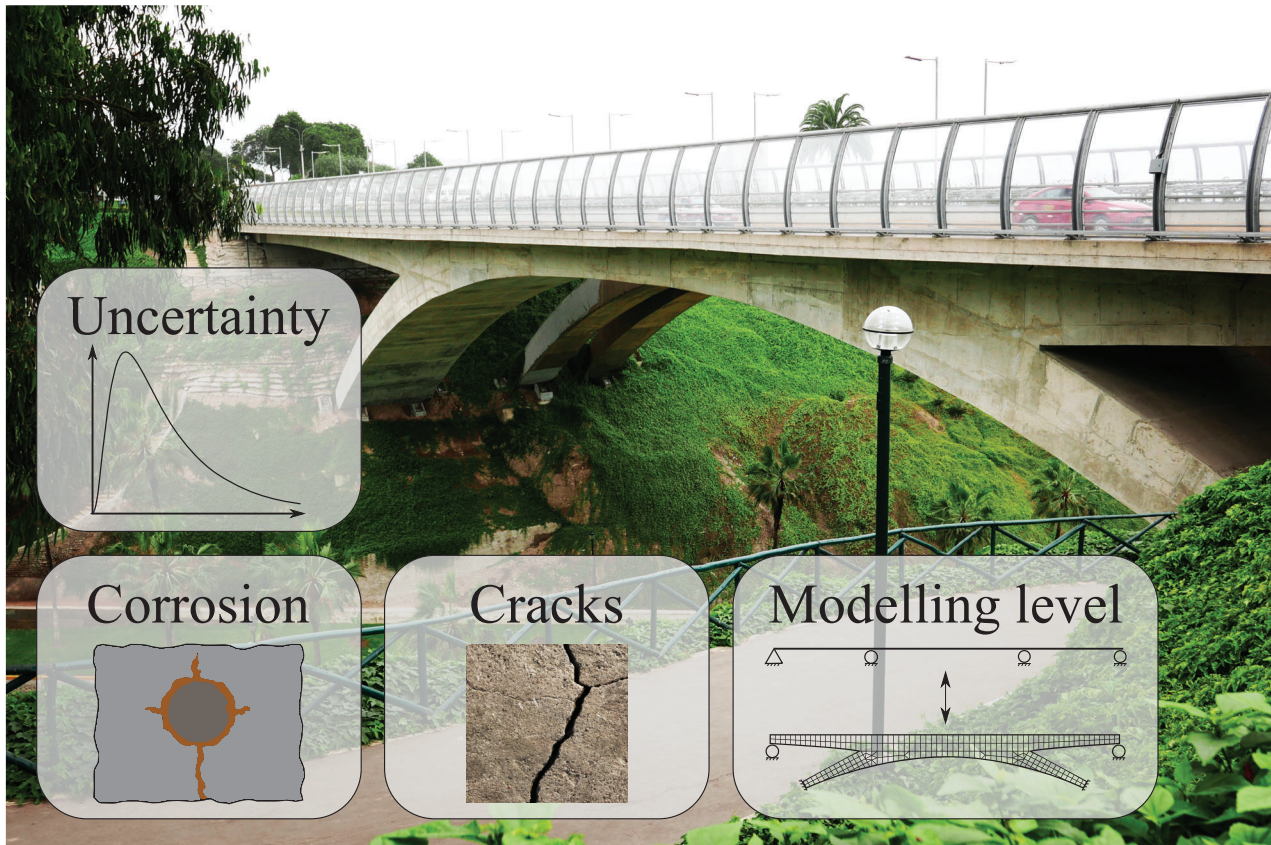




CHALMERS



Assessment of Concrete Structures Including Corrosion and Cracks

MATTIAS BLOMFORS

THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Assessment of Concrete Structures Including Corrosion and Cracks

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Concrete Structures

CHALMERS UNIVERSITY OF TECHNOLOGY

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Cover:

Illustrations of aspects treated in this thesis, such as uncertainty, modelling level, cracks and corrosion, are shown in the foreground. In the background a photograph of an example reinforced concrete bridge located in the Miraflores district of Lima, Peru, is shown.

