THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING EDUCATION

Mathematical Modelling and Problem Solving in Engineering Education

TABASSUM JAHAN

Department of Communication and Learning in Science

CHALMERS UNIVERSITY OF TECHNOLOGY

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Department of Communication and Learning in Science
Hörsalsvägen 2
Chalmers University of Technology
SE-412 96 Gothenburg
Sweden
Telephone + 46 (0)31-772 1000

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Social problem solving emphasizing guidance by the teacher
in developing metacognitive skills.
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Abstract

Background

Innovation and technology have made the 21st century engineering workplace problems more diverse and challenging. Mathematical modelling is being increasingly used as the primary form of engineering design and is fundamental in everyday engineering problem solving. The demand on engineering education is to prepare graduates to meet the challenges and needs of this rapid changing society. One of the keys to do so is to develop skills to be able to integrate knowledge from multiple domains and adapt to solving novel, complex problems in workplace. This means engineering education needs to create learning environments that are effective and mindful of the authentic practices of engineers.

Purpose

The goal of this thesis is to contribute to research efforts of improving engineering education, focusing on developing students' ability to solve mathematical modelling problems. In pursuit of this goal, this thesis examines an alternative learning design in a mathematical modelling and problem-solving course for engineers and understand how the learning design contributes to students' learning.

Scope/Method

The two empirical studies presented in this thesis employed a qualitative case study methodology. The case under investigation is a course in mathematical modelling and problem solving offered to undergraduate engineering students at Chalmers University of Technology. The first study aimed to understand how engineering students approach mathematical modelling problems early in the course and how the course impacts their learning. The second study aimed to contribute to the knowledge base of authentic learning by examining students' perceptions of different elements of authentic learning in the course.

Findings

The results show that students had little experience of mathematical modelling and solving realistic problems that lead to experiencing challenges early in the course. Many were unaware of the importance of understanding the problem and exploring alternatives which related to their lack of self-regulation or metacognitive skills and was impeded by different types of beliefs, attitude and expectations shaped by their prior experiences. The most important impact of the course was on students' metacognitive development. In the analysis of students' perception of this alternative learning environment, the results showed that students experienced elements of authentic learning in the course. Even though the tasks were not entirely 'real', the student experience them authentic and 'bought in'. Students engaged in deep reflective thinking in the course and presented several mechanisms of learning that linked elements of authentic learning and the course.

Conclusion

The findings in this thesis demonstrates the importance of self-regulation and beliefs in developing students' mathematical problem-solving abilities and exemplifies how the learning environment in the course contributed to developing students' mathematical modelling and realistic problem-solving skills as well as metacognitive skills. Furthermore, the thesis presents interesting outlook on students' perception of authenticity in the course's learning environment contributing to the knowledge base of authentic learning in engineering education. Finally, we recommend expanding this course's concept to other engineering programs and offer pointers to design courses that intend to provide authentic learning experience.

Keywords: Engineering education, Engineering education research, Mathematical modelling, Case study, Problem solving, Authenticity, Authentic learning, Metacognition, Self-regulation, Beliefs

List of publications

Appended papers

Paper A

Investigating and developing engineering students' mathematical modelling and problem-solving skills

Dag Wedelin, Tom Adawi, Tabassum Jahan & Sven Andersson

Published in *European Journal of Engineering Education 2015, 40(5), 557-572*

This paper builds on and is a further development of a conference paper that TJ presented at the CISPEE 2013 conference, where it won the best paper award.

Author contributions

All authors jointly conceived the initial idea for the study and contributed to the overall conceptualisation (RQs, aims, problem statement). TJ designed and carried out the data collection and analysis with guidance from the other co-authors. TJ wrote the initial draft under supervision from the co-authors. TA and DW drafted particular sections of the paper. TJ, TA and DW contributed to the review and revision of the manuscript.

Paper B

Student perceptions of authentic learning in a mathematical modelling and problem-solving course: a case study

Tabassum Jahan, Christian Stöhr & Dag Wedelin

To be submitted to *International Journal of STEM Education* in April 2020

Author contributions

TJ conceived and developed the idea of the paper with input from CS. TJ and CS contributed to the overall conceptualisation (RQs, aims, problem statement) of the study. The data collection and analysis were performed by TJ. TJ wrote the manuscript with consultation from CS. TJ, CS and DW contributed to the revision and completion of the final manuscript.

Other conference contributions

Teaching and learning mathematical modelling and problem solving: A case study

Dag Wedelin, Tom Adawi, Tabassum Jahan & Sven Andersson

Published In *2013 1st International Conference of the Portuguese Society for Engineering Education (CISPEE)* (p. 1-6)

Evaluating the design of a course in mathematical modelling and problem-solving from the students' perspective

Tabassum Jahan, Dag Wedelin & Tom Adawi

Published in *the 18th SEFI Mathematics Working Group seminar on Mathematics in Engineering Education* (p. 111)

Preface

I have always had this innate desire to explain what I know to others. Especially, if the subject is appearing to be difficult for someone and I happen to feel otherwise. Mathematics is one of those subjects that children dread. So did I, for some time in primary school. I did not understand the meaning of crunching numbers without knowing why. I have seen many students that I have helped with mathematics feel the same way. There is a chronic mathematics anxiety in school children, that lead to the fear of attending higher education related to science and technology, which heavily rely on mathematics. I believe this is because of the perception of mathematics that society and formal education provide. We have failed to give meaning and life to mathematics. The tedious textbook exercises, the time bound assessment and ineffective teaching is what I feel need to change.

Chalmers Läxhjalpsprojekt (homework help project) gave me the opportunity and freedom to reach out to school students here in Gothenburg and show them the harmony in mathematics. The project aimed at helping students who struggle with mathematics at school. I managed the project and designed the after school coaching sessions for several schools for about two years. The children who struggled the most were the ones who believed mathematics is a hard subject and they aren't smart enough. During my experience from the project and my previous experience as a high school teacher, I have never seen any connection between being smart and performing mathematical tasks. Children feel motivated and perform better when they find tasks meaningful and relevant. In the project, we taught children mathematics and its application, by means of playing, storytelling, solving puzzles, etc. Rarely did we resort to using the textbooks. It is such a wonderful feeling to watch children overcome their fear of math and start seeing it as language to express ideas and thoughts.

My curiosity in education research began there. While working with the children in the project, I realised that I needed to know more about learning and understanding mathematics to find new productive ways of guiding students to learn the language of mathematics. This desire to learn and teach lead me to the division of engineering education and this research project. I must admit that research in the field of education has been quite daunting, the traditions and the paradigms are very different than conventional scientific practice and research. It is a challenge, and I am taking small steps in learning and developing in this research arena every day. This thesis represents my efforts towards becoming the researcher I dream to become one day.

Contents

Chapter 1

Introduction

Nowadays we attempt to educate 21st century engineers with a 20th century curriculum taught in a 19th century institution

-Grasso and Burkins, 2010

The nature of engineering practise has drastically changed in the past 20 years. Introduction of innovative tools for computation, conceptualisation, design and communication has led to the requirement of different levels and types of mathematical understanding and abilities to successfully perform in the workplace (Lesh, Hamilton & Kaput, 2020). The ubiquity of computational based technologies in working practices has reduced the need to perform routine calculation and simultaneously increased the requirement for understanding, constructing, explaining and manipulating complex systems as well as analysing, interpreting and solving complex and nonroutine problems—all activities related to modelling (Gainsburg, 2006). This means engineering problems require diverse knowledge and skills beyond what is taught in the traditional classroom. It is currently expected that engineering programmes ensure that graduates not only have substantial understanding of essential principles in engineering science, mathematics and technology, but also the ability to apply that knowledge to solve problems and design systems in complex engineering settings (International Engineering Alliance, 2017).

Along with the widespread integration of innovative technology, sustainability and climate change is a major issue that concerns engineering education. The United Nation (UN) has formulated 17 sustainable development goals, in which engineering and technology are noted as vital in many of these goals, such as energy, employment and economic growth, responsible production and consumption as well as climate action (UNESCO, 2017). In order for engineering education to match the industry needs and sustainable development goals, increased interdisciplinary collaborations across programmes and disciplines are required. This is challenging for the education system since it involves integrating topics that are multi-disciplinary, containing theories from natural sciences, engineering and technology requiring mathematics, physics and information science (Hadgraft & Kolmos, 2020).

While engineering schools have embarked on the journey for educational reform to meet the demands of the 21st century, significant opportunity remains to address the integration of knowledge, skills and attitudes in developing engineering graduates for future practice. Traditionally engineering has not been taught in an integrative manner. The European education system continues to be dominated by the objectivist perspective (Arbaugh & Benbunan-Finch, 2006), whereby it is expected that students will be able to absorb the skills and knowledge recited by the teacher, and developing complex problem-solving abilities remain problematic (Nordstrom & Korpelainen, 2011). Nevertheless, there has been a progressive change, at least in theory and intent, from traditional teaching towards more student-centred pedagogies that create active and collaborative student engagement, promote deep and meaningful learning, as well as sustained knowledge and skill development (Biggs & Tang, 2007; Ramsden, 2002). Recent research literature shows that engineering education is giving importance to teaching integration of knowledge and skills combining theory with practice, otherwise called ‘design thinking’ in the classrooms. In the call for a design-based approach, Conceive-Design-Implement-Operate (CDIO) is one reform initiative to put ‘design thinking’ into practice that recognises the need to match engineering education to engineering practice (Crawley, 2001). With the introduction of new technology in the engineering field, mathematical modelling is increasingly being used as the primary form of design and include application, adaptation and creation of mathematical models. The CDIO syllabus version 2.0 includes complex problems solving and in terms of engineering reasoning and problem solving, as well as emphasises modelling (ibid).

Research investigating engineering practice also confirms that mathematical modelling is instrumental, and its use ranges from designing high-tech systems to basic representational activities (Lesh & Doerr, 2003). Packer (1998) contends that in the engineering field, the user of mathematics must understand the broader perspective of using and modifying mathematical systems, rather than knowledge about distinct pieces and components of the system that traditional mathematics classes teach. Mathematical modelling facilitates students in gaining a deeper comprehension of mathematical procedures and ideas to interpret, plan, reflect and perform nonroutine, complex problems in the workplace (Zawojewski, Deifes-Dux & Bowman, 2008). As a way to introduce authentic engineering problems in the classroom, scholars recommend introducing mathematical modelling to solve open-ended problems. Furthermore, practising modelling in the context of authentic problems encourages students to see mathematics as a tool to solve problems, which enhances their mathematical thinking and develops their identity as a professional user of mathematics (ibid; Boaler, 1999). The mathematics education community has been stressing the use of authentic context for teaching mathematics for decades. The urgency of mathematical literacy in the workplace, including mathematical models, is a worldwide concern (Hoyles, Wolf, Molyneux-Hodgson & Kent, 2002; De Lange, 1996).

All told, the research suggests the importance of mathematical modelling and its application in nonroutine problem solving for workplace performance, and the existing literature shows that the teaching of this knowledge and skills in the classroom is inadequate (Gainsburg, 2008). There is also a paucity of research in the area of problem solving and mathematical modelling in the context of developing engineering students' ability to perform authentic problems. The motivation for this research is to fill in that research gap with the intention of providing useful pointers for engineering educators to consider when designing courses, instructions and learning environments, that intends to meet the demands of the engineering practice. Hopefully, the outcome of this research will provide promising directions for the future research agenda related to the questions posed here.

1.1 Research aims and questions

The goal of this licentiate thesis is to contribute to the efforts of improving engineering education, specifically in the area of engineering problem solving, with the focus on developing students' ability to perform mathematical modelling problems. There is also a personal motivation associated with this goal. In my experience as a student and a teacher, I have noticed that mathematics courses tend to focus heavily on learning and training mathematical procedures and facts without a connection to real problems. Learning without a purpose and connection to practice often makes students feel mathematics is irrelevant, difficult and impacts their motivation to pursue further studies requiring mathematics.

In pursuit of this goal, this thesis aims to examine an alternative learning design in a mathematical modelling and problem-solving course for engineers and understand how learning design contributes to student learning. The two empirical studies in this thesis have been conducted in the context of this course. The purpose and rationale for the choice and description of the course are discussed in Chapter 4, Section 4.4.

Three research question have been invested in this thesis. In the first study (Paper A), the following research questions have been addressed:

RQ1: *How do engineering students approach mathematical modelling problems early in the course?*

This question was explored by investigating the challenges students experienced while solving the problems in the course and strategies they used to deal with those challenges.

RQ2: *How does the course impact students' learning to deal with such problems?*

This question was investigated by identifying the main impacts of the course and analysing the aspects of the course that contribute to those impacts. Both RQ1 and RQ2 were explored through students' self-reported reflections on the course.

Paper A showed us the uniqueness of the course's alternative learning environment and provided the motivation for designing the second study (Paper B). The research question that drove the investigation is:

RQ3: *How do engineering students experience different elements of authentic learning in the course?*

The research question was addressed by evaluating students' perceptions of the learning environment of this course in relation to the *authentic learning framework* (Herrington & Oliver, 2000) (the framework is discussed in Chapter 3).

As introduced earlier, the aim of this thesis is to examine an alternative learning design in a mathematical modelling and problem-solving course for engineers and gain an understanding of how the learning design contributed to students' learning. In order to do so, two empirical studies were conducted that are appended at the end of this thesis. The purpose of writing this thesis, also referred to as *the coat* in Sweden, is to discuss the findings of the studies to address the research goal and aim, as well as introduce relevant theories and past research to position my research in connection to them. The thesis is structured as follows:

Chapter 1: Introduction provides background and frames the purpose of this project followed by the presentation of the research questions.

Chapter 2: Overview of problem-solving and mathematical modelling research, provides the background literature review on past and recent literature, setting the stage for the rest of the thesis. This chapter draws heavily on the research in the field of Mathematics Education since much of the research in problem solving and mathematical modelling originates there.

Chapter 3: Theoretical frameworks and essential concepts introduce and discuss the theories, frameworks and concepts used in this research.

Chapter 4: Methodological considerations and method, discusses the methodology, approach and tools used in this research together with underlying philosophies and theoretical perspectives.

Chapter 5: Summary of the papers, provides a summary of the research papers appended in this thesis. The two articles are closely related; in fact, Paper B serves as a progression of Paper A.

Chapter 6: Concluding discussion integrates and discusses the research findings and their implication in teaching and learning, followed by the limitations and an outlook to future research.

