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Minimum trap separation for acoustical levitation using phased ultrasonic transducer arrays

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Abstract

Acoustic levitation for interactive visualizations is an emerging field which uses ultrasonic transducer arrays to induce radiation forces on small beads. To fully utilize the interactive and responsive aspect of the system, each bead must be controllable freely in the interaction space without interfering with the other beads. When beads are placed far enough apart it is possible to design a sound field that traps all the beads at their desired positions, taking any potential limitations of the hardware into account. The underlying physics limit how close in space two traps can be without interfering with each other. In this paper, we investigate the minimum spacing required between two beads for them to successfully and independently levitate in acoustical traps. Multiple methods for the sound field design are considered and compared with regards to the overall separation required as well as the gracefulness in the breakdown region.

Keywords: Acoustic levitation, Ultrasound, Phased array

1 INTRODUCTION

Acoustic levitation of multiple beads in close proximity requires knowledge of how close the two beads can be without interfering. If the minimum separation is known it is possible to either avoid the problematic regions while still utilizing the maximum performance of the system, or to develop advanced techniques to achieve a closer spacing of the beads.

To investigate and quantify how close it is possible to levitate individual beads using existing methods, we have chosen three procedures capable of levitating multiple beads: two versions of quiet zone based superposition of multiple sound fields, see section 3.1, and a variance based method, see section 3.2. All three methods are based on numerical minimization of cost functions O with respect to the array element phases φ_j and amplitudes a_j. Each method is used to create two levitation traps separated by various distances from 0.5 mm to 30 mm, covering a range approximately from $\lambda/17$ to 3.5λ. One trap is always at a fixed location while the other trap is placed at points along a straight trajectory. The trap centers in the final sound fields are compared to the desired trap centers, and each method is evaluated based on the minimum separation between the traps as well as the gracefulness in the breakdown region.

2 RADIATION FORCE

The fundamental quantity of interest in levitation is the force exerted on the levitated object. For an object to levitate at a point in space the total force must be zero, and to obtain stability the forces in a region around the levitation point needs to converge to said point. These conditions can be formulated mathematically as

$$\vec{F}(\vec{r}) + m\vec{G} = 0, \tag{1}$$

$$\frac{\partial F_x(\vec{r})}{\partial x} < 0, \qquad \frac{\partial F_y(\vec{r})}{\partial y} < 0, \qquad \frac{\partial F_z(\vec{r})}{\partial z} < 0, \tag{2}$$

where $\vec{r}$ is the levitation point, $\vec{F}$ is the force created to levitate the object, $m\vec{G}$ is the gravitational force, and x, y, z are Cartesian coordinates. Comparing the derivatives terms above with Hooke's law for linear springs, we identify the negative partial derivatives of the radiation force as the stiffnesses of the trap.

The radiation force on an object can be calculated from the linear scattering of an incident sound field of the object in question [1]. For simple geometrical shapes this can be done analytically under appropriate approximations. In this paper we only consider the levitating objects as small spherical beads, which allow for a simple formulation of the radiation force using only the sound pressure p of the incident field and its spatial derivatives, evaluated at the center of the sphere. This can be written as [2]

$$F_q = -\frac{\pi\kappa_0}{k^5}\Re\left\{ik^2\Psi_0 p\frac{\partial p^*}{\partial q} + ik^2\Psi_1 p^*\frac{\partial p}{\partial q} + 3i\Psi_1\left(\frac{\partial p}{\partial x}\frac{\partial^2 p^*}{\partial x\partial q} + \frac{\partial p}{\partial y}\frac{\partial^2 p^*}{\partial y\partial q} + \frac{\partial p}{\partial z}\frac{\partial^2 p^*}{\partial z\partial q}\right)\right\}$$

for each Cartesian coordinate $q = x, y, z$, and

$$\Psi_0 = -\frac{2(ka)^6}{9}\left(f_1^2 + \frac{1}{4}f_2^2 + f_1 f_2\right) - i\frac{(ka)^3}{3}(2f_1 + f_2),$$

$$\Psi_1 = -\frac{(ka)^6}{18}f_2^2 + i\frac{(ka)^3}{3}f_2, \quad f_1 = 1 - \frac{\kappa_*}{\kappa_0}, \quad f_2 = 2\frac{\rho_* - \rho_0}{2\rho_* + \rho_0}.$$

In these last equations subscript 0 indicate the medium (air) while subscript $*$ indicate the material of the object, $\kappa = 1/(\rho c^2)$ are the compressibilities, ρ are the densities, and c are the wave speeds respectively. The free-space wavenumber is indicated with k, and the radius of the spherical object is a.