Chalmers Reproservice

Gothenburg, Sweden 2020

To Lova and Sara

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Abstract

Reinforced concrete (RC) structures constitute a major proportion of the built environment and society relies continuously on their service. Many of these structures were built in the era following the Second World War and are thus approaching the end of their intended service life. The likelihood of deterioration increases with time and so damage caused by, say, corrosion is not uncommon. Also, increased demands are often laid on the load-carrying capacity of existing bridges, aimed at increasing utilisation of the road network by allowing heavier vehicles. Simply dismantling and re-constructing all bridges at the end of their designed service life, or taking needless strengthening measures, is unsustainable. Rather, improved methods of assessing the capacity of existing infrastructure are needed.

The current work has aimed to develop improved, reliable assessment methods. Its focus areas were structures with reinforcement corrosion and structures with cracks from previous loading. Both simplified and advanced methods of evaluating anchorage capacity were developed for concrete structures with corroded reinforcement. The simplified method modifies the bond stress-slip relationship and is calibrated against a large database of bond tests, with the safety margin ensured by deriving partial safety factors. The advanced method is based on finite element (FE) analysis, with tensile material properties altered for elements positioned at the splitting cracks along the reinforcement. The latter method was also investigated for RC without corrosion damage but with cracks from previous loading. Design results from advanced nonlinear FE analyses (meaning results with a proper safety margin) are obtained by applying a “safety format”. The current work investigated whether safety formats available in *fib* Model Code 2010 also ensured reliable design capacities for structures with somewhat complicated load application and geometry; in this case, a concrete frame subjected to vertical and horizontal loads.

The results indicate that the anchorage capacity may be reasonably well estimated by using the simplified method. The proposed partial safety factors also provided sufficient safety margin. Furthermore, in the advanced anchorage assessment, the capacity could be estimated solely from weakened tensile properties located at the position of the splitting cracks and without input concerning the corrosion level. Moreover, by including cracks from previous loading in advanced modelling, improved predictions of the failure mode, ultimate capacity and ductility were demonstrated. Lastly, in the investigation of safety formats for nonlinear FE analysis, the method of *estimating a coefficient of variance of resistance* (ECOV), did not reach the intended safety level. However, the *global resistance factor method* (GRF) and *partial factor method* (PSF) did.

This work has the potential to improve both simplified and advanced assessment methods, providing more sustainable infrastructure management in the future.

Keywords: reinforced concrete, corrosion, reinforcement bond, nonlinear FE analysis, cracks, assessment

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Sammanfattning

Betongkonstruktioner utgör en stor del av bebyggelsen i den industrialiserade världen och samhället förlitar sig ständigt på deras funktion. Många av dem byggdes under återhämtningen efter andra världskriget och börjar därför närma sig slutet på sin tekniska livslängd. Sannolikheten för skador ökar också över tid, så exempelvis korrosionsskador är inte ovanligt. Vidare efterfrågas med tiden generellt sett ökad bärförmåga hos befintliga broar, så att tyngre fordon kan användas och därigenom öka vägnätets resursutnyttjande. Att helt sonika riva äldre broar vid slutet av deras tekniska livslängd och bygga nya är varken miljömässigt eller ekonomiskt hållbart. Istället behövs bättre metoder för att bedöma kapaciteten hos befintliga betongkonstruktioner.

Målet med denna avhandling var att utveckla metoder för tillförlitliga tillståndsbedömningar. Arbetet fokuserades på konstruktioner med armeringskorrosion och på konstruktioner med sprickbildning på grund av tidigare belastning. För betongkonstruktioner med rostande armering utvecklades både förenklade och avancerade metoder för att bedöma förankringskapaciteten. Den förenklade metoden innebar förändrad vidhäftning som funktion av glidning och kalibrerades mot en stor testdatabas. Vidare tryggades säkerhetsmarginalen hos beräkningsresultaten genom att säkerhetsfaktorer för modellen togs fram. I den avancerade metoden, som baseras på finita element (FE) analys, försvagades betongens dragegenskaper hos de element som innehöll de spjälksprickor som den korroderade armeringen orsakat. Den avancerade metoden användes också för betongbalkar utan korrosionsskador, men med sprickor på grund av tidigare belastning. För att erhålla resultat i enlighet med specificerad säkerhetsnivå från icke-linjära FE analyser används så kallade säkerhetsformat. I detta arbete undersöktes om säkerhetsformaten som finns tillgängliga i *fib* Model Code 2010 ger önskad säkerhetsnivå även för konstruktioner med tämligen komplicerad geometri och belastning, närmare bestämt en betongram utsatt för vertikal och horisontell last.