Chapter 2

Overview of problem-solving and mathematical modelling research

When we mean to build
We must first survey the plot, then draw the model;
And when we see the figure of the house,
Then must we rate the cost of the erection,
Which if we find outweighs ability,
What do we then but draw anew the model
In fewer offices, or, at least, desist
To build at all?
-Shakespeare: Henry IV, pt 2

This chapter is intended to provide the background literature review and set the stage for the rest of the thesis. As alluded to in the 'Introduction', the primary goal of this thesis is to contribute to the efforts of researchers and educators in engineering education in developing engineering students' ability to perform mathematical modelling problems. For this purpose, it is helpful to provide a brief review of relevant literature in mathematical problem-solving research and mathematical modelling research as a basis of what follows. Section 2.1 begins with defining a problem and problem solving, followed by a brief literature review on problem-solving research that leads to mathematical modelling in Section 2.2. Section 2.3 discusses models and modelling and the purpose of their use in education. Finally, Section 2.4 discusses characteristics of engineering workplace problems and some recent approaches for preparing engineering students for the future workplace.

2.1 Problem and problem solving

The variety of definitions of a 'problem' has one key component in common; for a situation to be a problem, there must not be a straightforward solution path. Likewise, a situation is considered a problem when it challenges someone intellectually, and the method/procedure/algorithm to the solution is not readily evident (Blum & Niss, 1991). Tallman and Gray (1990) support this characterisation and define problems as *nonroutine events*. Also, the notion of a *problem* is somewhat subjective; it depends on the person addressing it. A problem to one person can be merely an exercise to another. As to *mathematical* problems, there are two kinds: a) applied mathematical problem—a problem where the situation defining it belongs to the 'real-world' although it may allow and involve mathematical procedures and concepts and, b) pure mathematical problem—a problem where the situation is entirely within the mathematical domain. An applied mathematical problem can have a purely mathematical problem embedded within, but if that problem is extracted out of the problem situation, it is no longer applied (ibid).

This thesis uses the definition by Lesh and Zawojewski (2007), who defines a mathematical problem as:

> A task, or goal-directed activity, becomes a problem (or problematic) when the "problem solver" (which may be a collaborating group of specialists) needs to develop a more productive way of thinking about the given situation (p. 782).

There is an element of 'modelling' involved in the above definition of a 'problem'. By 'productive way of thinking,' the authors mean that the individual engages in an iterative process, which is similar to 'modelling' (Section 2.3 discusses iterative process of modelling) in the context of mathematics. Following this definition, *problem-solving* refers to the process of approaching and attempting the problem in order to solve it. This process is often an iterative cycle of interpreting the problem mathematically that includes, but is not limited to: "expressing, testing and revising mathematical interpretations—and of sorting out, integrating, modifying, revising or refining clusters of mathematical concepts from various topics within and beyond mathematics" (Lesh et. al., 2007, p. 782).

2.2 A brief review of past research: from problem solving to modelling

Trends in research in applied mathematical problem-solving in the field of mathematics education have evolved tremendously in the last forty years (ibid). The main strands of research that stand out are a) *determinants of problem difficulty*, b) *individual problem-solving performance,* and c) *problem-solving instruction*. From the early/mid 1970s to the early 1980s, a significant amount of research on *determinants of problem difficulty* emerged, focusing on the types of problems solved in schools. Four classes of variables were identified that determine problem-solving difficulty, *content and context*, *structure, syntax*, and *heuristic behaviou*r variables. All these variables are inherent to the problem, and many of the research projects were to design school problems to test students' mathematical competence (Kilpatrick, 1985). This line of research was deemed inefficient since it only considered the problem solver's interpretation of the external factors of a problem and ignored the internal factors such as how individuals experience mathematical problems. Following this view, research focus shifted and moved to the investigation of the *interaction of these task variables* and the *characteristics of the problem solver* (ibid). In recent times, it has become widely accepted that problem difficulty and problem-solving performance do not depend as much on the various task variables as it does on the problem solver's characteristics and competence (Greeno, Collins & Resnick, 1996).

From the late 1970s until the mid-1980s problem-solving research concentrated on *individual problem-solving competence and performance*. Such as distinguishing 'weak methods' and 'strong methods' (Newell, 1980) or 'expert' and 'novice' problem solvers (Schoenfeld, 1982, 1985). Strong methods are the ones that are well organised, relevant and can solve a new problem efficiently. Problem solvers who use strong methods usually have good domain knowledge in comparison to those who use weak methods. Schoenfeld (1985, 1988) conducted a substantial amount of research in the same line and identified several important attributes that characterise 'experts' and 'novices'. For example, experts not only have good domain knowledge and *do* things differently, but they also *think* differently; experts focus more on the structural features of a problem whereas novices focus on surface features, furthermore experts are good at *controlling* and *monitoring* their activities in comparison to novices. *Control* and *self-monitoring* are two of the most important aspects of *metacognition,* that is, the ability to reflect on one's thinking and hence consciously monitor and control it. The notion of *metacognition* became a 'buzzword' in the 1980s and appeared increasingly over that decade. Lesh (1983), Schoenfeld (1982) and Silver (1982) were among the first to draw attention to metacognition as "the driving force" in problem solving. Later metacognition was also connected to other non-cognitive factors such as beliefs and attitude (see Section 3.5).

Research in *problem-solving instruction* began to gain importance and attention at that time. Lester (1985) identified that most problem-solving instruction design was being developed based on George Pólya's seminal work on the use of heuristic strategies to solve problems (Polya, 1945) rather than the vast array of research done in the area. His book *How to solve it* (ibid) discusses four basic principles of problem solving: 1) understand the problem, 2) devise a plan, 3) carry out the plan, and 4) look back, together with numerous heuristic strategies (e.g., draw a picture, divide the problem, look for similar problems) to address them. These principles together with some strategies were, and still are, frequently advocated as important abilities for students to develop. Later Schoenfeld (1985) developed a number of more specific and detailed strategies drawing from Pólya's general problem-solving principles. Although teaching such a descriptive list of what to do was generally deemed helpful, there has been concerns that it only provides support in 'doing' and carrying out procedures and rules, and not so much in "developing systems for interpreting and describing situation (i.e., "seeing mathematically")" (Lesh et al., 2007, p. 769).

Some of the critical outcomes of past problem-solving research that scholars and experts believe to have value in instruction design are: a) problem-solving ability develops by solving as many problems as possible, b) problem-solving ability develops over time, c) students must feel that their teachers believe that problem solving is beneficial for them in order to benefit from instruction, d) students benefit the most from structured problem-solving instructions, e) teaching students' problem-solving heuristics and strategies, makes minimal improvement in developing students' ability to solve a mathematical problem (Lesh & Doerr, 2003), f) metacognition and related disposition (e.g., beliefs and affects) have considerable influence in problem-solving ability, and g) when presented with novel problems, students apply knowledge and skills drawn from their familiar domains (Salomon & Perkins, 1989). Lesh et al. (2003) argue that these outcomes are necessary for designing effective problem-solving instructions but have not been entirely put to action.

Since the beginning of the 21st century, the use of innovative technology and computer tools in problem-solving, especially in the area of problem representation and idea generation, became prevalent. This called for a shift in the way problem-solving instructions have been carried out to a more student-centred approach in a socio-cultural context (Lesh et al., 2003). Still, interestingly in the last decade, there has been a noticeable decline in mathematical problem-solving research (ibid). Currently, it is more common for researchers to refer to previous literature than furthering research in this area. Concurrently research on applications and modelling began to amalgamate with problem solving in the field of mathematics education (Blum et al., 1991). This trend of unification in the fields was particularly visible in the International Congress on Mathematical Education (ICME) publications and discussions (Alsina et al., 1998; Fujita et al., 2004). The contribution in the area of applications and modelling in mathematical problem solving began to increase and become more and more prominent at the conferences of International Community of Teachers of Mathematical Modelling and Application (ICTMA) — a dedicated community of teachers, scholars and researchers who are concerned with research, teaching and practice of mathematical modelling (Haines, Galbraith, Blum & Khan, 2007; Stillman, Blum & Biembengut, 2015). One recent perspective on mathematical problem solving that has gained significant attention *is the model and modelling perspecti*ve (MMP) (Lesh et al., 2003). MMP recognises that problem-solvers who require heavy use of mathematics, for example, engineers and bankers, tend to organise their mathematical thinking around problem context, using, adapting and revising mathematical constructs and models that they have learnt in their professional education (e.g., university courses, internship, teachers). Hence, MMP in learning assumes that, in a problem-solving episode, a student engages in cycles of "expressing, testing, and revising their own mathematical ideas until their mathematical way of thinking meets goals and addresses the constraints of the given problem" (Zawojewski, Diefes-Dux & Bowman, 2008, p. 6), similar to the cyclical process of modelling (see the following section for a description of a modelling cycle). Moreover, they recommend students engage in modelling and design activities in their educational programmes to successfully perform applied mathematical problem solving in the workplace.

2.3 What is a mathematical model and modelling?

There is no homogenous definition of mathematical models and modelling in mathematics education research. The definitions vary depending on the author's theoretical perspective (Kaiser & Sriraman, 2006; Lingefjärd, 2006). Standard definitions tend to focus on the representational aspect of models, for example in technology literature a model is: "a representation of essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form" (Eykhoff, 1974, p. 1). Among the many, I find the following definition of mathematical models where models are viewed as both internal (in mind) and external (in representational media) constructs by Lesh et al. (2003) appropriate for this thesis:

> Models are conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation system, and that are used to construct, describe, or explain the behaviors of other system(s) – perhaps so that the other system can be manipulated or predicted intelligently. A mathematical model focuses on structural characteristics (rather than, for example, physical or musical characteristics) of the relevant systems. (p. 10)

Conceptual systems are ideas or concepts that are internally constructed and fundamentally social in nature. Borromeo Ferri (2006) refers to them as subjective mental representations that are naïve re-constructions of the situation. Conceptual systems or mental representations, when externally represented using different mathematical notations in a representational media, becomes a mathematical model.

Generally, mathematical modelling refers to the process of using mathematics to develop models to solve real-world problems. Research on mathematical modelling in mathematics education typically refers to a cycle that schematically illustrates how the modelling process takes place (Blum & Leiβ, 2007; Blomhøj & Jensen, 2003; Blum & Niss, 1991). One of the two prevailing general descriptions of mathematical modelling is called the modelling cycle. Figure 1 shows an example of one of the early illustrations of the modelling cycle described by Mason (1988). The first step of the process is to translate the real-world formulation in mathematics—which usually consists of defining variables and using equations to relate those. The mathematical problem is then worked on and perhaps solved. These steps can carry on for several rotations until the modeller is satisfied with the mathematical model. Lastly, the model is validated in the context of the real-world situation, using it to answer the question originally posed. It is possible to go through stages 1-7 in one go, but often the modelling process is not as straightforward, and usually, it requires several rotations to reach a realistic result. It is likely that after every iteration, the model becomes better at capturing the real-world situation. The modeller, at some point decides how well the model fits.

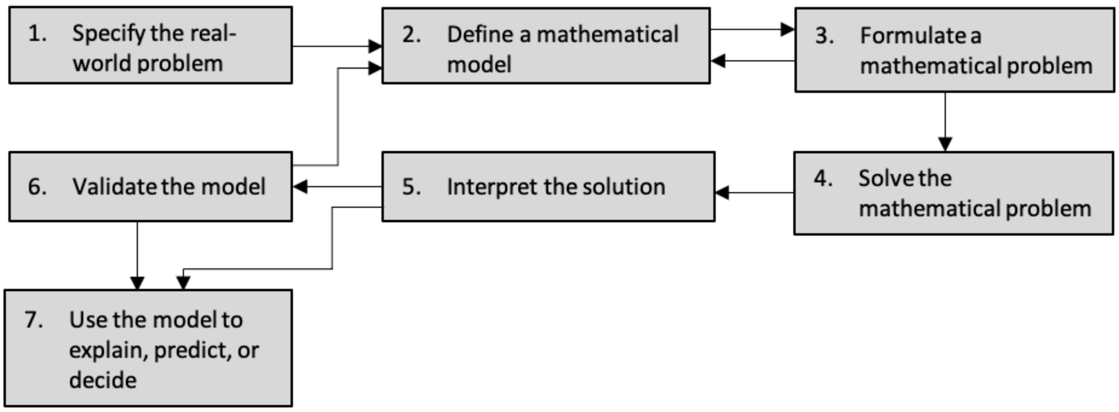

Figure 1 Main stages in modelling (adapted from Mason, 1988, p. 209)

In more recent modelling literature, the modelling cycle is commonly illustrated as two domains, the extra-mathematical world (reality domain) and the mathematical world (mathematical domain) (Blum, Galbraith & Niss, 2007). Figure 2 below illustrates a widely used modelling cycle by Blum and Leiβ (2007).

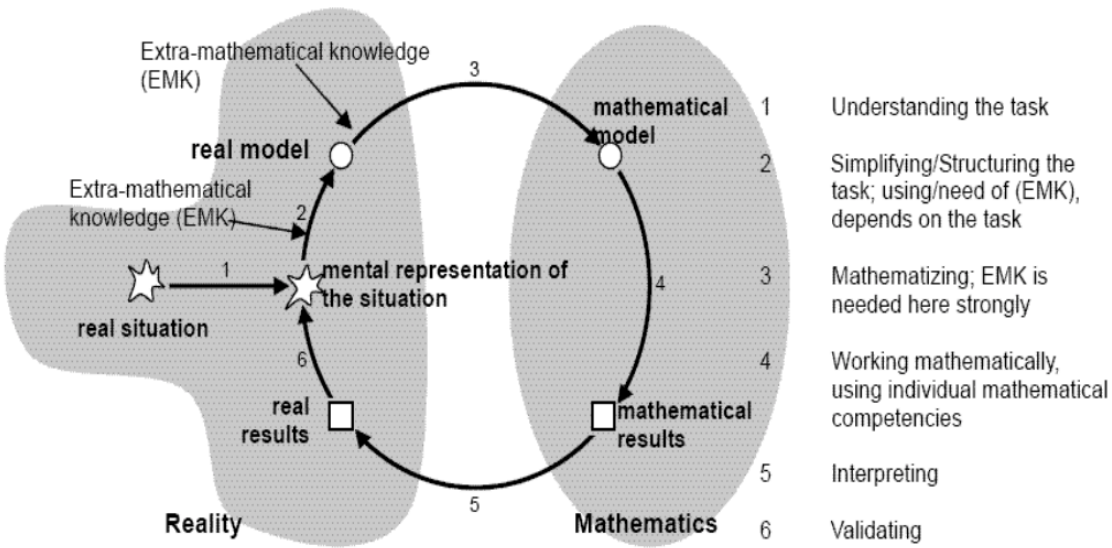

Figure 2 The modelling cycle by Blum and Leiβ (2007) as adapted and presented by Borromeo Ferri (2006, p. 92)

This particular cycle describes six phases, 1) real situation, 2) mental representation of the situation, 3) real model, 4) mathematical model, 5) mathematical results, and 6) real results, and six transitions between these phases. The *real situation* is the mathematical problem that resides in the reality domain. When this problem is understood, a mental model is created, which is then structured and simplified to a *real model.* The *real model* is then converted to a *mathematical model* by the process of mathematising. Note that, at this point, the domain has changed from reality to mathematics. The modeller then works mathematically on this model to generate mathematical results. Next, the mathematical results are interpreted into real results in the context of the initial problem by moving back to the reality domain and lastly; the model is validated in the context of the original situation. If the real results are not valid or do not answer the original problems satisfactorily, the modelling cycle is run through once again.