The sound pressure p and its spatial derivatives are calculated using linear superposition of the transducer elements in the array. Each element is modeled as a point source with the directivity of a circular ring, i.e.

$$p_j = \frac{e^{ikr_j}}{r}J_0(kr_j\sin(\theta_j))$$

where r_j is the distance between element j and the levitation point, and θ_j is the corresponding angle from the element normal. The required spatial derivatives are calculated analytically from the above expression.

3 METHODS

3.1 Superposition with quiet zones

This method uses the superposition of two sound fields, each with a levitation trap and a so called quiet zone [3]. The first sound field has a levitation trap at location A and a quiet zone at location B, while the second sound field has a levitation trap at location B and a quiet zone at location A. The quiet zones are necessary due to side lobes in the two fields which would, if not handled, interfere with the levitation trap in the other sound field and potentially weakening or even destroying the trap.

To obtain a sound field with a levitation trap at $\vec{r}_T$ and a quiet zone at $\vec{r}_Q$ we use a variation of the method from [3], optimizing the array phases and amplitudes in four stages to improve the convergence, stability, and speed. In the first stage the cost function is

$$O_1(\vec{r}_T) = w_S\nabla\cdot\vec{F}(\vec{r}_T) + w_p|p(\vec{r}_T)|^2,$$

i.e. to maximize the trap stiffness in the Cartesian directions and to minimize the squared pressure magnitude. This initial stage is seeded with array element phases known to produce a trap close to the desired levitation position [4], and the amplitudes are kept fixed at the maximum allowable level. The second stage use the squared net force magnitude (including gravitational force) instead of the squared pressure magnitude as

$$O_2(\vec{r}_T) = w_S\nabla\cdot\vec{F}(\vec{r}_T) + w_F|\vec{F}(\vec{r}_T) + m\vec{G}|^2,$$

and also optimizes the transducer amplitudes. The purpose of this stage is to fine-tune the position of the trap center, since velocity gradient terms can move the trap center slightly from the position of a pressure minimum. The third and fourth stages also include the quiet zone as

$$O_{3,4}(\vec{r}_T, \vec{r}_Q) = O_2(\vec{r}_T) + O_Q(\vec{r}_Q),$$

optimizing only the phases in stage three but the phases and amplitudes combined in stage four. Stages two, three, and four are started from the results found in the previous stage.

Two formulations of the quiet zone conditions O_Q were investigated. The first formulation “pressure & velocity” resembles the original approach by minimizing the squared pressure magnitude and the squared particle velocity magnitude, i.e.

$$O_{Q,pv}(\vec{r}_Q) = w_{Q,p}|p(\vec{r}_Q)|^2 + w_{Q,v}|\vec{v}(\vec{r}_Q)|^2.$$

The other formulation “force & stiffness” instead minimizes the trap force and the trap stiffness in the quiet zone, i.e.

$$O_{Q,FS}(\vec{r}_Q) = w_{Q,S}|\nabla \cdot \vec{F}(\vec{r}_Q)|^2 + w_{Q,F}|\vec{F}(\vec{r}_Q)|^2.$$

This can be seen as a weaker requirement since it can allow higher sound pressure in the quiet zone, as long as the field is such that it will not create any force on beads in the quiet zone.