Resultaten visar att försäkringskapaciteten kan uppskattas med hjälp av förenklade metoder. Vidare gav de framtagna säkerhetsfaktorerna tillräcklig säkerhetsnivå i resultaten. Med den avancerade metoden kunde förankringskapaciteten uppskattas enbart genom försvagade element vid spjälksprickornas positioner, utan att använda korrosionsnivån i analysen. Dessutom, genom att sprickor på grund av tidigare belastning beaktades i den avancerade modellen, förbättrades bedömningar av brottmod, kapacitet och seghet. Slutligen, ett av säkerhetsformaten (förkortat ECOV) för icke-linjär FE analys gav för låg säkerhetsnivå medan de två andra gav tillfredsställande nivå (förkortade GRF och PSF)

Arbetet har möjliggjort förbättringar av både förenklade och avancerade metoder för tillståndsbedömning och tagit ett steg mot mer hållbar infrastrukturförvaltning i framtiden.

Nyckelord: armerad betong, korrosion, vidhäftning, icke-linjär FE analys, sprickor, tillståndsbedömning

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APPENDED PAPERS

Paper I: Evaluation of safety formats for non-linear finite element analyses of statically indeterminate concrete structures subjected to different load paths

Paper II: Engineering bond model for corroded reinforcement

Paper III: Partial safety factors for the anchorage capacity of corroded reinforcement bars in concrete

Paper IV: Incorporation of pre-existing longitudinal cracks in finite element analyses of corroded reinforced concrete beams failing in anchorage

Paper V: Incorporation of pre-existing cracks in finite element analyses of reinforced concrete beams prone to shear failure

BIBLIOGRAPHY, CONCRETE STRUCTURES

Doctoral Theses

Licentiate Theses

Preface

The work presented in this thesis was conducted mainly within two research projects. The first was conducted between October 2015 and September 2018, at the Division of Structural Engineering at Chalmers University of Technology in Gothenburg and the CBI Swedish Cement and Concrete Research Institute in Borås, Sweden. This research project was funded by the Swedish Road Administration, CBI Swedish Cement and Concrete Research Institute's A-consortium and SBUF. The work on the second research project was conducted between October 2018 and September 2020, at Chalmers University of Technology and with financial support from FORMAS.

Firstly, I would like to thank my supervisor Assoc. Prof. Kamyab Zandi for his continuous feedback, his positivity and his responsiveness to my wishes throughout the years. I also want to thank my co-supervisor and examiner, Prof. Karin Lundgren for providing countless valuable ideas and comments. Her extraordinary ability to bring clarity and reassurance in times of need has made this journey much more enjoyable. Thanks also to Oskar Larsson Ivanov who, in his role as co-supervisor for the first part of this work, provided much-appreciated input. I would also like to acknowledge Dániel Honfi and Per Kettil, for their good ideas and comments. And I would like to thank all reference group members, in both research projects, for sharing their knowledge and helping guide the work. Lastly, I would like to thank Morten Engen, for being a source of inspiration and awakening my interest in research six years ago.

To my current and former colleagues at the Division of Structural Engineering, thanks for creating an open and friendly environment and for sharing your thoughts and experiences. Carlos Gil Berrocal, I'm glad to have worked more closely with you this last year. Thanks for sharing your knowledge, experience and humour. Ignasi Fernandez Perez, our lives have changed quite a lot since we went racing with our bikes, but I think you'll agree that it's mostly for the better. Adam Ściegaj, your sharp mind and sense of humour makes every discussion interesting. Samanta Robuschi, it has been a pleasure to get to know the most untraditional Italian and observe the culinary progress you have made. Sebastian Almfeldt, sincere thanks for your help during the experimental part of this work. You always took the time to explain things in an informative manner, with one or two jokes along the way.

This work would not have been possible without the support of my family; in particular from my beautiful wife, Sara. I'm so happy we found each other; we are so different but at the same time so very much alike. Thank you for showing me what is truly important in life. My beloved daughter Lova, you effectively prevented me from worrying too much about work, when at home. Stories, dance and laughter filled our evenings instead. Oh, I am so happy I've got you!