The other of the two general descriptions of mathematical modelling is the notion of *modelling competence* or *modelling competency*. Modelling competency often refers to, or draws on, a modelling cycle as a tool to assess a modeller's competence. Blomhøj and Højgaard Jensen in 2007 called *competence* the 'buzzword' of recent years in mathematics education and noted, "by mathematical modelling competence we mean being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context." (Blomhøj & Jensen, 2003, p. 126).

Mathematical modelling generally has two major purposes in the domain of education, one is to learn *modelling as a content* (Kaiser & Sriraman, 2006) and the other to use *modelling as a vehicle* to learn mathematics (Blum, Galbraith & Niss, 2007). *Modelling as a content* refers to learning to model for the purpose of gaining the knowledge of modelling (Julie & Mudaly, 2007), and *modelling as a vehicle* refers to doing modelling in order to better understand mathematics in a context (Niss, 2007). Apart from these two, at a general level, teaching students modelling in the context of problem solving in higher education has informed several important educational benefits, it helps students to a) solve complex problems that require higher order thinking (Lesh, Lester & Hjalmarson, 2003), b) learn application of mathematics (Galbraith, 2007; Zawojewski et al., 2008), c) link knowledge between different domains, increasing motivation and improving students' engagement with tasks (Haines et al., 2007), d) gain knowledge about different disciplines (e.g., mathematics and engineering) (Carpenter, Romberg & Carpenter, 2004), and e) learn reflective and metacognitive thinking (Lesh et al., 2003).

In a literature review on studies in teaching and learning mathematical modelling, Blomhøj noted (2009) seven purposes of using the modelling cycle in the educational setting:

1. Used for analysing a real-life practice or problem situation (Pollak, 1979).
2. Used in designing a meaningful context (Lesh & Doerr, 2003).
3. Used for designing and analysing modelling tasks with respect to particular intentions for the students' learning (Blum & Niss, 1991).
4. Used for defining mathematical modelling competency as a learning goal.
5. Emphasising mathematisation and the "model of – model for" transition (Treffers, 1987).
6. Used to structure the modelling process so as to identify the cognitive skills needed to model a given situation.
7. To structure the critique and reflections in relation to the modelling process and the application process.

The models and modelling perspective concentrates on using modelling as a vehicle for learning real-life problem solving and is pertinent to all the above listed benefits as well as additionally stressing using authentic engineering problems as modelling activity in classrooms. MMP uses "open-ended, realistic and client-driven problems that require the creation or adaptation of a mathematical model for a given solutions" called model-eliciting activities (MEA) (Zawojewski et al., 2008, p. 17). MEAs are team-based activities designed in a way that invokes students' conceptual development in acquiring, integrating, assimilating and applying engineering content. The focus is not only on the 'product' (i.e., the model) but also on the 'process' (i.e., how students are interpreting the situation, mathematising, and communicating with the team). This is precisely why they are called 'model-eliciting', because they require students to explicitly elicit the processes, they are involved in. The activities strive to create appropriate simulations of real-life/authentic modelling problems to foster students' abilities for success in the engineering workplace.

2.4 Engineering workplace problems

Research shows that engineering workplace problems are substantially different from traditional textbook and classroom problems (Regev, Gause & Wegmann, 2008; Jonassen, Strobel & Lee, 2006). Classroom problems are often routine, well-structured, and the solution process is usually known either by the teacher or can be found in the textbook. In contrast, workplace problems are nonroutine, ill-structured and more complex in nature with no straightforward route to the solution. In a study interviewing more than 100 professional engineers, Jonassen et al. (2006) described twelve characteristics of engineering workplace problems:

1. Workplace problems are ill-structured.
2. Ill-structured problems can include aggregates of well-structured problems.
3. Problems may have multiple, often conflicting goals.
4. Success is rarely measured by engineering standards (e.g., client's satisfaction can be a criterion).
5. Ill-structured problems can be solved in many different ways.
6. Most constraints in problem solving are non-engineering (e.g., budget limitations).
7. Problem-solving knowledge is distributed among team members (different team members bring in a different set of knowledge and skills that is shared).
8. Most problems require extensive collaboration (diverse nature of problems).
9. Engineers primarily rely on experiential knowledge (experience often determines expertise).
10. Engineering problem can often encounter unanticipated problems (e.g., human error).
11. Engineering problems can require multiple forms of problem representation (e.g., drawing mental representation on paper or using software application).
12. Problems require communication (most problems are team-based, requiring communication with teammates).

Similar renditions can be seen in a study about students' experience in a Requirement Engineering Education course, by Regev et al. (2008) who formulated a summary of attributes of workplace problems and classroom problems illustrating how they differ. Table 1 below illustrates the difference between classroom and workplace problems.

Table 1 Critical differences between classroom and work problems adapted from Regev, et al., 2008, p. 87

Experience	Classroom	Workplace
1. Problem definition	Well defined.	Ill-defined. Half of the challenge is just defining the problem. Often, in fact, a solution is implied by a mutually acceptable definition.
2. Problem approach	Strongly indicated by most recently presented classroom material. Problems tend to be carefully compartmentalised to reinforce specific methodologies.	Few hints as to how to approach the problem. In small companies, there will likely be no one to go to for help. You will, nearly always, be required to go beyond past studies and methods and may be required to invent new methods.
3. Problem solutions	Professor always knows the solution. If the problem is an odd numbered problem, the solution is in the back of the book.	A solution to the problem will only be apparent when it has been accepted by management.
4. Problem scope	Many problems are 'scoped' so that they can be solved by one person (student) in a few days or weeks.	The scope of the problem will not be recognised and you will be expected to produce the resources and time necessary to achieve the end result. In general, problems require a team of several people working over a period of many months.
5. Social environment	Working as an individual with implied competition.	Working as a team member, cooperation being essential.
6. Information levels	Accurate, well defined, explicitly stated.	Vague, unrecognisably and ambiguous. Occasional hidden agendas. Credibility of the source

		and timeliness of the information is always an issue.
7. Solution methods	Given by an authority figure, usually to reinforce material recently presented. Veracity and efficacy never an issue.	May have to invent a new method as part of the problem-solving process. Authority figure often projects his/her solution as the method of approach.
8. Design team	Same group members from beginning to end of project.	New members join the team and old, experienced members leave the team, sometimes at the worst possible times.
9. Stability of problem statement	Once stated, the problem statement is rarely, if ever changed.	The problem statement changes frequently, new information becomes available and new clients are brought into the picture.
10. Information channels	Heavy use of well-documented, written form.	Some documentation but much critical information is conveyed in ‘expedient’ verbal) sometimes, off-hand) forms such as one-on-one meetings, telephone and other informal conversations.

From the above table, it is evident that there is a stark difference between classroom problems and engineering workplace problems. Furthermore, research indicates that learning to solve well-structured problems do not necessarily support solving complex, ill-structured engineering problems in the workplace (Dunkle, Schraw & Bendixen, 1995). In order to better prepare students for workplace problems, Jonassen et al., (2006) provided a few guidelines for revising current educational practices to a constructivist learning environment, such as, providing an opportunity to solve ill-structured problems in teams, providing an opportunity for authentic workplace problem solving through internship and including problem-based learning (PBL) in the curricula, prompting students to self-monitored and self-directed learning. Similarly, Cardella (2008) emphasises the importance of understanding the characteristics of workplace problems that students will encounter and design the curriculum and instruction accordingly. Swan and Burkhardt (2014) noted that the purpose of problem-solving lessons “are not primarily about developing an understanding of mathematical ideas, but rather about students developing and comparing alternative mathematical approaches to nonroutine tasks for which students have not been previously prepared” (p. 14). With the intention of developing students’ ability to solve nonroutine real-world problems, Lesh et. al. (2003) developed model-eliciting activities which require students to interpret and mathematise an authentic problem provided by a fictional client. Since the problems are client driven with real-life context, the students learn to use mathematics, modelling and problem-solving skills in the context of workplace problems. MEAs have six design principles that closely aligns with attributes of engineering workplace problems:

1. Model Construction—involves constructing a model that is often a procedure for carrying out a task or design a product.
2. Reality—the problem is situated in an authentic engineering context.
3. Self-assessment—students work in a team to assess the usefulness of their model from the perspective of the client, their own experience in the context and problem situation.
4. Model documentation—the model should be documented by the team. Often it is in the form of a procedure description with a spreadsheet or computer programme.
5. Model shareability and re-usability—the model should be shareable (requires documentation), re-useable with other data set. Local generalisability of the model is expected.
6. Effective prototype—the model should be globally generalisable or modifiable. The model should facilitate the development of other models.

It is evident that problem solving, and mathematical modelling influence the way mathematics is used to solve problems, more specifically engineering workplace problems which differ significantly from classroom problems. Research also shows that engineering workplace problems are significantly different then classroom activities and problems, and knowledge about practices in these two areas are important to comprehensively design problem-solving activities and equivalent learning environments in educational setting (Jonassen, 1999).

Chapter 3

Theoretical frameworks and essential concepts

Without a framework we have to rely only on intuition, experience and common sense. This can take us far, and indeed it often does. But without a framework guiding our constructions or focusing our evaluation, we will never really know exactly what we are doing and why it failed, or why it worked so well.

-Lithner, 2008

Theoretical framework(s) are essential for all kinds of research projects. Lithner (2008) notes that theoretical frameworks help guide the research design and evaluate the research findings. Niss (2007) includes a few more roles of theory in the research process: *explanation*, *predictions*, *guidance for action or behaviour*, *a structured set of lenses, a safeguard against unscientific approaches,* and *protection against attacks*. This chapter discusses the theoretical frameworks and related concepts that have guided this thesis. The chapter commences with discussing *situated learning* which provides the basis for the frameworks and concepts that follow. Situated learning is the perspective has been consistent with the design of the course that was investigated in this thesis and guided the research interest. Section 3.2 discusses the *cognitive apprenticeship* model that supported the interpretation of the results in Paper A. Sections 3.3 and 3.4 discuss the concept of *authenticity* and the *authentic learning framework* that has been used as an analytical lens in Paper B. We chose to use the concept of authenticity for Paper B as a follow-up research perspective from Paper A and selected the *framework for authentic learning* (Herrington & Oliver, 2000) since it is reasonably extensive and well developed among the other frameworks reviewed. Lastly, Section 3.5 discusses *Schoenfeld's theory of exploring mathematical cognition* and the concept of *metacognition* and related dispositions that have been used in Paper A to support the findings.

3.1 Situated learning

The *situated* perspective of learning draws on the concepts of individual knowledge construction as well as the sociocultural context of it. It is a radical shift from the cognitive perspective of learning; here, learning is understood within the interaction in the sociocultural web rather than merely a change in mental structures. The core idea is that knowledge and learning cannot be separated from the context. It views human knowledge as "arising conceptually through dynamic construction and/or reinterpretation within a specific social context" (Johri, Aditya & Olds, 2011, p. 160). Hence, knowledge and learning exist within people and their environment, and the community they are part of (Greeno, Collins & Resnick, 1996). Among others, Brown, Collins and Duguid (1989) and Lave and Wenger (1991) have been instrumental in developing the ideas of situated perspective of learning into a theory.

Following this view, Collins (1988) defined *situated learning* as "the notion of learning knowledge and skills in contexts that reflect the way knowledge will be useful in real life" (p. 2). A 'context' can be people, machines, environments, artifacts and other objects or agents that create connections and interact with the learner and what is being learnt. It could also include shared culture, practices, understanding and motivation (Young, 1993).

Furthermore, learning happens when individuals participate in meaningful activities of a *community of practice* (Lave & Wenger, 1991). The concept of *community of practice* refers to a group of individuals who share a common concern and purpose who come together to explore, share and develop ideas to grow their practice. An example (taken from Lave (2011)) would be a group of tailor apprentices working together and learning under the supervision of a master tailor. Their common goal is to produce suits and garments to sell. With a common shared goal, the tailors contribute to the business based on their abilities. The community provides a relevant context where the participants can learn through observing and participating in that community's practices. In this respect, Jean Lave put forward the concept of *legitimate peripheral participation* (Lave & Wenger, 1991). *Legitimate peripheral participation* is a socialisation process through which a newcomer associates with the community. It begins with the newcomer observing from the 'boundary' then slowly, through meaningful participation, the newcomer's role changes from being an observer to a 'fully functioning agent'. From his experience as a beginner mathematician working with expert mathematicians, Schoenfeld (1987) makes the case that through meaningful participation, a novice mathematician can develop a mathematical point of view which is more than just a set of skills, but also the sense of what it means to be a mathematician:

> I remember discussing with some colleagues, early in our careers, what it was like to be a mathematician. Despite obvious individual differences, we had all developed what might be called the mathematician's point of view—a certain way of thinking about mathematics, of its value, of how it is done, etc. What we had picked up was much more than a set of skills; it was a way of viewing the world, and our work. We came to realise that we had undergone a process of acculturation, in which we had become members of, and had accepted the values of, a particular community. As the result of a protracted apprenticeship into mathematics, we had become mathematicians in a deep sense (by dint of world view) as well as by definition (what we were trained in, and did for a living). (p. 213)

Meaningful participation in practices as that of an apprentice is one of the premises of situated learning that Brown et al., (1989) extended to design a model of instruction called the *cognitive apprenticeship*. They supported their model with the argument that the traditional education system is contrived and focuses on activities that are far from the "ordinary practice of the culture" (ibid, p. 34), and "by ignoring the situated nature of cognition, education defeats its own goal of providing useable, robust knowledge" (Brown et al., 1989, p. 32). In their view, learning that takes place in a de-contextualised manner often remain 'inert' prohibiting learners to apply it later in real-life tasks. For meaningful learning to occur, learners should engage in authentic activities or activities that are related to the ones performed by practitioners in their daily work. The following Section 3.2 discusses the design of the model of *cognitive apprenticeship* that draws on these ideas.

Even though situated learning has been widely acclaimed, there has also been some debate and concerns around it. A common criticism has been that situated learning is not feasible for classroom application (Tripp, 1993), and that the abstract knowledge taught in the classrooms (commonly referring to lectures) is reasonably useful and effective as well as easily implemented in a classroom setting. However, the principal theorists of situated learning argue that this nascent theory with continued research has the potential to produce a model suitable for classroom application (Brown et al., 1989). The use of computer-based or media-based education is an example of that (McLellan, 1994). Herrington and Oliver (2000) assert that the reproval regarding the application of situated learning rise from naively comparing it to traditional apprenticeship, whereas the theory is much broader and considers apprenticeship at a cognitive level. For those who argue that classroom and computer-based representations are not authentic contexts, McLellan (1994) pointed out that context can be "1) the actual work setting; 2) a highly realistic or 'virtual' surrogate of actual work environment; or 3) an anchoring context such as a video or multimedia program, or even a subway ride or a bus ride—a 'real world' anchor" (p. 12) and provide a similar learning experience.