This procedure is performed twice per trap separation distance, resulting in two sets of phases and amplitudes

$$\begin{Bmatrix} \varphi_{j,A} \\ a_{j,A} \end{Bmatrix} \text{ from } \begin{Bmatrix} \vec{r}_T = \vec{r}_A \\ \vec{r}_Q = \vec{r}_B \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \varphi_{j,B} \\ a_{j,B} \end{Bmatrix} \text{ from } \begin{Bmatrix} \vec{r}_T = \vec{r}_B \\ \vec{r}_Q = \vec{r}_A \end{Bmatrix}.$$

These two sets of phases and amplitudes are superposed on the array element level to create the superposed sound field, as

$$c_j = \frac{a_{j,A}\mathrm{e}^{\mathrm{i}\varphi_{j,A}} + a_{j,B}\mathrm{e}^{\mathrm{i}\varphi_{j,B}}\mathrm{e}^{\mathrm{i}\Phi}}{2}$$

where $c_j = a_j\mathrm{e}^{\mathrm{i}\varphi_j}$ is the complex representation of the superposed element phase and amplitude. Note how the two fields are divided by two to not oversaturate the transducers, i.e. the total available power is split between the two traps. The additional phase Φ is added equally to all transducer elements in one of the fields. Phase shifting a single sound field by a constant addition does not change the structure of said field, since the absolute phase of the individual sound fields does not hold any physical significance other than as a point of reference. However, changing the relative phase between the fields will influence how the two fields interact in the regions of significant interference.

3.2 Trap stiffness variance

This method for two-bead levitation creates a field with two levitation traps directly by maximizing the trap stiffness and minimizing the squared pressure magnitude at the two point simultaneously, as

$$O_D(\vec{r}_A, \vec{r}_B) = w_S \nabla \cdot \vec{F}(\vec{r}_A) + w_p|p(\vec{r}_A)|^2 + w_S \nabla \cdot \vec{F}(\vec{r}_B) + w_p|p(\vec{r}_B)|^2.$$

Applying this idea directly will typically converge to one very strong trap and one very weak trap. To prevent this imbalance between the traps, the cost function is extended to also minimize the variance between the two trap stiffnesses, as

$$O_V(\vec{r}_A, \vec{r}_B) = O_D(\vec{r}_A, \vec{r}_B) + w_V \frac{(\nabla \cdot \vec{F}(\vec{r}_A) - \nabla \cdot \vec{F}(\vec{r}_B))^2}{2}$$

where the last term is a simplified expression for the variance of two values.

4 SIMULATED CASES

The two quiet zone methods described in section 3.1 and the variance based method described in section 3.2 were used to calculate the array element phases and amplitudes for two different array geometries, singlesided and doublesided, and two different trajectories of the moving trap position, horizontal and vertical, for a total of four different cases per method.

The singlesided array was chosen as a 16x16 element square matrix with 10 mm spacing between the elements, modeled as point sources operating at 40 kHz. Note that at 40 kHz the wavelength is approximately 8.5 mm, which mean that the array is undersampled in space and spatial aliasing will occur. One of the traps was always placed 80 mm above the center of the array.

The doublesided array consists of two arrays of the same geometry as the singlesided array, facing each other so that one is pointing up and the other is pointing down. The two halves are separated by 20 wavelengths, i.e. 17 cm. The fixed trap is in the center of the array both in vertical and horizontal directions, i.e. in the middle of the line between the centers of the two sides of the doublesided array.

For the horizontal test cases the second trap is placed at the same height as the first trap but at a distance away along the x-axis, i.e. along one of the major axes of the array grid. The vertical test cases instead has both traps positioned above each other with the second trap placed either above or below the first trap, i.e. along the z-axis.

5 RESULTS

5.1 Doublesided Arrays

Figures 1 to 3 show the positions of the two trap centers in the final sound fields for the three methods using the doublesided array. It is clear that when the trap separation is larger than the wavelength all three methods manage to successfully create the two traps in the correct positions. For trap separations smaller than one wavelength the physical limitations of the sound field restricts the solution space and the optimizer cannot successfully create the two traps in the positions specified. This is the so called breakdown region, and the three methods behave differently.