Mattias Blomfors

Gothenburg, 2020

List of publications

This compilation thesis consists of an introductory part and five appended publications, specifically:

Journal Papers

- I. Blomfors, M., Engen, M., Plos, M. (2016), Evaluation of safety formats for non-linear finite element analyses of statically indeterminate concrete structures subjected to different load paths, *Structural Concrete*, 17: 44-51. doi: 10.1002/suco.201500059.
- II. Blomfors M., Zandi K., Lundgren K., Coronelli D. (2018), Engineering bond model for corroded reinforcement, *Engineering Structures*, 156: 394-410. doi: 10.1016/j.engstruct.2017.11.030.
- III. Blomfors M., Larsson Ivanov O., Honfi D., Engen M. (2019), Partial safety factors for the anchorage capacity of corroded reinforcement bars in concrete, *Engineering Structures*, 181: 579-588. doi: 10.1016/j.engstruct.2018.12.011.
- IV. Blomfors M., Lundgren K., Zandi K. (2020), Incorporation of pre-existing longitudinal cracks in finite element analyses of corroded reinforced concrete beams failing in anchorage, *Structure and Infrastructure Engineering*. doi: 10.1080/15732479.2020.1782444.
- V. Blomfors M., Berrocal C.G., Lundgren K., Zandi K. (2020), Incorporation of pre-existing cracks in finite element analyses of reinforced concrete beams prone to shear failure. *Submitted for publication*.

AUTHOR'S CONTRIBUTION TO JOINTLY PUBLISHED PAPERS

The appended papers were prepared in collaboration with co-authors. The contribution of the author of this doctoral thesis to the appended papers is described below.

In **Paper I**, the author was partly responsible for planning the study. He conducted the literature review, conducted the non-linear finite element analyses and applied the reliability methods. The author also took the lead in writing the paper and the co-authors assisted in planning the study, discussing the results and writing the paper.

In **Paper II**, the author planned the main part of the paper, developed the bond model and took the lead in writing the paper. The co-authors participated in discussing the model development and results and in writing the paper.

In **Paper III**, the author planned the main part of the paper, derived the partial safety factors and conducted the reliability analyses. The author also took the lead in writing the paper. The co-authors participated in discussing the methods and results and in writing the paper.

In **Paper IV**, the author planned the main part of the paper, developed the methods for implementing cracks and conducted the finite element analyses. The author also took the lead in writing the paper. The co-authors participated in the discussion of the methods and results and in writing the paper.

In **Paper V**, the author was, with Berrocal, responsible for planning and executing the experimental study. The author planned the main part of the paper, further developed the methods for implementing the cracks and conducted the finite element analyses. The author also took the lead in writing the paper. The co-authors participated in the discussion of the methods and results and in writing the paper.

OTHER PUBLICATIONS RELATED TO THIS THESIS:

In addition to the appended papers, the author of this thesis has also contributed to the following publications.

Licentiate Thesis

Blomfors M. (2017). *Reliable Assessments of Concrete Structures with Corroded Reinforcement: An Engineering Approach*. Chalmers University of Technology, Licentiate Thesis, Gothenburg, ISSN 1652-9146.

Conference Papers

- C-I. Blomfors M., Zandi K., Lundgren K., Larsson O., Honfi D. (2016): Engineering Assessment Method for Anchorage in Corroded Reinforced Concrete, in *19th IABSE Congress Stockholm, Sweden 21-23 September 2016*, pp. 2109-2116.
- C-II. Blomfors M., Zandi K., Lundgren K. (2016): Development of engineering assessment method for anchorage in reinforced concrete, in *Nordic Concrete Research*, 2/2016, pp. 63–78.
- C-III. Blomfors M., Zandi K., Lundgren K., Larsson O., Honfi D. (2017): Reliable Engineering Assessments of Corroded Concrete Structures, in *XXIII Nordic Concrete Research Symposium, Aalborg, Denmark, 21st-23rd August 2017*, pp. 245-248
- C-IV. Blomfors M., Larsson O., Honfi D., Zandi K., Lundgren K. (2018): Reliability analysis of corroded reinforced concrete beam with regards to anchorage failure, in *6th International Symposium on Life-Cycle Civil Engineering, Ghent, Belgium, 28th-31st October 2018*, pp. 337-344.
- C-V. Blomfors M., Zandi K., Lundgren K. (2019): Incorporation of cracks in finite element modelling of existing concrete structures, in *12th International Workshop on Structural Health Monitoring, Stanford, USA, 10th-12th September 2019*, pp. 1611-1618.