The issue of *transfer* has been another widely debated concern about situated learning (Anderson, Reder & Simon, 1996). The argument is that if all learning is context-dependent, how can learners apply this knowledge in a different context. The debate on transfer has been going on for decades. The cognitivist perspective in a nutshell claim, transfer between tasks happen if they are identical or overlap or at least share some degree of commonality (Lobato, 2006). Since from the situated perspective, knowledge is bounded in a situation, transferring that knowledge becomes problematic. There is no agreed consensus on the notion of transfer yet. Proponents of situated learning argue that the role of the teacher is important in this issue. If teachers "tune the attention" of the learner to important aspects of the situation that are invariant across similar situations that they might encounter in the future, transfer will take place (Young, 1993). Furthermore, for students to learn to use knowledge in novel situations, teachers need to assist students in working on novel situations before they can independently do so. So, the important question is not whether transfer happens but the connection between the amount of transfer and the amount of practice of a certain kind of tasks. This calls for more research in a situated educational setting to understand how that transfer takes place.

3.2 Cognitive apprenticeship

In *cognitive apprenticeship,* similar to a traditional apprenticeship, novices learn from experts through interactions focusing on completing an authentic task, except the focus here is on developing cognitive skills through actively participating in an authentic learning experience. The meaning of the term 'authentic' in this context is discussed in the following section. Collins, Brown and Newman (1989) defined *cognitive apprenticeship* as "learning-through-guided-experience on cognitive and metacognitive, rather than physical, skills and processes" (p. 486). The central tenet of cognitive apprenticeship is to help students think about, and process information and ideas the way scholars in the discipline do. The role of the teacher in such an environment discontinues being an information provider and instead adopts the role of a guide, scaffolder or problem/task presenter. The teacher creates an environment where students are more in charge of their own learning and have the opportunity to think and explore. This goes beyond declarative knowledge such as facts, concepts and procedures of a subject to teaching various types of strategic knowledge required for expert performance: *heuristic strategies* or "tricks of the trade"; *control strategies* or *metacognitive strategies*; and *learning strategies*. Since cognitive processes are not directly observable, one needs to make the thinking process explicit deliberately. There are six important aspects of cognitive apprenticeship: *modelling, coaching*, *scaffolding*, *articulation*, *reflection* and *exploration* (Collins & Kapur, 2006).

In *modelling*, teachers demonstrate their strategies for handling a certain task, making their tacit knowledge visible to the students. One technique is to use the 'think aloud protocol', where teachers explicitly verbalise, *what they are thinking and doing*, *why they are doing what they are doing*, as well as their *self-correcting* processes.

Coaching is the main thread connecting all the apprenticeship processes. The teacher's role is to be 'the guide on the side' and facilitate students to carry out a task by providing hints, bringing attention to un-noticed aspects of the task, evaluating activities, addressing weakness, encouraging and motivating them to explore. In short, overseeing and guiding students' learning.

Scaffolding involves the teacher supporting the students by executing parts of the task that the students cannot yet manage, critical to which is identifying the students' current level of difficulty and finding an appropriate intermediate step in carrying out the activity. As learners become increasingly capable, the teacher slowly fades away, giving the learner more and more responsibility.

While carrying out a task, it is also essential for the students to *articulate* their reasoning, knowledge and thinking process. By doing so, the students get to visualise and better understand their own as well as others' processes.

Reflection is "those intellectual and affective activities in which individuals engage to explore their experiences in order to lead to new understanding and appreciations" (Boud, Keogh & Walker, 1985, p. 19). It enables students to think back and evaluate their learning compared with others and perhaps take new and improved approaches.

Exploration is an important step that enables students to solve future problems independently. It involves the teacher pushing them to devise questions or problems that are interesting and that they are able to solve. As students learn to independently use this strategy, teachers slowly reduce their involvement. It is a natural way of fading support.

The cognitive apprenticeship model does not provide a specific formula to design teaching or learning environments. It scaffolds student centred activities, independent and self-directed, where students can visualise expert performance, challenge their solutions and eventually transform their roles. It is important that students are actively engaged in solving authentic problems within a realistic instructional context (Young, 1993).

3.3 Authenticity

The term *authentic* in engineering and mathematics education research literature is often used synonymously with 'realistic', 'real-life' and 'genuine'. Young (1993) contends that in order to be *'authentic'*, tasks and situations must have at least some of the important attributes of real-life tasks and situations. Vos (2015) notes that this rendition of the term authenticity implies 'imitation' or 'simulation' of real-world activity, which is essentially a copy but not the real thing. Hence, using the term in such a manner might complicate the operationalisation of it.

Lesh and Lamon (1992) recognise this ambiguity and offer the following definition of authenticity in the domain of mathematics education:

> Authentic mathematical activities are actual work samples taken from a representative collection of activities that are meaningful and important in their own right. They are not just surrogates for mathematical activities that are important in "real-life" situations". (p. 17)

Vos (2015) takes this understanding of authenticity further and argues that authenticity is a social construct that is agreed upon between the involved parties, that is, the teacher and the students. According to her, *authenticity* can be assigned to a task or activity if it has an 'out of school' origin and has been certified by a knowledgeable expert in the area. In the context of engineering education, an example would be incorporating a modelling activity in a task that has an engineering workplace origin and has been certified by researchers in the area of mathematical modelling. Petraglia (1998) puts the perspective of the learner as another decisive factor for authenticity. He notes that:

> "authenticity is not an intrinsic property possessed by an object, but rather a *judgement*, a decision made on the part of the learner constrained by the sociocultural matrix within which he or she operates" (p. 100).

In this respect, Honebein, Duffy and Fishman (1993) contend that authenticity is a subjective concept and asserts the importance of considering learners' perception in order to achieve its benefits. It might as well be a downsized authentic activity that is easily implemented in a classroom setting and offers the students an experience the way professionals encounter in the real world.

Authentic learning is not a learning theory nor a pedagogical approach but rather a philosophy that originated from the ideas of the situated perspective and apprenticeship model. The term is commonly used to refer to learning that happens in a situated environment, which aligns learning objectives with real-life tasks, content and context.

As the trend in engineering education shifted towards promoting the development of more professional skills and competencies, classroom instruction and design took a turn to incorporate authentic learning (Jonassen, 1991; Strobel, Wang, Weber & Dyehouse, 2013). David Jonassen is one of the notable researchers who has been persistent in the quest for authenticity in engineering education. His research mainly includes ill-structured problem solving, workplace problem solving (Jonassen et. al., 2006), designing effective problem-solving environments (Jonassen, 1999) with an undying passion for fighting traditional teaching methods (Jonassen, 1991) and introducing the complexity of real life in classrooms. Another design model that made its way from mathematics educations that has been particularly developed to provide an authentic learning experience is *model-eliciting activities* (MEAs). The activities strive to create appropriate simulations of real-life modelling problems to foster students' abilities for success beyond school. They are designed in a way that invokes students' conceptual development in acquiring, integrating, assimilating and applying engineering content.

In recent years, the concept of authentic learning has been included in developing many learning environments. For example, the inclusion of the problem-based learning (PBL) approach using "real-world" problems/projects that include concepts relevant to the subject domain and require expert like activities (Savery & Duffy, 1995). Even though PBL first originated in the context of clinical education, it is now widely used in engineering education and other domains.

Models to conceptualise authenticity in education are prevalent. Strobel et al. (2013) devised one model to conceptualise different dimensions of authenticity in engineering education. Their model constitutes of four dimensions of authenticity gathered through a systematic literature review of studies related to *authenticity* in the field of engineering education research. The dimensions are:

1. Context authenticity—context or situation is cultivated from or resembles real life.
2. Task authenticity—tasks or activities that resemble real-life activities.
3. Impact authenticity—output or solution to the tasks and activities have a purpose outside of school.
4. Personal/Value authenticity—tasks have connections to personal life, and they satisfy students' personal needs.

The model was designed with the intention to inform pre-university curriculum design. It is quite broad and does not provide any practical instructions to design learning environments. The authors acknowledge the elusiveness of authenticity in their model and call for more research in the area in order to make these dimensions of authenticity operationalisable for educational purposes.

3.4 Authentic Learning Framework

In order to operationalise situated learning into practice, Herrington and Oliver (2000) developed a framework called the *authentic learning framework*. As alluded to in the above sections, the merits of authentic learning are based on the premises of situated learning theory. The challenge is to identify those critical aspects of the theory and inculcate it for classroom application. Herrington and Oliver (2000) distilled those essential aspects of situated learning through critical reading of the work of the principal theorists and compiled nine elements to produce a framework for authentic learning environments. Table 2 below lists the nine elements with corresponding guidelines for implementation adapted from Herrington and Oliver (2000). The authors used this framework to design and implement a multimedia programme for pre-service mathematics teachers. They also studied the experience of the students attending this innovative programme. The findings suggested that the framework provided effective instructional design guidelines for designing an authentic learning environment for the acquisition of advance knowledge. In Paper B, this framework has been used as an analytical lens to understand students' perceptions about an authentic learning environment in a course.

Table 2 Elements of authentic learning framework with corresponding guidelines for implementation (Herrington & Oliver, 2000, p. 30)

Elements of situated learning	Guidelines for design and implementation of learning environment
1. Authentic context that reflects the way the knowledge will be used in real life	• a physical environment reflecting real use • a non-linear design • adequate number of resources • no attempt to simplify
2. Provide authentic activities	• activities that have real-world relevance • ill-defined activities • an opportunity for students to define the tasks • a sustained period of time for investigation • the opportunity to detect relevant information • the opportunity to collaborate • tasks that can be integrated across subject areas
3. Access to expert performances and the modelling of processes	• access to expert thinking and modelling processes • access to learners in various levels of expertise • sharing of stories • access to the social periphery
4. Provide multiple roles and perspectives	• different perspectives on the topics from various points of view • the opportunity to express different points of view • the opportunity to crisscross the learning environment
5. Collaborative construction of knowledge	• tasks that are addressed to a group rather than an individual • classroom organisation into pairs or small groups • opportunity for learners to compare with experts
6. Reflection to enable abstractions to be formed	• opportunity for learners to compare with other learners • collaborative groupings of students
7. Articulation to enable tacit knowledge to be made explicit	• a complex task incorporating inherent opportunities to articulate • groups to enable articulation • publicly present argument to enable defence of learning

8. Coaching and scaffolding at critical times	• a complex, open-ended learning environment • non-linear design • guidelines for the use of the programme in a variety of contexts • collaborative learning • recommendations that the lecturer is available for coaching
9. Authentic assessment of learning within the tasks.	• fidelity of context • the opportunity for students to craft polished, performances or products • significant student time and effort in collaboration • complex, ill-structured challenges • assessment seamlessly integrated with the activity • multiple indicators of learning • validity and reliability with appropriate criteria for scoring varied products

In addition to the theory and concepts already discussed, the next section discusses a framework for *mathematical thinking* in the context of mathematical problem solving followed by a discussion of *Metacognition* and related dispositions.

3.5 Schoenfeld's framework for mathematical thinking

As alluded to in the previous chapter, problem-solving research in mathematics education in the 1980s and 1990s showed that teaching abstract mathematical concepts and problem-solving heuristics deductively does not necessarily improve students' problem-solving abilities. In connection to that, research literature strongly accentuates teaching applied problem solving exclusively and using it as a vehicle for learning mathematics (e.g., Lester & Lambdin, 2004).

To understand and improve students' problem-solving abilities, Schoenfeld did substantial research on *how experts and novices* perform problem solving. Accumulating the results of the research, he claimed that an individual's mathematical behaviour could be determined by four factors, 1) *resources,* 2) *heuristics,* 3) *monitoring* and *control,* and 4) *beliefs*. According to his theory, these four factors dictate a person's success or failure of problem-solving attempts (Schoenfeld, 1985).

The term *resource* is used to refer to the repertoire of mathematical content knowledge such as theorems, algorithms and different kinds of models; basically, the mathematical 'tools' one requires to solve a particular problem. Later in 1992, Schoenfeld re-named this term as *knowledge base. Heuristics* describes the literature of mathematical problem-solving strategies. George Polya's (1945) problem-solving strategies in his famous book, *How to solve it* is a good starting point. Examples of some strategies Schoenfeld (1992) has derived from Pólya's problems solving heuristics are *working back*, *searching analogies, decomposing* and *recombining*. *Monitoring* and *control* refer to how students manage and utilise their resources and their knowledge of heuristics. They are both closely related to *metacognition and self-regulation* (Schoenfeld, 1987). Schoenfeld hails metacognition as the most important 'driving force' for problem solving. Lastly, one's *belief system* consists of one's internal thoughts, feeling and expectations about oneself, mathematics or problem solving, which affects one's abilities of problem solving and utilising metacognitive strategies. He highlighted the impact of individuals' belief systems on allocating and utilising resources (Schoenfeld, 1985). For example, despite having enough knowledge base, students often fail to solve problems due to negative *self-belief* (e.g., not being smart enough). A similar conclusion was drawn by Zan (2000), who analysed the problem-solving performance of university students and observed that students' "anti-mathematical" beliefs (e.g., every problem has one correct solution) hindered their problem-solving process even though they had appropriate *resources* and *heuristics* at their disposal.

Much of Schoenfeld's work in developing the theory has been influenced by George Pólya's (1945) problem-solving research in cognitive science and social constructivism. Schoenfeld's views extended from a purely cognitive sphere to a social sphere. His interest began in understanding students' cognitive processes and short-term and long memory utilisation when they are involved in problem-solving, which moved to how classroom activities and teachers' facilitation affects students' mathematical thinking and problem solving (Schoenfeld, 1992).

> Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns — systematic attempts, based on observation, study and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically ("pure mathematics") or models of systems abstracted from real world objects ("applied mathematics") (Schoenfeld, 2016, p. 3).

Schoenfeld employs several elements of cognitive apprenticeship in this teaching (Collins, Brown & Newman, 1988). For example, modelling experts' problem-solving processes in the classroom where students collaborate in small groups and Schoenfeld facilitates their work as a 'consultant'. For Schoenfeld, the main objective of a mathematics faculty is to teach the students how to think. Mathematical skills cannot just be transferred, they must be coached to the students so that they can construct the necessary knowledge by themselves. Schoenfeld advocates social problem-solving sessions, where groups solve problems 'semi-independently' facilitated by the teacher. Furthermore, discussions are encouraged so that students' metacognitive actions and internal beliefs come forth and perhaps seeing other students' struggle may alleviate some of their difficulties and insecurities (ibid).

Metacognition, beliefs and attitude

Metacognition, or 'thinking about thinking', consists of two separate but related aspects: a) knowledge and beliefs about cognitive phenomena, and b) the regulation and control of cognitive actions (Garofalo & Lester, 1985). Flavell's (1976) definition of the term includes these two facets:

> Metacognition" refers to one's knowledge concerning one's own cognitive processes and products or anything related to them (e.g., the learning-relevant properties of information or data). Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective (p. 232).

