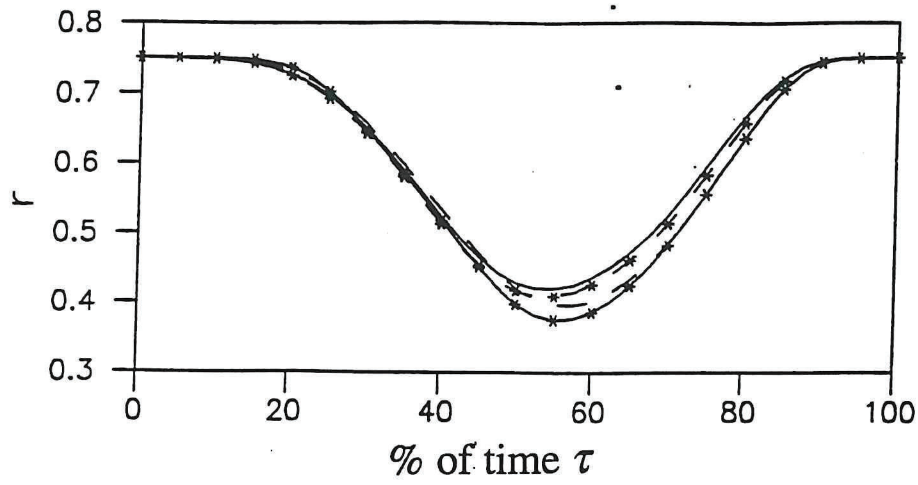
0.8
0.7
0.6
0.5
0.4
0.3
r
0
20
40
60
80
100
% of time τ

Fig.7

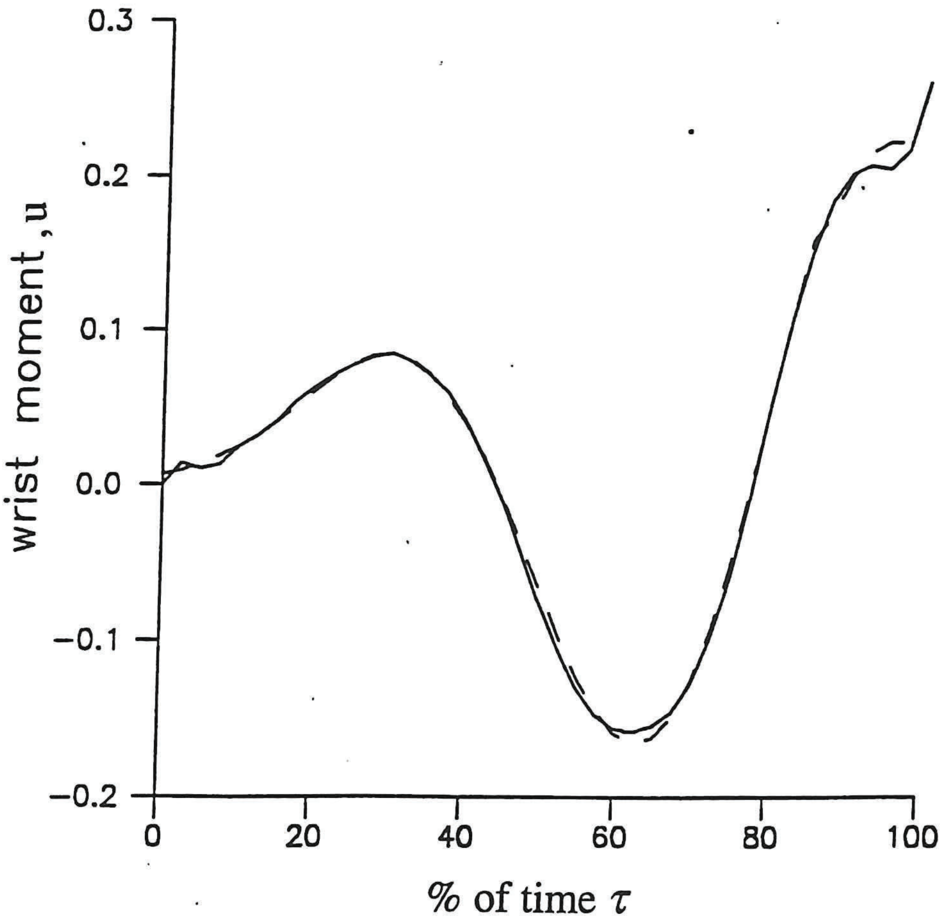
0.3
0.2
0.1
0.0
−0.1
−0.2
wrist moment, u
0
20
40
60
80
100
% of time τ