Popular Science Papers

- P-I. Tahershamsi M., Blomfors M., Fernandez I., Lundgren K., Zandi K. (2016): Modellering av förankringskapaciteten i betongkonstruktioner med rostande armering, *Bygg & Teknik*, 7/16, pp. 16-20.
- P-II. Blomfors M., Lundgren K., Zandi K. (2019): Digitala tvillingmodeller underlättar bedömning av risker, *Samhällsbyggaren*, 4/19, pp. 24-25.

1 Introduction

1.1 Background

Reinforced concrete (RC) structures constitute a large part of the built environment and society continuously relies on their service. Core societal features such as transportation, shelter and electricity depend on well-functioning concrete structures such as bridges, houses, dams, and so on. Many concrete structures were built during the economic expansion following the Second World War [1] and are now approaching the end of their designed service life. Enabling a safe service-life extension of those structures, or accommodating increased loads often requires an assessment of capacity [2]. As long-term exposure to the environment increases the likelihood of deterioration, the structural models used in these assessments must be able to account for potential deterioration. Reinforcement corrosion is the most common type of deterioration in concrete structures [3]. Advanced corrosion may result in considerable loss of reinforcement bar cross-section and reduced bond properties due to loss of confinement and rib area, plus the weak frictional properties of the corrosion products [4]. Unlike the ductile failure modes typically sought during design, loss of bond might lead to an abrupt failure [5], meaning that users of the structure are unable to avoid imminent danger. Thus, bond capacity may be a major concern when assessing corroded structures. This phenomenon has been studied in recent decades, cf. [6–10] and advanced models have been developed to simulate the phenomenon, cf. [11–13]. Nevertheless, when practical assessments of corrosion-damaged structures are conducted, advanced methods are typically unfeasible due to the large amount of time they take. Simply disregarding the contribution of corroded reinforcement to capacity is a quick and practical approach but may lead to overly conservative estimates of structural capacity. Thus, a need has been identified for an engineering method of anchorage assessment. To enable practical use of the results obtained from such a method, they must be able to provide a sufficient safety margin (or reliability) [14]. To this end, “safety formats” are used to ensure that the failure probability is less than acceptable. Semi-probabilistic safety formats in the form of partial safety factors are most commonly used [15]. For a model to assess the anchorage of corroded reinforcement (and with professional engineers as the intended users), the author believes the well-known partial safety factor approach to be preferable in ensuring safety.

The use of simplified models of anchorage assessment of corroded bars is a step forward from overly conservative assumptions. There are also other types of structural analyses (of, say, shear capacity) for which increased complexity may be justified. The complexity of structural analyses varies, from simple hand calculations to detailed numerical analyses, often conducted by nonlinear finite element (NLFE) analysis. The underlying principle is that, as the complexity of the analysis increases, so does the accuracy of the estimate and the effort from the analyst [16]. In other words, a structure’s capacity might be more accurately assessed if a more elaborate structural analysis is used. However, this increases demand on the analyst’s competence, time and resources. In structural analyses, complex methods (such as NLFE analyses) are typically used to capture realistic behaviour of a structure. When conducting an NLFE analysis, choices need to be made concerning force equilibrium, kinematic compatibility and constitutive relations [17]. These choices influence the “modelling uncertainty” (accuracy of prediction), which have been studied in [18, 19] for example. Guidelines on how to conduct NLFE analyses of concrete structures have also been published [20]. The author regards such

recommendations as an important step towards more consistent, and hopefully also decreased, modelling uncertainty. Furthermore, since NLFE analyses evaluate capacity on the global level, safety formats based on global resistance have been put forward which provide a design resistance with a sufficient safety margin, cf. [16, 21]. Since the modelling uncertainty needs to be considered when determining the design resistance, an underestimate may lead to insufficient reliability. However, concerning the NLFE safety formats suggested by *fib* [16], it is unclear how the loading sequence should be addressed. It is, therefore, of interest to investigate whether different choices regarding the loading sequence may influence the resulting design resistance, particularly for more complex loading and geometry.