Although there are other terms for (e.g., reflective intelligence) for and descriptions of metacognition, one common denominator is the separation of metacognition and cognition in a hierarchical manner. Schoenfeld (1992) demonstrates this hierarchical nature and refers to metacognition as "executive control" (p. 355) that enables problem solvers to divide a complex task into smaller sub-tasks, prioritise them and then attempt them accordingly. In other words, metacognition helps in *planning what to,* and cognitions helps in *doing*.

In general, the first aspect of metacognition (i.e., *knowledge and belief)* concerns what an individual knows about cognitive abilities, processes and resources and the second aspect (i.e., *regulation and control)* concerns the variety of decisions and strategic activities that an individual engages in during a cognitive task (Garofalo & Lester, 1985).

An individual's belief system also plays an important role in metacognition. In the field of mathematics education, there exist many different variations of the concept of belief (e.g., Mcleod, 1989; Schoenfeld, 1985, 1992), although most refer to it as *subjective knowledge* that influences cognitive behaviour. Schoenfeld noted belief system as "individual's understandings and feeling that shape the ways that the individual conceptualises and engages in mathematical behaviour" (Schoenfeld, 1992, p. 358). Similarly, Goldin (2002) defines beliefs as internal representations that an individual attributes as true, and validate information based on what he or she believes.

Connected to beliefs is the notion of *attitude*. Attitude is defined as a behavioural disposition that, when acquired, affects the cognitive feeling, which in turn influences an individual's behaviour (Eagly et al., 1993). McLeod (1989) brings beliefs, attitude and emotions coupled together as the *affective domain* and noted that "when students work on non-routine problems, their effective responses are more intense; we see more evidence of emotions and the influence of attitudes and beliefs". In line with Schoenfeld, Mcleod stresses that the *affective domain* has "extraordinarily powerful consequences" on how students' mathematical practices are shaped (Mcleod, 1989, p. 359). Attitude can be both positive and negative. When referred to behaviour, a 'positive' attitude generally leads to being 'successful' in an endeavour. One example could be, having a confident approach while problem solving may lead to a successful outcome. In contrast, a negative attitude is often expressed in feelings such as frustration or feeling insecure which can lead to failure.

Despite long research in metacognition in the field of mathematics education, what invokes metacognitive behaviour and how metacognitive skills develop is not fully understood. However, research has shown that metacognition and related dispositions (beliefs, attitude and emotions) have a significant effect on productive learning outcome and problem-solving persona (Stein et al., 2003; Lesh, Lester & Hjalmarson, 2003). There is an indication that metacognitive skills develop along with successfully performing problem solving and are affected by social interaction during the problem-solving process (Lesh et al., 2003).

Chapter 4

Methodological considerations and method

I think; therefore, I am.
-Descartes

The purpose of this chapter is to discuss the methodology, approach and tools used in this research. In doing so, I will also explore some of the underlying philosophies and theoretical perspectives. I will try to portray the separation between methodology and methods and inform the philosophical beliefs that influence them.

Ernest (1998) describes methodology as “a theory of methods – the underlying theoretical framework and the set of epistemological (and ontological) assumptions that determine a way of viewing the world and, hence, underpin the choice of research methods (p. 35)”. Consequently, the choice of methods is linked to research methodology, which in turn is influenced by the researcher’s theoretical perspective governed by the researcher’s philosophical orientation (Figure 3 illustrates this relationship (adapted from Gray, 2013)). The latter may not always be explicit but implicitly shapes and orients the former choices.

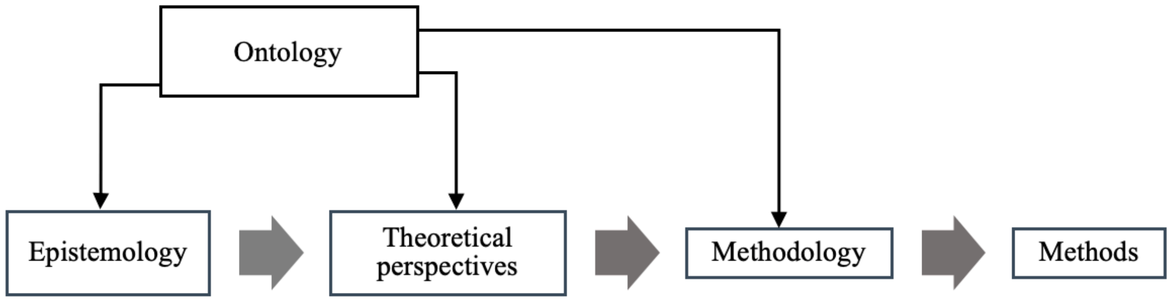

Figure 3 Relationship between ontology, epistemology, theoretical perspective, methodology and research method

This thesis aims to examine an alternative learning design in a mathematical modelling and problem-solving course for engineers and understand how the learning design contributes to student learning. The hope is that the implication of this research will help engineering teachers, programme developers and engineering education researchers to better understand students’ thinking around learning and the learning environment, and subsequently develop appropriate classroom practices. The research methodology and methods are aligned to the aim and the questions this research posits. For the purpose of clarity, I would like to revisit the research questions below:

RQ1: How do engineering students approach mathematical modelling problems early in the course?
RQ2: How does the course impact students’ learning to deal with such problems?
RQ3: How do engineering students experience different elements of authentic learning in the course?

4.1 Philosophical foundations

This section briefly discusses the philosophical grounds. Gray defines *ontology* as “the study of being, that is, the nature of existence”, this constitutes the understanding of *what is,* whereas *epistemology* “tries to understand *what it means to know”,* it deals with the nature of knowledge and what it means to know *(Gray, 2013, p. 35)*. In my naïve words, ontology refers to what kinds of things exist in this social world and epistemology deals with what we can know and ways of knowing it.

Broadly speaking, western philosophy discusses two opposing ontological positions that Chia (2002) labels as the ontology of *being* and the ontology of *becoming*. The *being* ontology emphasises the permanent and unchanging reality. It claims that there is an external reality independent of human consciousness. A reality that consists of ‘clear cut’ entities that have identifiable properties and that truth about this reality can be discovered. This ontology has a major influence on western philosophy. *Objectivism* has its roots in this ontology. It claims that an objective reality exists ‘out there’. The theoretical perspective *positivism* is associated with objectivism, which argues that the independent reality must be investigated through scientific inquiry (Gray, 2013).

The *becoming* ontology emphasises the changing and emerging reality. It underlines the chaos, formlessness and fragmented aspect of it (Chia, 2002). It claims that reality can only be understood as a part of human consciousness, and the object’s meaning is invented by the investigator. *Subjectivism* is linked to the *becoming* ontology. Subjectivisms claims that meaning is enforced by the subject on the object. *Postmodernism* is a theoretical perspective associated with subjectivism, which claims that there is no objective reality, meaning is created by the investigator’s subjective beliefs (Gray, 2013).

On the other hand, *constructivism* gives the possibility of having multiple realities when approaching the world. It takes the view that truth and meaning do not reside in the same reality and meaning is individually constructed, not discovered. Even though constructivism and objectivism have different foundations, they originate from the same *being* ontology.

Interpretivism (the theoretical perspective chosen for my research) is linked to constructivism. It holds the view that knowledge is neither discovered nor invented but socially constructed. As researchers, we are seeking to understand these socially derived interpretations of the world. This view contrasts with the philosophy of positivism. Gray (2013) contends that positivism/post-positivism and interpretivism are the most influential theoretical perspectives available. Figure 4 shows different epistemologies and their related theoretical perspectives with some examples of methodologies and methods. The following section discusses these two perspectives, and hopefully, the choice for interpretivism will be more evident.

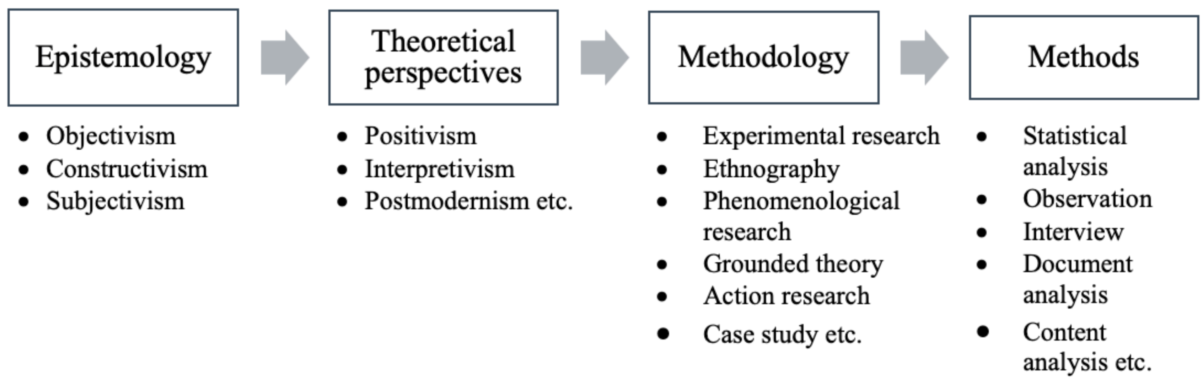

Figure 4 Depiction of the different epistemologies and their related theoretical perspectives with some examples of methodologies and methods

4.2 Theoretical Perspectives

The purpose of a theoretical perspective is to aid and promote the generation of knowledge; it shapes our way of doing research. As alluded to in the above section, the choice of theoretical perspective is based on the investigator's basic beliefs about the nature of reality, both epistemologically and ontologically. Several theoretical perspectives exist within the field of education that are referred to in numerous ways by different scholars, for example, *paradigm of inquiry* (Denzin & Lincoln, 1994, Kuhn 1962), *theoretical traditions* (Patton, 2002), or simply *theoretical perspectives* (Crotty & Crotty, 1998). For decades, a significant amount of work has been published by different scholars from varied disciplines around these classifications, some of which are overlapping. Guba and Lincoln (1994), in their seminal work describe four alternative paradigms of inquiry: positivism, post-positivism, constructivism and critical theory. Table 3 shows examples of emerging theoretical perspectives and their general characteristics within the field of engineering education (adapted from Koro-Ljungberg and Douglas, 2008). The discussion is focused mainly on interpretivism since it is the perspective taken in this thesis and as a contrast positivism/post-positivism is used.

Positivism has dominated the research activity in physical and social sciences for hundreds of years (Guba & Lincoln, 1994). It holds the objective view of reality, and the goal is to perform scientific examination and studies of evidence to discover the truth about that reality. The aim of inquiry from this perspective is to be able to predict and control a phenomenon. Post-positivism shares the same view but also recognises the limitations on the certainty of that knowledge. Commonly, post-positivistic research is used to test a hypothesis and understand cause and effect relationships between variables (Creswell, 2007). Effective research in this paradigm is often based on a control and a treatment group, assignment of random participants to those groups, focus on sample size, and identification of variables so that unbiased comparisons can be made. The role of the researcher in this kind of investigation is to strives to be as objective as possible and try not to influence the results in any way.

In contrast to positivism, interpretivism holds the subjective view of reality; that is, reality is interpreted through the subjective experiences of the human mind within a social context.

The interpretive paradigm is closely linked to *constructivism* in terms of epistemology and is often used interchangeably. They share the core belief that reality is pluralistic, interpretive, and context-dependent (ibid). It accounts for "culturally derived and historically situated interpretations of social life-world" (Crotty & Crotty, 1998, p. 67) and allows development and construction of knowledge as a social endeavour, emerging from people's interactions and making sense of their common experiences. In other words, interpretive research allows different meanings for the same thing constructed by different people.

Interpretivism is often conflated with other related theoretical perspectives (see Table 3). Koro-Ljungberg and Douglas (2008) refer to related theoretical perspectives such as social constructionism, phenomenology and postmodernism, as *situational* theoretical perspectives. Although they each have different objectives, they collectively differ from positivism in their dependency on specific situations. The purpose of situational perspectives they note is "to provide descriptions or critiques of particular situations in order to understand, criticize, emancipate, or deconstruct specific phenomenon" (p. 165). This requires researchers to investigate the phenomenon from the eyes of the study subjects rather than simply their own subjective interpretations. The research inquiry is commonly done inductively allowing insights and findings to emerge throughout the data analysis and collection. The researcher brings his/her subjectivity to the inquiry and most definitely influences the interpretation of the data. However, it is important to recognise the researcher's role and subjectivity and understand how that influences the interpretations. Table 3 illustrates the comparison between different attributes of theoretical perspectives.

Table 3 Comparison between theoretical perspectives adapted from Koro-Ljungberg and Douglas, 2008

Theoretical Perspective	**Post-positivism**	**Interpretivism (social constructionism, hermeneutics, phenomenology)**	**Postmodernism**
View on reality	Single falsifiable reality	Multiple subjective realities	Multiple fragmented realities
Purpose	To find relationships among variables, to define cause and effect	To describe a situation, experience, or phenomenon	To deconstruct existing 'grand narratives'
Methods	Methods and variables defined in advance; hypothesis driven	Methods and approaches emerge and are to be adjusted during study	Methods and approaches generated during the study
The role of researcher	Researcher is detached	Researcher and participants are partners	Researchers and participants have various changing roles
Outcome or research product	Context-free generalisations	Thick situated descriptions	Reconceptualised descriptions of the phenomenon

In order to comprehend student perspectives, individual experiences in the context of the course needed to be investigated. Students' ways of thinking, social influences, feelings, and beliefs required to be accounted for. Consequently, I adopted interpretivism and invested myself in the students' experiences as their partner. In order to produce an in-depth description of their experiences, a qualitative approach was taken. The next section describes the properties of qualitative research in brief, followed by a discussion on the methodology chosen for this inquiry.

4.3 Qualitative research

Even though qualitative research is common in interpretivism, it can also adopt various other theoretical perspectives and methods. Because of its multi-faceted nature, *qualitative research* is difficult to define. Koro-Ljungberg and Douglas noted that "qualitative research cannot be defined; it can be only described since the qualitative research community presents a large spectrum of different theoretical perspectives, methodologies, and methods" (2008, p. 164). Drawing from literature, key terms that are frequently used to describe qualitative research include 'emergent', 'participant perspective', 'focus on meaning', 'holistic', 'inductive analysis', 'naturalistic', 'descriptive', and 'flexible design' (Denzin & Lincoln, 1994; Hatch, 2002; Creswell, 2007; Miles & Huberman, 1994; Gray, 2013; Ernest, 1998).