For horizontal separations the two quiet zone methods tend to create two separated traps shifted away from the target positions, but still separated by approximately one wavelength. This behavior is strongly influenced by the relative phase shift between the two fields, and if not chosen properly the two traps will instead merge to one large trap in between the two desired positions. For separations of less than 2 mm the quiet zone method with ”force & stiffness” formulation only produces a single trap. The variance method behaves like the quiet zone methods with incorrect relative phase, creating one larger trap between the two desired positions.

For vertical trap separations the quiet zone method with ”pressure & velocity” quiet zones tend to shift the positions randomly in the horizontal directions while still maintaining a vertical separation of the beads. The ”force & stiffness” formulation is much more stable in this orientation, introducing a slight shift in the vertical position of the traps. The variance method manages to create two separated traps down to approximately half of the wavelength, although with some shifts in the horizontal directions. For trap separations smaller than half of the wavelength the variance method completely fails to produce any traps.

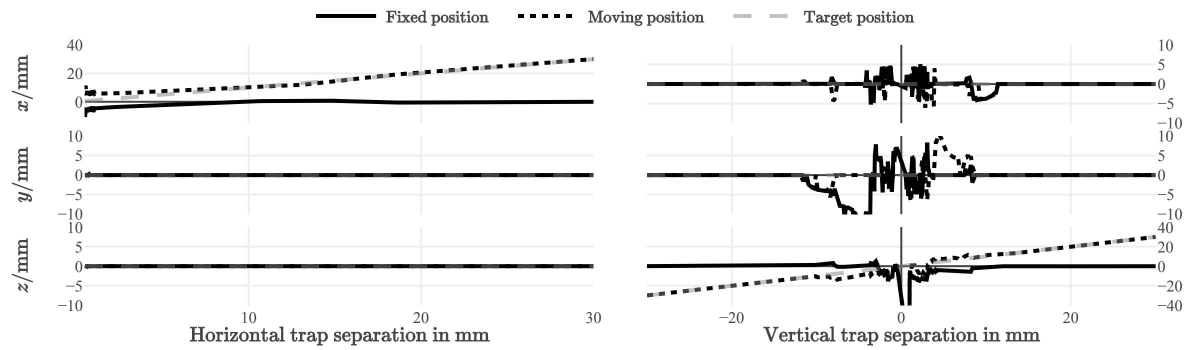

Figure 1. Final positions of the two traps when using a doublesided array and superposition of two fields with ”pressure & velocity” quiet zones.

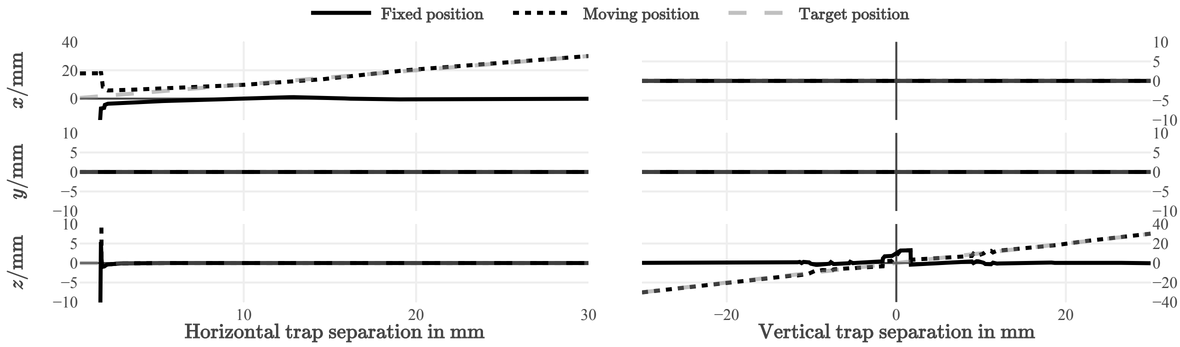

Figure 2. Final positions of the two traps when using a doublesided array and superposition of two fields with ”force & stiffness” quiet zones.