Fig.8

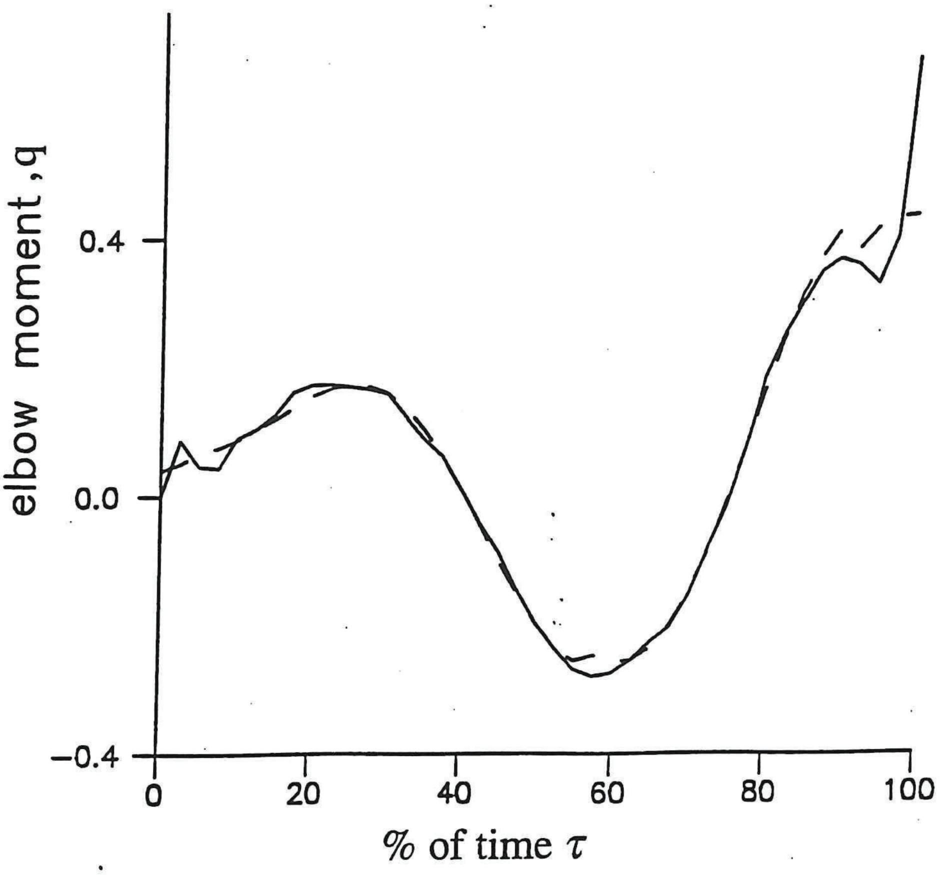
elbow moment , q
0.4
0.0
−0.4
0
20
40
60
80
100
% of time τ

Fig.9

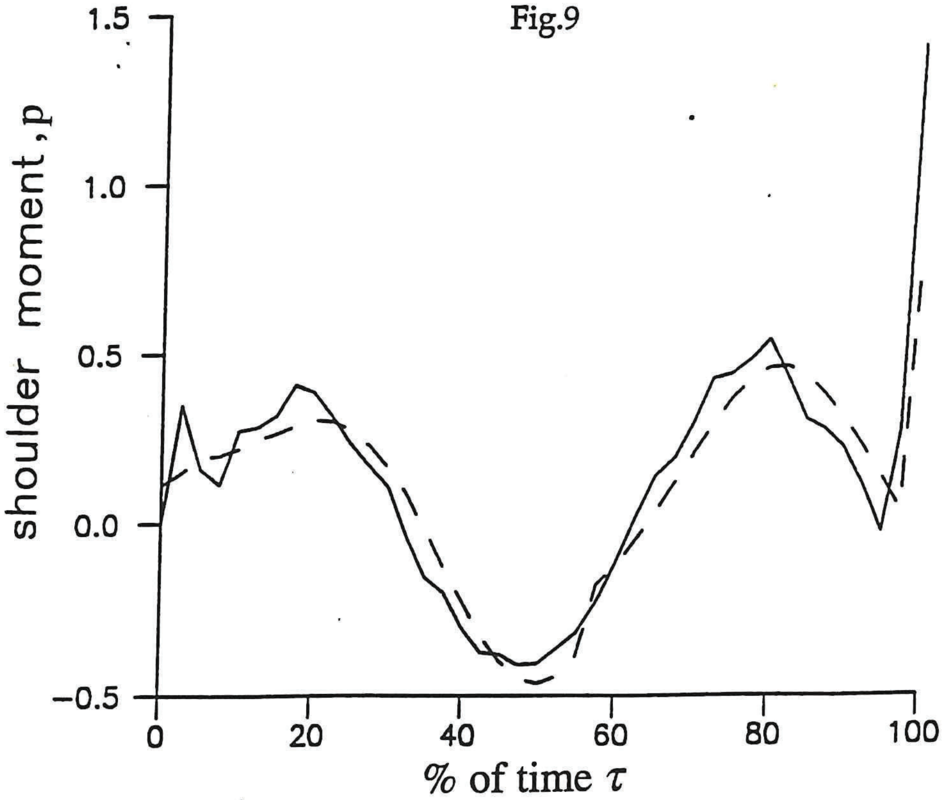
shoulder moment , p
1.5
1.0
0.5
0.0
−0.5
0
20
40
60
80
100
% of time τ

Fig.10