In assessing structures, their behaviour at the time of the assessment should be evaluated [22]. Furthermore, cracking in RC structures is common and not necessarily undesirable, as cracking is needed if mild reinforcement steel is to carry any significant tensile force. The causes of cracking in concrete members are numerous. A non-exhaustive list includes (apart from cracking induced by reinforcement corrosion) cracking due to other types of internal and external restraint and due to live loads acting on the structure [23]. Design codes stipulate crack control measures that meet durability and aesthetic requirements, cf. [24, 25]. However, cracking may also affect the ductility and structural capacity of an RC member [26], thus posing a risk to the integrity of the structure. Although the design codes are followed, some cracks may grow past the specified limits. Therefore, there is a need for improved assessment methods for reinforced concrete infrastructure, that are capable of incorporating cracks formed in the structure at the time of assessment. These are referred to as pre-existing cracks since they are present at the start of the NLFE analysis (as opposed to cracks forming during the analysis). One way of facilitating an assessment with updated structural information is to use Digital Twin (DT) models [27]. A DT is a virtual copy of the structure in which information collected during its service-life (through, say, different types of sensors) is stored. In a proposal for a DT of civil infrastructure [28], FE analysis provides insights on the structural capacity based on updated information about the structure. The latest advancements in techniques of data collection, such as the application of inspection and monitoring drones [29–31] and fiberoptic sensing [32, 33], allow for crack data to be collected with unprecedented accuracy and frequency. Therefore, to leverage information on pre-existing cracks, the FE simulations used in structural assessments need to include this information.

There is a need for improved assessment methods, to make better use of current concrete infrastructure and avoid unnecessary repair and decommissioning. Some avenues for improving current assessment methods are pursued in this thesis.

1.2 Aim of the research

This research aimed to develop reliable structural assessment methods, for concrete structures with corrosion damage and pre-existing crack patterns. Accordingly, the following questions were formulated and addressed:

- might available safety formats for nonlinear analyses also be used for structures with more complicated global failure mechanisms than a simple concrete beam? For example, an indeterminate structure subjected to non-proportional loading;
- is it possible to obtain representative estimates of the anchorage capacity of corroded reinforcement using a simplified model that is suitable for an engineering context?
- might such a model be used in conjunction with a semi-probabilistic safety format to ensure a sufficient safety margin?
- might the influence of cracks on the structural behaviour be represented without modelling their formation (load history)? Specifically:
 - the effect of corrosion-induced splitting cracks (along the longitudinal reinforcement) on the anchorage capacity;
 - the effect of cracks (formed under previous external loading) on the moment and shear capacity.

1.3 Methodology

The above research questions were answered by completing the following tasks, listed in the same order:

- investigate the applicability of various safety formats to a statically indeterminate frame, subjected to vertical and horizontal loading;
- develop and validate a model for anchorage assessment of corroded reinforcement, intended for use in practical applications;
- equip the proposed model for anchorage assessment with a semi-probabilistic safety format, to ensure a proper safety margin for the results;
- investigate methods of incorporating corrosion-induced longitudinal cracks in structural analysis;
- explore the extendibility of a selection of the aforementioned methods, to incorporate cracks due to previous external loading.

1.4 Limitations

A summary of the main limitations of the present work is presented below:

- the evaluation of safety formats in **Paper I** considered one type of structure with a certain load configuration. An exhaustive evaluation would also consider other types of structures and, say, other types of loading;
- only failures due to loss of anchorage were considered for corroded structures. In other words, reduction of reinforcement cross-section leading to other failure modes was not studied in **Papers II-IV**;
- in **Paper II**, the calculation model for anchorage was calibrated based on experimental tests of corroded specimens. These specimens, and those used for validation in **Paper IV**, were artificially corroded. Although the transferability to natural corrosion is

unclear, artificial corrosion induced by relatively low-current densities was considered acceptable due to the lack of experimental results with natural corrosion;

- in **Paper III**, when calibrating the partial safety factors and verifying the reliability levels, no time-dependence of the resistance due to damage propagation was considered. A reference period of one year was chosen, to reduce the influence of this simplification;
- the incorporation of cracks in **Paper V** only considered cracks due to previous external loading. Since this thesis also includes work with corrosion damage, it is important to clarify that, in the case of corrosion-induced cracks, the corrosion level (and possibly also the distribution and shape of the pits) needs to be accounted for when assessing bending and shear failure modes. This was not addressed in the current work.