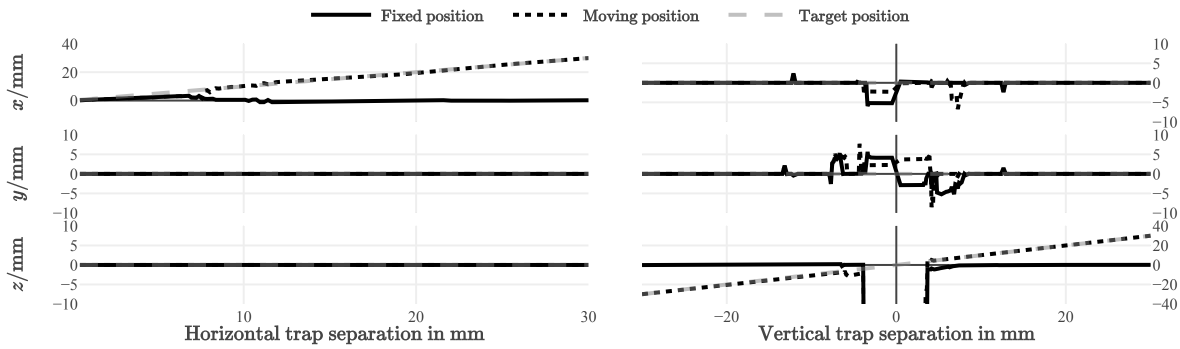

Figure 3. Final positions of the two traps when using a doublesided array and the stiffness variance based method.

5.2 Singlesided Arrays

Figures 4 to 6 show the positions of the two trap centers in the final sound fields for the three methods using the singlesided array.

The quiet zone method with the ”pressure & velocity” formulation creates traps which horizontally behaves similar to the doublesided versions, keeping the separation of at least a wavelength by moving the two traps slightly. It does not, however, manage to create the traps at the desired height. This is linked to the fact that most of the vertical force comes from the traveling nature of the wave, and the force gradient is much smaller than for the doublesided array. This causes small changes in the overall sound field amplitude to change the vertical position of the trap. The quiet zone method with the ”force & stiffness” formulation creates horizontally separated traps down to a requested separation of one wavelength, but again with a shift in the vertical position of the traps. For smaller separations it does not manage to create two traps, regardless of the relative phase shift between the two fields.

The variance based method only manages to create traps at the correct positions when the horizontal separation is larger than 18 mm, and produces no traps at all for separations smaller than one wavelength.

For vertical separations of the traps all three algorithms fail to keep the traps apart for most of the range, instead merging the two traps to a single trap. If the vertical trap separation is very large the traps are created at different heights as intended. This occurs for smaller separations closer to the array, when the traps are approximately 50 mm and 80 mm away from the array, but for the ”pressure & velocity” quiet zone formulation it also works for trap positions around 80 mm and 110 mm.

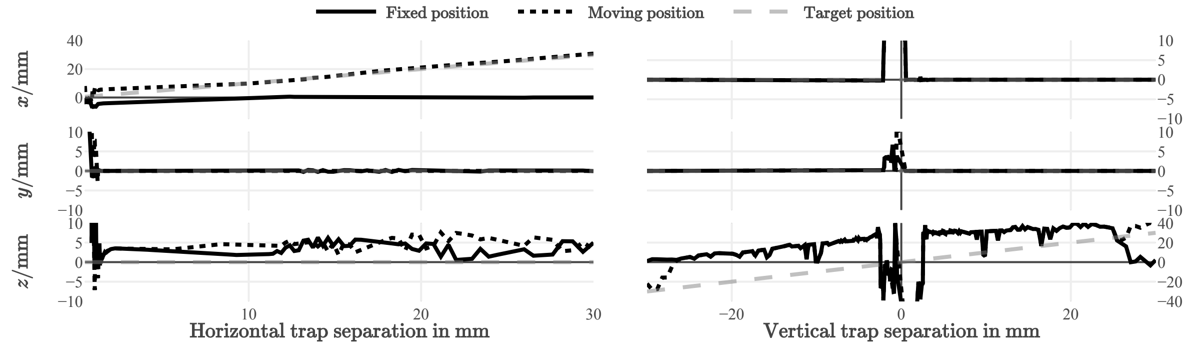

Figure 4. Final positions of the two traps when using a singlesided array and superposition of two fields with "pressure & velocity" quiet zones.