Qualitative research provides the opportunity to voice the participants, allowing them to define factors and emphasise issues that they find meaningful and important to describe their experiences. It is primarily concerned with individual interaction with the world, interpretation, intersubjectivity, individual truth (Ernest, 1998). It has the capacity to gather the complexity of human behaviour that surpass the studies based on hypothesis and randomised control. This is not to say that qualitative research cannot be used to test hypotheses or check if theoretical propositions can be supported with evidence. Historically, positivist oriented qualitative research approaches were prevalent in the 1980s and 1990s (Koro-Ljungberg et al., 2008). However, as the theoretical perspectives began to evolve with an epistemological shift from a single, fixed, and agreed upon reality (positivism) to a multiple, constructed and interpretive reality (postmodernism), qualitative researchers have also begun moving from positivism to other situational theoretical perspectives. Researchers are going beyond testing hypotheses to understanding *how* and *why* things happen in a more complex but contextual setting. Understanding how different individuals experience and interact with their social reality and how they construct meaning is regarded as *interpretive* qualitative research.

Case and Light (2011) noted that, historically quantitative approaches had been a preference in the field of engineering education research. One of the reasons being, "engineering educators who have been trained primarily within the quantitative tradition may not be familiar with some of the norms of qualitative research". Koro-Ljungberg and Douglas (2008), in a review of Journal of Engineering Education (JEE) articles, reported that there is paucity in situational theoretical perspectives in the publications. They postulate that researchers find the assurance of rigour in these perspectives, which are based on different epistemological views, difficult to embrace. To promote understanding of the basis of different methodologies, Case and Light (2011) describe several promising yet *emerging* qualitative research methodologies in the engineering education research community. Case study is one of those emerging methodologies, which has been chosen for the studies included in this thesis. Description and rationale for case study methodology is elaborated on in Section 4.4.

Assuring quality in qualitative research can be tricky. Krefting (1991) contended that frequently, similar criteria are used in evaluating both quantitative and qualitative research, which can be problematic. Qualitative researchers argue that it is not appropriate to use the same criteria to judge worthiness in two different modes of inquiry where nature and purpose are contrasting. Guba (1981) presented a model describing four aspects of trustworthiness relevant to both quantitative and qualitative research. These include, a) *truth value*—determines whether there is confidence in the evidence, b) *applicability*—refers to the extent findings can be applied, c) *consistency*—whether the findings can be replicated, and d) *neutrality*—whether the research procedure and results are unbiased. Guba defined these criteria differently for quantitative and qualitative research using terms appropriate for each. Krefting (1991) prescribed several strategies to enhance trustworthiness throughout the qualitative research process. Table 4 illustrates Guba's criteria for trustworthiness in qualitative research along with some strategies to address them.

Table 4 Overview of Guba's (1981) quality criteria and strategies to enhance them

Quality criteria in qualitative research (Guba, 1981)	**Strategies to enhance quality** (Krefting, 1991)
Credibility (to the extent the findings are believable by others in the field)	Prolonged and varied field experience, Time sampling, Reflexivity, Triangulation, Member checking, Peer examination, Interview technique, Establishing authority of researcher, Structural coherence, Referential adequacy
Transferability (if the findings can describe similar situations)	Nominated sample, Comparison of sample to demographic data, Time sample, Dense description
Dependability (the extent the findings are consistent in the context of the study)	Dense description of the research methods, Thick description of the findings in context, Stepwise replication (splitting data and replicating the analysis), Triangulation, Peer examination
Confirmability (concerns how well the findings represent the study participants and settings)	Relate and compare with existing literature, Triangulation, Reflexivity (reflect on researcher's role and influence)

4.4 Case study methodology

A special form of inquiry in qualitative research paradigm is the *case study methodology*. Among others, three seminal authors Yin (2002), Merriam (1998), and Stake (1995), have done extensive work in developing this methodology. Their methodological suggestions and procedures have provided the 'road map' for designing the case studies for this research work.

Merriam defines case study as "an intensive, holistic description and analysis of a single instance, phenomenon of a social unit" (Merriam, 1998, p. 21). By focusing on a particular phenomenon or entity, the intention is to provide a 'thick' and an in-depth description of the phenomenon. It is particularly useful in asking 'how' and 'why' questions to a contemporary event. In line with Merriam, Stake (1995) adds two notions to the description, "integrated system" and "bounded system". Conversely, Yin (2009) defines case study as a research process, "a case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (p. 13). However, Merriam and Stake emphasise the importance of *delimiting* the study object. Merriam further points out that if the *unit of analysis* is not intrinsically bounded then it cannot qualify as a case study. In simple words, one should be able to 'fence in' the object of study. One way to assess the *boundary* is to see if there is a limit to data collection or if there are a limited number of participants to interview or a limited amount of time for observation. Stake (1995) elucidates further,

> The case could be a child. It could be a classroom of children or a particular mobilization of professionals to study a childhood condition…An innovative program may be a case. All schools in Sweden can be a case. But a relationship among schools, the reason for innovative teaching, or the policies of school reform are less commonly considered a case. These topics are generalities rather than specifics. The case is a specific, a complex, functioning thing (p. 2).

Stake noted that the way a case study investigation contributes depends a lot on the researcher's epistemological orientation. He purported that constructivism essentially informs qualitative case study research since "most contemporary qualitative researchers hold that knowledge is constructed rather than discovered" (Stake, 1995, p. 99). He further added that qualitative case study researchers are merely interpreters and organisers of interpretations which require them to report their interpretations/renditions of the constructed reality that they have gathered through their inquiry. The process should essentially take the researcher to another level of reality or knowledge construction besides that of the reader of the report. Similarly, Merriam contended that constructivism guides qualitative case study research and the main assumption that qualitative research should entail is that "reality is constructed by individuals interacting with their social worlds" (Merriam, 1998, p. 6). Her ontological disposition that "reality is not an entity; rather, there are multiple interpretations of reality" (Merriam, 1998, p. 22) braces the foundation of her epistemological commitment. Moreover, the conclusion of a qualitative case study would be an interpretation by the researcher of others' experience of the phenomenon under study, filtered through his/her lens.

However, case study and qualitative methodology in general has been questioned on the issue of generalisability. For decades, scientific inquiry has been concerned with describing replicable situations with a general law/theory. The positivist view on generalisabity is that general context-independent laws are 'valuable' and useful in making predictions, and often dismisses the generalisability of context dependant knowledge that qualitative research generates. Qualitative researchers argue that it is a different kind of generalisability compared to quantitative research. In interpretivism the focus is on context, interactions and hermeneutics and the aim is to have a theoretical understanding of the case under investigation; in that generalization is not achieved through statistical or any other quantitative means for a population, but generalisation is "interpreted as generalisation towards a theory" (Carminati, 2018 p.2097). By a thick, valid and rich description of sufficient depth following a well-established criterion of quality of the findings, transferability if achievable by the reader. The reader as an agent can justify extrapolation and application of the findings (or partial findings) to other situations and context.

The case: a course in mathematical modelling and problem solving

The *case* chosen for the studies in this thesis is the course titled 'Mathematical Modelling and Problem Solving". It is a second-year course given in the department of Computer Science and Engineering at Chalmers University of Technology. The course is well acclaimed by the students, and it was awarded the Chalmers Pedagogical prize in 2010. The unique and innovative pedagogy in the course was one of the reasons that it was chosen as the unit of study. At the beginning of my PhD education, I attended the course as a student and experienced first-hand how transformative and effective the learning in the course was. Later, I took part as a teaching assistant for some of the problem-solving sessions during two consecutive study periods.

Description and design

The course runs in six weekly modules and gives 7.5 ECTS credits. The course is available for all engineering students at the university. The prerequisite is the compulsory courses in mathematics for the Software Engineering programme (i.e., discrete mathematics, linear algebra, analysis and mathematical statistics). Most of the students on the course are enrolled in the same department the course is given at, and a few come from other engineering or science departments. The class size is usually around 100, which is optimal for available physical resources and supervision. There is one main teacher and around 3-4 teaching assistants for every class.

The aim of the course is to teach mathematical modelling and structured problem solving, directed at developing generic skills and attitudes to deal with real-world problems in science and technology. The learning outcomes of the course, as mentioned on the course page[1] are:

[1]https://student.portal.chalmers.se/en/chalmersstudies/courseinformation/Pages/SearchCourse.aspx?course_id=24640&parsergrp=3

1. A systematic view of different kinds of mathematical models and how they can be used in different areas of application. Attention not only to classical mathematical modelling but also to models common in computer science.
2. Ability to create, use and evaluate mathematical models in different and possibly new areas of application.
3. Improved general ability to solve mathematical problems.
4. Perspective on the role of mathematical modelling and mathematics in general and for the professional engineer.

The course is divided into six weekly modules based on model types and application. Each module with a short description is listed below:

1. Functions and equations—introduces the significance of different kinds of mathematical expressions, modelling simple relations using variables, functions and equations.
2. Optimization models—includes modelling problems intended to develop the ability to model optimization problems from different areas such as economics and decision support.
3. Dynamic models—represents modelling with time-dependent aspects, such as simulation in biology, physics and engineering.
4. Probabilistic models—intends to introduce and develop models based on probabilities, such as stochastic simulation, Markov models, and Bayesian inference.
5. Discrete models—discusses how discrete mathematical concepts are used in computers science and how problems can be translated into standard problems.
6. Modelling language—includes application of modelling language to facilitate the use of complex structured models.

The modules contain a total of 30 reasonably realistic problems that are solved in pairs. The problems are simplified real-world problems, which have been carefully designed, keeping the essential realistic aspects intact and allowing variation in applications, models and problem-solving approaches. The problems are of different character and complexity, and some are more structured than others. Usually, the modules begin with problems that have more straightforward and well-defined answers, followed by problems that are ill-structured and open-ended.

Each module begins with an introductory lecture that provides general information about the module, the kind of models and problems involved. As the course proceeds, the modelling cycle, problem-solving steps and heuristic strategies are also discussed.

The problem-solving sessions come next, where students solve the problems in pairs until the end of the week. Supervision is available during these sessions at given times. Students are required to inform ahead whether they will require supervision or not. The main instructor and other teaching assistants provide supervision. The supervision is done in 'Socratic' style, where students are probed with questions to their queries without giving them direct answers.

The students submit their work report in the course's online digital learning system called FIRE by the end of the week, receiving feedback on the reports the week after.

A follow-up lecture takes place soon after, where students discuss their solutions with the instructor, receive feedback, and the instructor demonstrates an example problem-solving process to the class. Here the students get to reflect on their work, clear any misunderstandings and view an expert problem-solving process.

The course ends with a final reflective report, where the students are asked to reflect on their learning over the whole course period. The report does not have any strict structure, and the students are encouraged to write as freely as possible. The overall structure of the course is illustrated in Figure 5 below.

There is no formal assessment, such as written examination or oral presentation in the course. The students are assessed based on their attempt to solve the problems. Grades are based on level of participation in the course and submission of the modules and final report. Students can re-submit their solutions after having received teacher feedback.

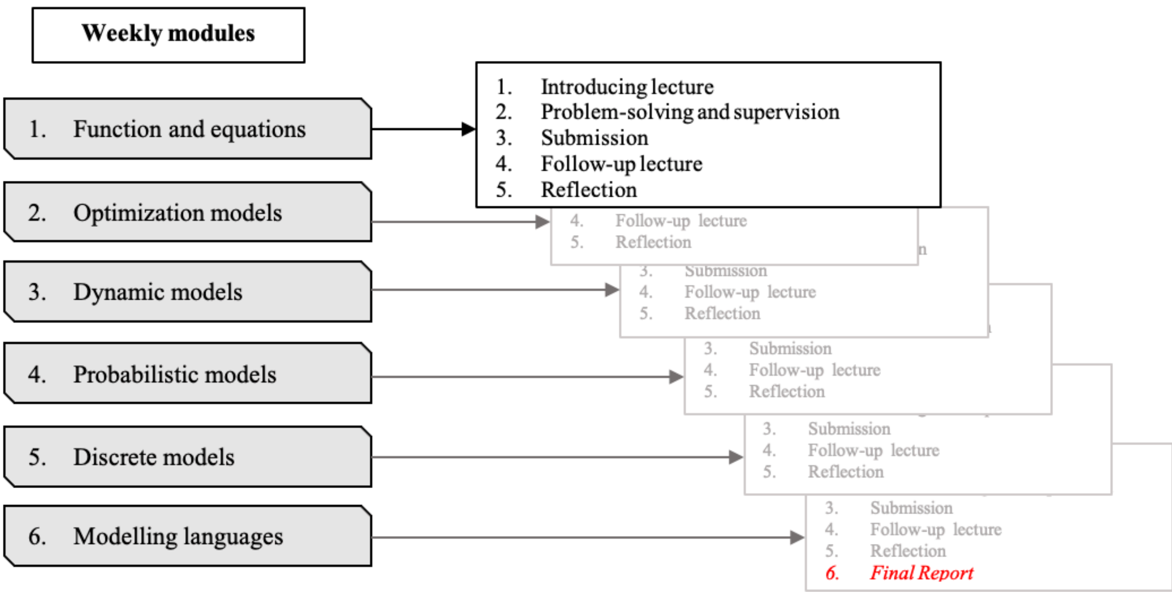

Figure 5 Overall structure of the course Mathematical Modelling and Problem Solving

Even though the course has been designed intuitively by the course instructor, the philosophy and rationale has been supported by *inquiry-based learning* or IBL (Prince & Felder, 2006). IBL is a pedagogical approach that emphasises student's ability to explore, pose questions, generate ideas, test and analyse those ideas, and formulate an appropriate conclusion. Teaching and learning through IBL use questions or problems as a context for learning. The approach is rather *inductive* since learning begins with a task or a problem and then concludes with a theory. The course uses realistic modelling problems as the context of learning. In other words, the problems are central to this course, and the learning surrounds them.

Data collection and analysis

Traditionally, data collection and analysis necessitated "objectivity". Corbin and Strauss (2008) contemplate that "objectivity in qualitative research is a myth" (p. 32). In contrast to objectivity, they suggest recognising the subjectivity that researchers bring into the work and focus on *sensitivity* to the participant's experience. Sensitivity is "the ability to pick up on subtle nuances and cues in the data that infer or point to meaning" (ibid, p. 19). We have made an effort to be sensitive during the data collection and analysis process, so we do not miss any relevant issue, event and cue and that the students' perceptions come forward through the 'eyes' of the researcher. Corbin and Strauss (2014) stresses that researcher should be aware of their subjectivity at every step so that they do not force their ideas on to the data but let the data speak for itself.

In the first study (Paper A), we have used three sets of data, 1) a pilot group interview with five students, 2) individual semi-structured interviews with eight students, and 3) reflective reports submitted by 103 students at the end of the course.

The pilot group interview was mostly exploratory to gain an overall understanding of students' early experiences in the course. The interview was carried out in a *semi-structured* (Gray, 2013) way, that means there was a pre-defined progression with a set of questions to keep the interview focused but probing questions were used to explore certain aspects in detail. Semi-structured interviews provide a flexible way to get detailed responses from the respondents on pre-determined questions.

The individual interviews were designed based on the students' experience in solving two problems in the course. The questions were devised in the context of those two problems allowing probing of views, opinions and clarifications whenever needed.