1.5 Original features

The original features of the present work may be summarised as follows:

- a bond model, based on a 1D bond stress-slip relationship, was developed and calibrated for corroded reinforcement bars in concrete;
- partial safety factors were derived for the above model;
- methods for incorporating pre-existing cracks into NLFEM analysis were developed, based on the visually obtained crack pattern.

1.6 Outline of the thesis

This thesis consists of an introductory part and five appended papers. The former is divided into six sections:

Section 1 gives a background to the work, and presents its aims, methods and limitations, plus its original features.

Section 2 presents an overview of structural assessments.

Section 3 describes the structural consequences of damage from reinforcement corrosion and also from cracks induced by previous loading.

Section 4 provides an overview of structural reliability analyses.

Section 5 briefly summarises the appended papers.

Section 6 states the main conclusion drawn from this work and puts forward suggestions for future research.

2 Overview of structural assessment

This thesis deals with structural assessments of RC structures. An overview is given below and describes where the proposed models and methods fit into the framework of assessment. It is based on [22] which, in turn, is based on [14, 34 and 35].

The design of new structures and assessment of existing ones are fundamentally different, in that the latter has already been realised. At the design stage of a building process, many circumstances are unknown and will be determined in the future. For example, any differences between the original and the as-built design, or any structural damage sustained during the service life of the structure. However, for an existing structure, this information is known (to some extent at least). The original design of the structure, the as-built design, or both, may have been subjected to changes. The structure may also have been damaged due to misuse or deterioration (with reinforcement corrosion being the most common example [36] for concrete structures). A proper structural assessment will use the current structural state and loading conditions.

The safety of a structure is typically assessed for one (or more) of the following reasons:

- changes in use: to demonstrate the safety of a structure if the conditions of use (and thereby the associated loads) have changed;
- addition of new structural members: to study the consequences for structural behaviour if new elements are added to the structural system;
- in the case of repair: to determine suitable repair measures. These may differ depending on the damage (caused by, say, accidents, natural phenomena or environmental impacts). The safety level of a repaired structure is also of interest;
- doubts as to safety: to study the structural safety relating to concerns arising due to other reasons;
- other circumstances: to accommodate any requirements by insurance companies, official bodies or owners.

Two main principles are generally followed in reliability assessments:

- use of current codes: the codes valid at the time of assessment should be used. Earlier codes, such as those valid at the time of the structure's design, should serve only as guidance;
- use of actual structure: the actual geometries and actual applied loads plus in-situ material properties should be used. Given that the structural behaviour at the time of assessment should be estimated, design documentation should only be used for guidance.

An overview of the processes relating to structural reliability assessment is given in Figure 2.1.