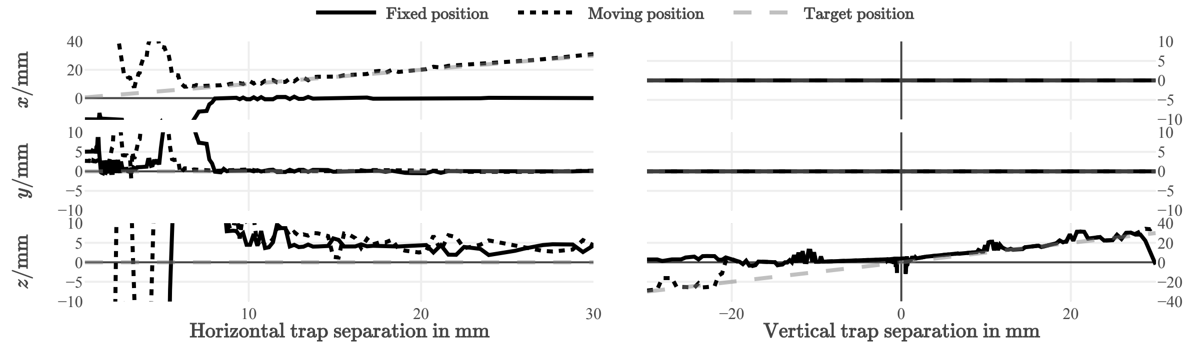

Figure 5. Final positions of the two traps when using a singlesided array and superposition of two fields with "force & stiffness" quiet zones.

6 CONCLUSIONS

The creation of two acoustic levitation traps in close proximity to each other were investigated in the context of using ultrasonic transducer phased arrays. Three different methods based on numerical optimization were used, as well as both doublesided and singlesided arrays. Traps were separated by various distances, either in the same horizontal plane or on the same vertical axis.

For doublesided arrays all three methods manage to create separated traps down to one wavelength horizontal separation or half of a wavelength vertical trap separation. Singlesided arrays manage to create separated traps down to one wavelength separation in the horizontal plane, although not always on the correct height. When placing two traps on the same vertical axis above a singlesided array the traps needs to be separated by at least 25 mm, depending on the design method used and the distance from the array.

For trap separations of less than one wavelength (one half wavelength in the vertical direction for doublesided arrays) none of the investigated methods is able to achieve the desired trap placement. If a quiet zone based superposition is used and a suitable relative phase between the two fields is found there will still be two separated traps, but shifted in position to maintain the required separation.

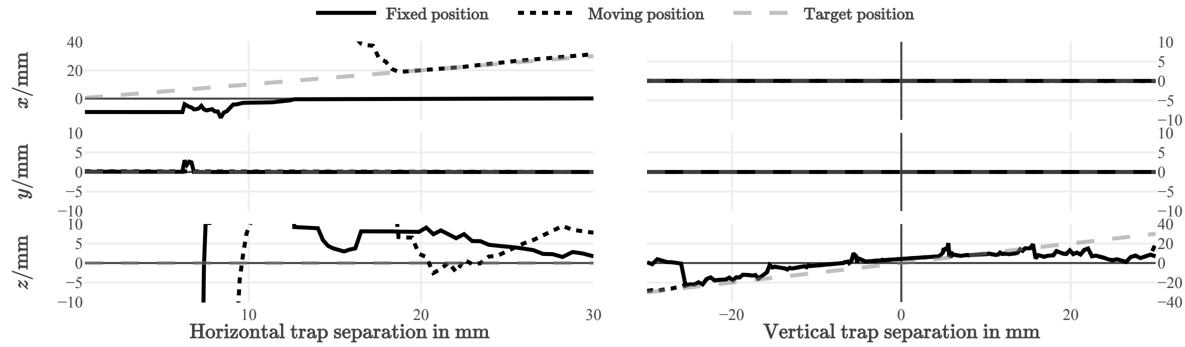

Figure 6. Final positions of the two traps when using a singlesided array and the stiffness variance based method.

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