The participation in the interviews were voluntary and all the students were offered confidentiality. Furthermore, the students were informed about their right to terminate, not to answer and withdraw their interview whenever they wanted. No written consent was taken, but all the students were asked for verbal consent at the beginning of the interview. The interviews were approximately an hour long. Both sets of interviews were audio recorded and transcribed soon after each event.

The majority of the data came from students' reflective reports. The reports did not have any strict criteria and students were encouraged to write freely about the course and their learning. The reports contained rich data about models, modelling, problem solving and students' personal reflection on their learning, the course and various other related aspects.

The data was analysed using the *general inductive* approach described by Miles and Huberman (1994). It fitted well since we wanted the data "to speak for itself" and capture the emerging themes. The analysis process consisted of five steps and several iterations within the steps: 1) getting familiar with the data, by reading the data several times and taking notes, 2) breaking up the data into segment of manageable pieces, 3) tagging the segments with short phrases that convey the meaning of the segment, 4) sorting, shifting and grouping the tags in an iterative way, looking for themes to emerge, and 5) identifying and reviewing the themes.

To ensure the trustworthiness of the findings, triangulation of data collection (i.e., using different data collection methods such as group interview, individual interview and reflective reports) was used. Furthermore, *peer examination* was employed at several stages in the analysis process (i.e., the preliminary findings were discussed with the other authors in order to double check the interpretations of the results).

In the second study (Paper B) we have used the reflective reports as data set, where the students were additionally asked to describe what they learnt from the course and what aspects of the course were important to them. The data analysis was done in two stages. In the first stage, a *general inductive* approach (Thomas, 2006) was used guided by Miles and Huberman's techniques of qualitative analysis (1994). At first the data was read several times to separate the meaningful segments. Then the segments were coded with words or phrases that represent its core meaning. Which was then sorted and shifted iteratively looking for categories. As the categories emerged, tables and diagrams were used to visualise the categories in connection to other categories. After finalising the categories, detailed memos were written that contained explanation and description for each category.

In the next stage, we did a deductive analysis using an analytical framework (framework for authentic learning by Herrington and Oliver, 2000) as the lens. The process was guided by constant *comparative analysis* (Strauss & Corbin, 1990), comparing the categories against the elements of the framework and reorganised accordingly. The memos of the categories and the guidelines for the elements of the framework were useful during this process. Conclusions were drawn and intermittently checked with the original data.

For both the studies, I joined the course as a participant observer and was introduced to the class. This provided the opportunity to closely observe and take notes during the lectures and the problem-solving sessions. To be able to closely observe the course and take notes, helped the process of interpreting the data with a sensitive eye.

Chapter 5

Summary of the papers

This chapter presents the summary of the two papers appended in this thesis. To avoid too much repetition from the previous chapters, the focus has been primarily on highlighting the empirical results in respective studies.

Paper A

Investigating and developing engineering students' mathematical modelling and problem-solving skills

The first study in this project was inspired by the intention to contribute to the relatively limited research on understanding how engineering students approach mathematical modelling problems and how to improve their ability in dealing with such problems. This study was placed in the context of a course in mathematical modelling and problem solving for engineering students (see Section 4.4 for a detailed description of the course) and posed two research questions: 1) how do engineering students approach mathematical modelling problems at the early stage in the course, and 2) how does the course impact students' learning to deal with such problems?

The study adopted a qualitative case study (Merriam, 1998) approach. The empirical data consisted of student interviews during the early stages of the course and students' reflective reports at the end of the course.

In terms of the first research question, the results of Paper A revealed that when approaching the problems early in the course, students had general difficulties in *simplifying* and *mathematising the problems*. We identified three major challenges students faced and strategies they used to deal with them (Table 5). Interestingly, these challenges had little to do with mathematical modelling but related to problem-solving skills.

Table 5 Challenges and solution strategies students used to address them

Challenges for students	Solution strategies
Understanding the problem	• Reformulate the problem in simpler words • Splitting the problem into parts • Visualisation • Discussing with peers
Exploring alternatives	• Asking suitable questions • Seeing the problem from different angles • Changing the representation of the problem
Having the right attitude	• Approaching problems with a more open mind • Expecting to try things out • Courage and daring to use common sense • Communication

Regarding the impact of the course, the study shows that the students developed both in *mathematical modelling* and *problem solving,* which coalesced into developing the ability to solve real-world problems. They recognised the importance of being aware of the challenges and actively reflecting and utilising the strategies to deal with them. They also reported having developed a positive *self-belief* about their abilities in problem solving and mathematics in general.

The paper continues with a discussion of the difficulties students experienced drawing on Schoenfeld's framework for *mathematical thinking* (Schoenfeld, 1985; 1992). It concludes that students have sufficient *resources* and knowledge about *heuristic* strategies but lacked skills in effective *self-regulation* and were hindered by their pre-existing *beliefs* about mathematics and related aspects.

Finally, to understand how the learning environment contributed to the students' developments, the paper draws on the model of *cognitive apprenticeship* (Collins et al., 1989), identifying two core elements: *authentic problems* and *making thinking visible*. The problems in the course were reasonably realistic in nature with varying character and complexity so that students found them challenging, stimulating and inherently encouraged them to explore. Further, the course instructor made his 'thinking visible' to the students by demonstrating the problem-solving process, including alternative approaches and other considerations, as well as by coaching them during their problem-solving attempts. Similarly, students made their 'thinking visible' by practising solving the problems and discussing with the teacher and teaching assistants. The Socratic questioning method during the problem-solving sessions helped the students to reflect on their own problem-solving process, which contributed to problem solving and developing metacognitive skills.

Based on the findings, the paper suggests that this course is an intermediate step between traditional engineering education and the ability to solve more complex engineering problems and argues that the course or similar teaching should be considered in the education of all engineers.

Paper B

Student perceptions of authentic learning in a mathematical modelling and problem-solving course: a case study

The second study in this project was inspired by and builds on the results of Paper A by examining student experiences of the course through the lens of authentic learning. Research indicates that learning in a decontextualised manner does not effectively prepare students for the workplace, and *authentic learning* attempts to address this issue. The study investigated the same course in mathematical modelling and problem solving for engineering students as above. The aim of Paper B was to better understand how engineering students experience different elements of authentic learning in the course and thereby contributing to the knowledge base of designing student-centred and authentic learning environments in engineering education.

The study employed a qualitative case study methodology (Merriam, 1998) and used the *framework for authentic learning environments* as an analytical lens that consists of nine elements for authentic learning: 1) *authentic context,* 2) *authentic activities*, 3) *expert performances,* 4) *multiple roles and perspectives*, 5) *collaboration,* 6) *reflection,* 7) *articulation,* 8) *coaching and scaffolding*, and 9) *authentic assessment* (Herrington & Oliver, 2000, p. 30). The data consisted of students' reflective reports where they were additionally asked to describe what they learnt from the course and what aspects of the course were important to them.

The results showed that students experienced and reflected on all nine elements of authentic learning in the course's learning environment to varying extent. Students were generally very positive about all elements and outlined different mechanisms to how they contributed to their learning and skill development. The problem-solving and modelling activities in the course provided the students with an authentic context for their learning. Students constantly compared the course structure and its activities with prior courses they had attended and perceived it to be more authentic. The unique and diverse characteristics of the problems made them challenging and appeared realistic to the students. Among others, collaboration played a very important role in experiencing the different authentic elements in the course environment. Similarly, reflection was another topic mentioned profoundly in the reports.

Based on the empirical results, the paper discusses three particular aspects of authentic learning environments: the concept of *authenticity* (How authentic is authentic enough?), *collaboration* as a vehicle of bringing everything together, and the role of the teacher.

Regarding authenticity, the study shows that even though the course design – unlike what the authentic learning framework suggests – used 'realistic' rather than 'real' problems of varying complexity, the students nevertheless experienced them as authentic. The paper ascribes this to the limited experience of first- and second-year students in this particular context and argue in line with other authors that 'cognitive realism' - rather than physical realism – is key to achieving the cognitive benefits of authentic learning so that students 'buy in' to the tasks and activities.

Further, the paper discusses the important role of collaboration in authentic learning. Beyond its multitude of benefits for students' learning, collaboration was shown to be a 'vehicle to operationalize' many other aspects of the course, such as reflection, articulation, and even coaching whenever needed.

Finally, we underline the critical but also the demanding role of the teacher as a part of the *cognitive apprenticeship* model that was applied in this alternative learning environment. The students outlined how the various teacher roles as a guide, scaffolder or problem/task presenter were crucial to promote multiple perspectives, exploration, reflective activities and help students develop their own problem-solving route.

The paper ends by concluding that the provided outcomes are generally consistent with research on authentic and situated learning and may be considered a further validation of constructivist learning designs. The framework for authentic learning enabled a deeper understanding of different aspects of the course that made the learning experience so rewarding to the students and provided pointers for designing authentic learning environments for engineering education.

Chapter 6

Concluding discussion

What I’ve come to realize is that a core aspect of my teaching is its human and moral dimension. My teaching is really about helping those I work with to become open-minded and inquisitive thinkers who are willing to—even hungry to—ask questions, gather evidence, and sort through ideas in a reasoned way. It’s about questions as much as answers, about values as much as facts.
It’s about preparing students of all ages to be engaged and productive citizens of our democratic society.

-Alan H. Schoenfeld, 2009

The goal of this thesis is to contribute to the efforts of improving engineering education with the focus on developing students' ability to perform mathematical modelling problems. The research proposition was to examine an alternative learning design in a mathematical modelling and problem solving course for engineering students and understand how the learning design contributes to student learning. For this purpose, two empirical studies have been carried out; the first study (Paper A) investigated how students approached mathematical modelling problems in the early stages of the course and how the course impacted their learning. The second study (Paper B) examined how the students experienced different elements of authentic learning in the course.

This chapter intends to consolidate the findings through a concluding discussion of the project's contributions to the knowledge base of the field, some implications for instruction design, and a brief retrospective reflection on mathematical modelling and problem solving indicating the need for a framework for teaching and learning mathematical modelling and problem solving. The section finishes with a short consideration of potential limitations of this thesis and an outlook for future research.

6.1 Knowledge Contribution

This thesis contributes to the knowledge base in engineering education and engineering education research both empirically and theoretically in the following ways:

1. Providing evidence of the importance of self-regulation, beliefs and attitude and their influence on students' problem-solving ability.
2. Exemplifying how the learning environment impacts students' learning.
3. Illustrating students' perception of authentic learning in an alternative learning environment.

Self-regulation, beliefs and attitude influence students' problems solving ability

One of the aims of this research was to understand how students approach mathematical modelling problems by investigating the students' challenges and strategies used to counter those challenges early in the course. Interestingly, despite approaching modelling problems, students' challenges mainly related to their problem-solving skills. From the study in Paper A, it was evident that the reason for experiencing challenges early in the course was due to the lack of effective *self-regulation* skills and counterproductive *beliefs* about mathematics, problem solving and themselves. Metacognition and related dispositions such as beliefs and attitudes have significant effect on an individual's cognitive actions and behaviour (see Chapter 3, Section 3.5), and this case study confirms this contention. Moreover, the results show that students' beliefs can override their ability to control and monitor (twin processes in self-regulation) their activities at times during problem solving. Figure 6 illustrates how self-regulation affects students' ability to understand the problem and explore alternatives and how students' beliefs can over-ride self-regulation and subsequently influence their problem-solving ability. Hence, even though the students possessed appropriate heuristics and control strategies, having negative beliefs and lacking adequate self-regulation skills blocked the ability to utilise them. Research shows that traditional mathematics classrooms suffer from a lack of attention to developing metacognitive skills. Students are rarely given the chance to ask critical questions about mathematical, conceptual thinking since the tasks and well-structured instruction in the classroom are designed to meet the assessments that determine competence in performing "correct" mathematical procedures (Schoenfeld, 1988; 1992). This limits students' opportunity to explore and consequently does not enhance metacognitive skills. Schoenfeld (1985) argues that successful problem solvers differ significantly in the four categories of his framework in comparison to less successful ones. For example, successful problem solvers emphasise assessing their choice of plan, have significant control over their solution process and are aware of their beliefs better than less successful problem solvers. Drawing from our research I agree that *resources, heuristics, self-regulation* and *monitoring (the four categories of Schoenfeld's framework)* influences successes in problem solving and stress on how beliefs can have the upper hand in reaching success during problem solving. Lesh and Doerr (2003) explain another reason which adds to the shortcomings of traditional teaching in that schools provide students with a narrow perspective of problem solving. In school, students usually learn to perform routine exercises they refer to as 'problems', which involve finding a mathematical procedure that links the 'well-specified givens' to 'well-specified goals' (ibid). The belief about what a problem and problem solving mean come from that experience. Contrariwise, nonroutine problems such as the ones used in this course, where finding the givens and the goals is a major challenge goes against their beliefs about problems and problem solving causing them to face difficulties. Similarly, in conventional teaching, heuristics and strategies are taught as techniques to perform when students get stuck. Even knowing the strategies, students seldom use them consciously when approaching nonroutine problems. They are caught between their beliefs and self-regulated control of heuristic strategies; a situation that Schoenfeld (1987) recommends countering by starting with "damage containment"—identifying negative beliefs and dealing with them on a case by case basis, then moving to a social problem-solving environment where students can collaboratively share and learn (p. 213). Addressing students' beliefs individually in a typical engineering classroom, where class size is relatively large can be tricky. Scholars suggest that authentic learning environment inherently develop students in affective domain, which we also see in the course. It is not clear exactly how the course develops students' beliefs and attitude, but the results suggests that the differently aspects of the learning environment collectively contribute to their development.

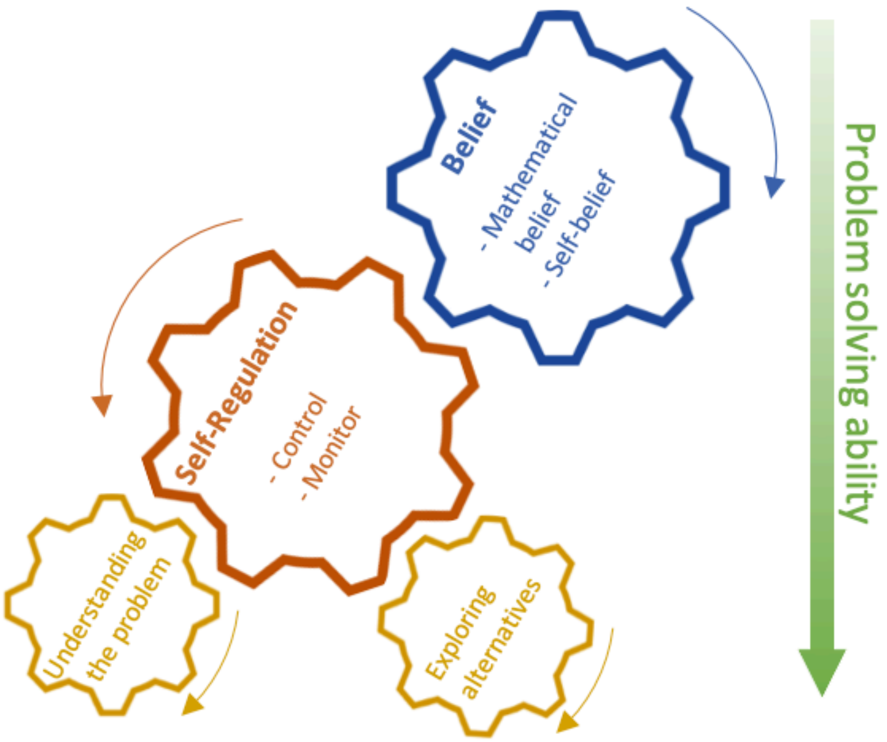

Figure 6 Illustration depicting the influence of self-regulation and beliefs on problem solving ability.

The impact of the learning environment on students' learning

One of the other aims of this research was to understand the impact of the alternative learning environment of the course on students' learning. The results demonstrate that, students developed both in *mathematical modelling* and *problem solving* as well as their overall ability to solve real-world problems. Additionally, the results showed development in effective self-regulation skills and productive beliefs, attitude and expectations. Students' development in metacognitive skills is particularly interesting because research literature specifically in mathematics education, view teaching metacognitive skills as a difficult endeavour (Lesh et. al 2003). Schoenfeld (1985, 1992) noted that one reason why it is difficult to teach self-regulation and control skills they often require behaviour modification including underlying negative control behaviour acquired through previous experiences. The intricate web of the learning environment in the course, for example, the social form of learning, with guided teaching promoting reflections, meaningful contextual problems and activities to develop exploration, and integrated assessment, all together contributed to students' cognitive and metacognitive development. Results from Paper A showed that Socratic questioning, 'making thinking visible' protocol alongside of providing authentic problems were critical elements of the learning environment that contributed to the impacts. This exemplifies and contributes to the knowledge of how problem solving skills, especially metacognitive skill development, can be addressed through instruction.

Furthermore, we used cognitive apprenticeship to understand these elements in the course. Cognitive apprenticeship emphasises the development of cognitive skills to facilitate students in thinking about and processing information and ideas, in ways experts in their discipline do. It provides the opportunity to visualise the cognitive processes experts use, gain a better understanding of the tasks in their field, and by practising similar tasks in a social context get to take part in the community of practise. We have seen in our empirical results how this is carried out by engaging in the meaningful activities of the course. As a result, students at the end of the course began envisioning themselves as future engineers, and with that students were beginning to see themselves as participants in the community of practising engineers. In Paper A, and in more detail in Paper B, it has been seen how the elements of cognitive apprenticeship have been operationalised. Among others, the role of the teacher was critical to adopt cognitive apprenticeship in the course, i.e., the teacher models expert performance during follow-up lectures, coach and scaffold during problem-solving sessions and also act as a task/problem presenter during introductory sessions. Making the teacher's role multifaceted is essential since in their role as experts, teachers demonstrate expert cognitive processes, and as guides they coach and scaffold to promote those relevant processes in order for students to reach their 'zone of proximal development'.

As alluded to earlier, in paper B we used a broader analytical framework—the framework for authentic learning to examine the course's learning environment. The cognitive apprenticeship model includes many of the essential elements of situated learning for instructional design, but the authentic learning framework includes critical elements of situated learning needed to operationalise its designing learning environment. Using an authentic learning framework as the analytical lens enabled us to identify manifestation of situated learning elements in the course. This contributes to the knowledge base of disciplinary case studies on manifestation of situated learning elements in a course.

Students' perception of authenticity in an alternative learning environment

Further contributions of this thesis relate to the concept and desired degree of authenticity in mathematical modelling and problem solving education. Working on authentic tasks purportedly helps students to learn to use and implement knowledge in a manner that practicing professionals would. Bruner (1962) argues that in order for students to utilize their problem solving skills in future novel problems, we need to provide them with opportunities for active and prolonged engagement in meaningful contextual activities. Agreeing to this renditions, Herrington and Oliver's (2000) guidelines to implement authentic context and tasks suggests using one complex task with no attempt to simply the environment. In that respect, the course design examined here showed a noteworthy deviation from this suggestion as it uses small reasonably realistic modelling problems of varying character in order to create '*forced variation'* (Wedelin & Adawi, 2014). Nevertheless, we show that the students experienced the context and the activities in the course as well as several other aspects of the learning environment to be *authentic*. Based on these findings and in line with scholars like Honebein, Duffy and Fishman (1993) and Herrington et al. (2003), this thesis stresses the notion of authenticity as a relative and subjective concept. An implication of our results is that irrespective of how authentic the instructor considers the course tasks and activities, to achieve the cognitive benefits, students must perceive them as authentic. Barab et al. (2000) use the term '*buy in'* in that respect. As we have argued in paper B, an important potential factor for finding the 'right' degree of authenticity appears to be related to the level of the students. This observation is echoed by scholars in the area of mathematics and modelling education arguing that it is useful to provide 'realistic' problems rather than absolute 'real' problems in the early stages of higher education, due to students' knowledge not being comprehensive enough at the outset to tackle the complexity of real problems they will encounter once they have finished the programme (Zawojewski et al., 2008). Supporting this, Jonassen (1991) notes that learners who are just entering an advanced level of knowledge acquisition, for example, learning to address ill-structured problems, are in a transition phase and require much support to acquire the skills required to solve complex real-world problems. At this stage, it is appropriate to introduce realistic problems in a constructivist environment, fostering teamwork so that as a team, the students can offer different skill sets and collaboratively solve the problems (Zawojewski et al., 2008; Jonassen, 1991). It can be postulated that this is precisely the reason why the design of the course works for students at their early stages. They have limited experience of engineering problems and engineering workplace setting and their perception of mathematics courses come from their previous experience of attending traditional lecture-based courses. Therefore, even though the problems in the course are cultivated from original engineering problems, several layers of complexity have been removed and the physical environment of the course does not resemble a physical engineering workplace it is still enough to create an authentic environment for the students. In summary, this understanding contributes two important points in respect to designing authentic learning experience, 1) it is possible to provide authentic experience without providing absolute authentic tasks depending on the students learning stage, and 2) it is important to consider students' perception of authenticity in order to design an effective authentic learning environment which can differ based on the students' learning stage and previous experience.

6.2 Implication for instruction design

The outcome of the two empirical studies and critical observations during those studies offered insights that may be useful for instruction design:

1. Paper A discusses several heuristic strategies students adopted during problem solving in the course, many of which were hinted by the teacher and teaching assistants during the problem-solving sessions. For example, to understand the problem better students sometimes *reformulate the problem* in their own words. The challenges and appropriate strategies[2] that students used may prompt educators to include discussing and introducing them in their courses. Schoenfeld (1985) notes that providing students with heuristic strategies help them when they are stuck as well as promote independent exploration.

[2] A large collection of problem-solving strategies can be found in Pólya (1945) and Schoenfeld (1985, 1992).

2. Paper B demonstrated that the course uses elements of situated learning in its learning environment and an authentic learning framework captures the essential aspects of the course that is consistent with the elements of situated learning. The authors of the framework provide general guidelines (Herrington & Oliver 2000) for implementing situated learning elements. Both the framework and its manifestation in the course can be used as a support for designing authentic learning environments.
3. Cognitive apprenticeship is a well-researched instruction model, and in conjunction with the course design, may provide useful pointers in operationalising it in the classroom. For instance, how to make teachers' thinking processes visible to students and vice versa, how to situate tasks in an authentic context and how to coach and facilitate students ensuring independent exploration and reflection.

4. All the elements in the authentic learning framework appeared in the students' reflections, although collaboration was significantly highlighted as an aspect of the course tremendously supporting students' learning as well as operationalising different elements of the framework. This is in line with research in situated learning which identifies collaboration as vital in a situated environment (Herrington & Oliver, 2000).

6.3 Retrospective reflections on mathematical modelling and problem solving

In this section, I would like to present some reflections on mathematical modelling and problem solving that has resulted from this research and literature review. On the basis of these reflections, I will argue for the need of further development efforts towards a framework for teaching and learning mathematical modelling and problem solving in engineering education.

Problem solving, as discussed in Chapter 2, is a goal-directed activity (often iterative) that requires a productive way of thinking, basically make systematic attempts and persistently explore to reach a reasonable outcome. One can use Pólya's (1945) four general principles of problems solving to assist this process. To solve an applied mathematical problem, the process follows a similar problem-solving pattern, except there is an intermediate step of mathematising, working the math and interpreting and validating the results in the context of the original problem—mathematical modelling. In a simple and routine mathematical problem, this step often passes unnoticed, but for a complex problem, this step might require more comprehensive skills and attention. In mathematics education research and mathematical modelling research, the modelling cycle is often used to emphasise the different actions in a modelling problem, it may also remind the modeller to perform these actions iteratively to improve the model. Critiques of the modelling cycle note that modelling is not likely a cyclic process but rather jumps from one activity to another (Ärlebäck & Bergsten, 2010). Similar indications have been observed in the course, that students usually define their own problem solving and modelling route. In that respect, looking at the modelling cycle, a problem-solving aspect can be observed within. It can be postulated that; this is because any nonroutine situation that requires a mathematical model would inherently require problem-solving skills to reach a resolution. Therefore, in the context of nonroutine, ill-structured and complex problems, similar to those engineers encounter in the workplace (characteristics of workplace problems are discussed in Chapter 2, Section 2.4), both mathematical modelling and problem-solving skills are essential. This contention is also supported by the works of Lesh and colleagues (Lesh et al., 2003; Zawojewski et al., 2008)

Then the question arises, how do we teach mathematical modelling and problem solving? As discussed in Section 2.2, research in problem solving generated a few valuable outcomes useful for designing problem-solving instruction, many of which have been intuitively implemented in the course. For example, structured instruction to facilitate the students, hinting heuristic strategies during problem solving, promoting reflective thinking, etc. Often research in problem solving considers traditional routine problems and views problem solving as the process of translating the problem to mathematics, and then working on it to reach a mathematical solution, which is then translated into the terms of the original problem. Researchers argue that this simplistic view of problem solving makes problem solving instruction ineffective (Lesh et al., 2003). A teacher who adheres to this view focuses on mathematisation and tends to deal with problems and application once mathematics concepts and procedures have been introduced and practised. Nonroutine, ill-structured problems usually appear at the end of the course, if not in the later stages of the programme when students have gained sufficient mathematics knowledge (ibid). Similar issues have been seen with regard to modelling problems. As discussed in Chapter 2, research has indicated two general purposes of learning mathematical modelling, *modelling as content* and *modelling as a vehicle* to learn and use mathematics. The course examined in this thesis addresses both since it intends to teach modelling as well as application of mathematics that students already know. I argue that both are essential for students, especially engineering students, to be able to solve workplace problems which nowadays increasingly use mathematical modelling as primary form of design. Workplace engineering problems in a stark difference to the problems practised in the classroom (see Table 1, Section 2.4.), require self-monitored and self-directed learning in a collaborative environment, and both modelling competence and problem-solving ability come into play. Thus, in order to use mathematics effectively to solve real-world problems, students must learn mathematical modelling and problem solving together in the context of authentic problems. Theories and frameworks concerning problem solving, for example, Schoenfeld's framework for mathematical thinking capture cognitive and metacognitive processes involved in problem solving well. Whereas the modelling cycle seems to focus on cognitive aspects of modelling but lack in capturing metacognitive aspects. Schaap et al. (2011) noted in a study identifying blockages experienced by students during mathematical modelling noted that some of the results (namely heuristics) were not captured by the modelling cycle but rather by a framework for problem solving. On the other hand, the modelling cycles consider problem representation and identification typical for addressing ill-structured problems as an important activity. From these observations, it is reasonable to conclude that, in order to optimally address teaching and learning mathematical modelling problems, a comprehensive framework that addresses problem solving activities (including metacognition and belief) as well as mathematical modelling activities in an authentic context is required.

6.4 Limitations and outlook for future research

I would like to finish this thesis with a few considerations regarding the limitations of two studies conducted in this thesis, followed by some suggestions for future research. Regarding the limitations, the majority of the data consisted of students' reflective reports written for the purpose of the course. Thus, the fact that there were no explicit questions regarding the research question addressed in the studies could be interpreted as weakness, even though in Paper B, we asked the students to clarify their learning and important aspects of the course. Nonetheless, we felt this open design provided students the opportunity to reflect on aspects they found important on their own terms. Further, since this research is based on one case (the course), generalizations and transfers to other contexts have to be done carefully and hence point to the need of future research.

The research in this thesis has contributed to answering the research questions that had been devised within the scope of this licentiate. The research endeavour gave rise to various new avenues and exciting questions for future exploration:

As alluded to in the previous section, I argued for the need to develop a framework for teaching and learning mathematical modelling problem that captures both problem solving and modelling activities. In order to identify the attributes for such a framework more theoretical and integrative research is needed in the area.

Similarly, during the literature review, it became apparent that several theoretical and disciplinary research fields such as model eliciting activities (MEAs), problem based learning (PBL) (Du, de Graaff and Kolmos, 2009) and authentic learning have emerged in parallel. They overlap in their aim to contribute to designing constructivist learning environments, though they tend to put emphasis on slightly different aspects of learning depending on the applied conceptual lens. Interestingly, the research literature on these different approaches draws relatively little on each other, and the field could benefit from further examinations of these approaches and their cumulative contribution to engineering education. For example, during the literature review we learnt about attempts to model and *model eliciting activities* (MEAs) that are designed to introduce authentic modelling problems to engineering students. Purdue University in the United States have used these activities in their engineering programmes, and research has revealed several benefits such as teaching students high-order skills, improving motivation and engagement (Zawojewski et al., 2008). It would be interesting to further investigate MEAs and explore their potential in my future work.

Further the findings in this thesis are based on studying a single case, that is the course Mathematical Modelling and Problem Solving, which is an example of good practice at the Chalmers University of Technology. However, in order to strengthen the transferability of the results to related contexts, further research should broaden its efforts to the investigation of multiple cases, both within the university and as well as at other institutions to gain a broader understanding and comparative insights through cumulative case study research.

The results of this thesis, as already discussed, have provided evidence for the significant learning development in the students and an overwhelming positive response to the course. However, less is known about to what extent students are actually capable of transferring their learning to other courses and the workplace. Longitudinal studies following students over the course of their study program and professional development could help to address these questions.

Finally, it became evident from our findings as well as the literature, that the teacher's role is paramount to the success of alternative, constructivist learning environments. In this course, like many other courses at Chalmers University of Technology and other universities, it is common practice to appoint doctoral students as teaching assistants that typically have received very limited training in education and teaching. Thus, further studies examining teacher perceptions and identifying specific teacher development and support needs could contribute to help educational institutions to provide the necessary conditions to implement and conduct courses as the one studied here in practice.

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