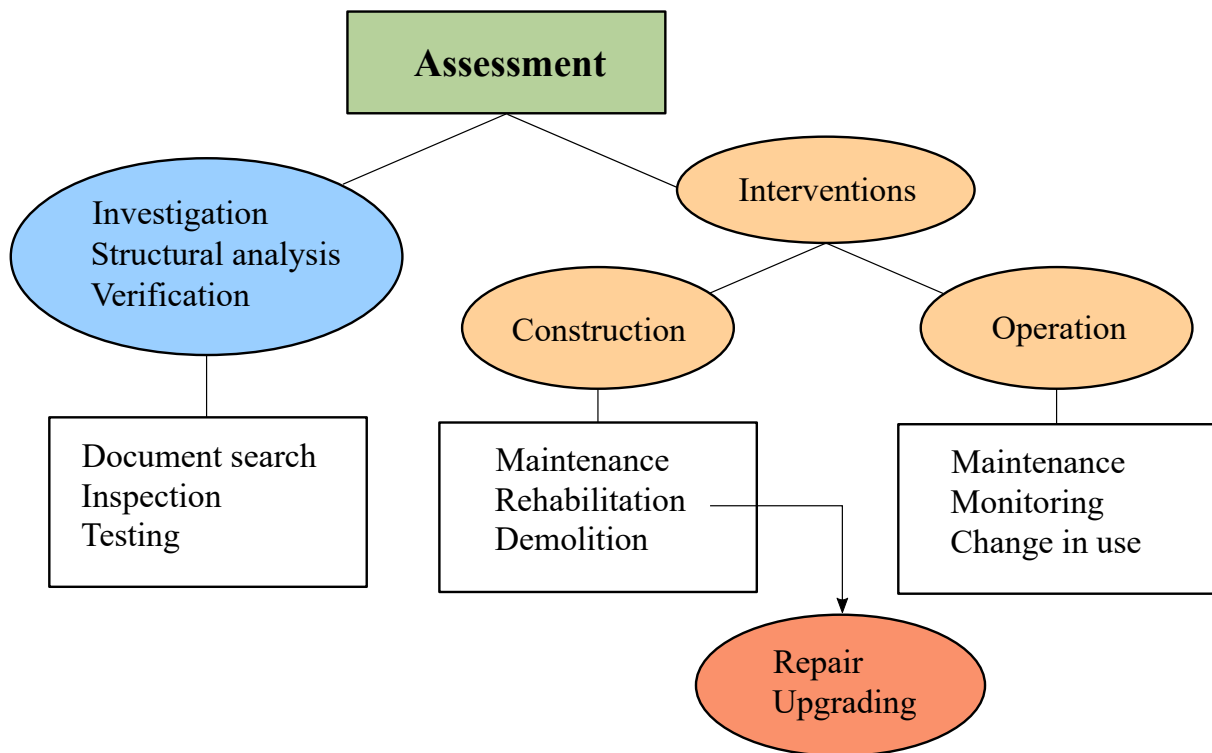


Figure 2.1. Overview of the processes related to structural reliability assessment of an existing structure, adopted from [14].

Not all parts of a structure have to be included in all assessments. Some parts may be excluded if unaffected by changes (due to repair or change of use, for example) and are neither damaged nor suspected of being insufficiently reliable. The processes involved in structural reliability assessment are illustrated in a flow chart in Figure 2.2.

At the beginning of an assessment process, the objectives (in terms of the structure's future performance) should be set by the client, the assessing engineer and any relevant authorities. In the next step, likely scenarios associated with changes to structural conditions or actions should be specified, to help identify any critical situations for the structure. The assessment and any interventions to ensure the structure's reliability are based on these identified scenarios.

A preliminary assessment is then started, in which the state of knowledge concerning the structure is established by studying available documents and other material. The structural system and any damage are identified by visual examination during a preliminary on-site inspection. Damage detectible by visible inspection typically comprises deformations, cracks, spalling and signs of corrosion. The damage is graded in qualitative terms (for example "none", "minor", "moderate", "severe", "destructive" or "unknown". It is worth noting that the corrosion level is difficult to quantify using non-destructive measures, as elaborated upon in Section 3.2.1. The information acquired in the preliminary assessment serves as a basis for initial checks to identify current and future deficiencies that are important to the structure's safety and serviceability. If these preliminary checks clearly indicate an unsafe condition for the structure, danger to the public should be mitigated by prescribing immediate measures. The preliminary assessment also forms a basis for determining whether further investigations are necessary. If so, a detailed assessment may be conducted.

A detailed assessment includes an in-depth study of all available documentary information. If concerns are raised regarding the trustworthiness of the information (regarding the structural

dimensions and the material properties, for example), this should be collected instead during the detailed inspection and through material testing. The quantitative results from the detailed inspection will yield updated values for relevant parameters in the structural analysis, plus the actions that have been determined for the structure. As noted, the structural analysis comprises information about the load effects from actions on the structure and the capacity of its structural components. It is, therefore, of the utmost importance that any deterioration of the existing structure is considered in the analysis and that suitable reliability assessment methods are used. **Papers II, IV and V** contribute to this area. Structural analysis, including reinforcement corrosion, was conducted in the first two of these papers and pre-existing cracks were included in the third. It is also possible to use non-destructive testing to estimate the load-bearing capacity and certain properties of a structure.

Verification of the structural safety is conducted by ensuring that the structure meets the target reliability level (which was the topic of **Papers I and III** in this thesis). The verification may be done using “adjusted partial safety factors”, which may consider alternative values for the reliability index, remaining service-life and updated information of, say, material properties [37]. Moreover, the verification basis may comprise the past performance of the structure. The assessment results should be documented in a report, providing conclusions and suggested interventions.

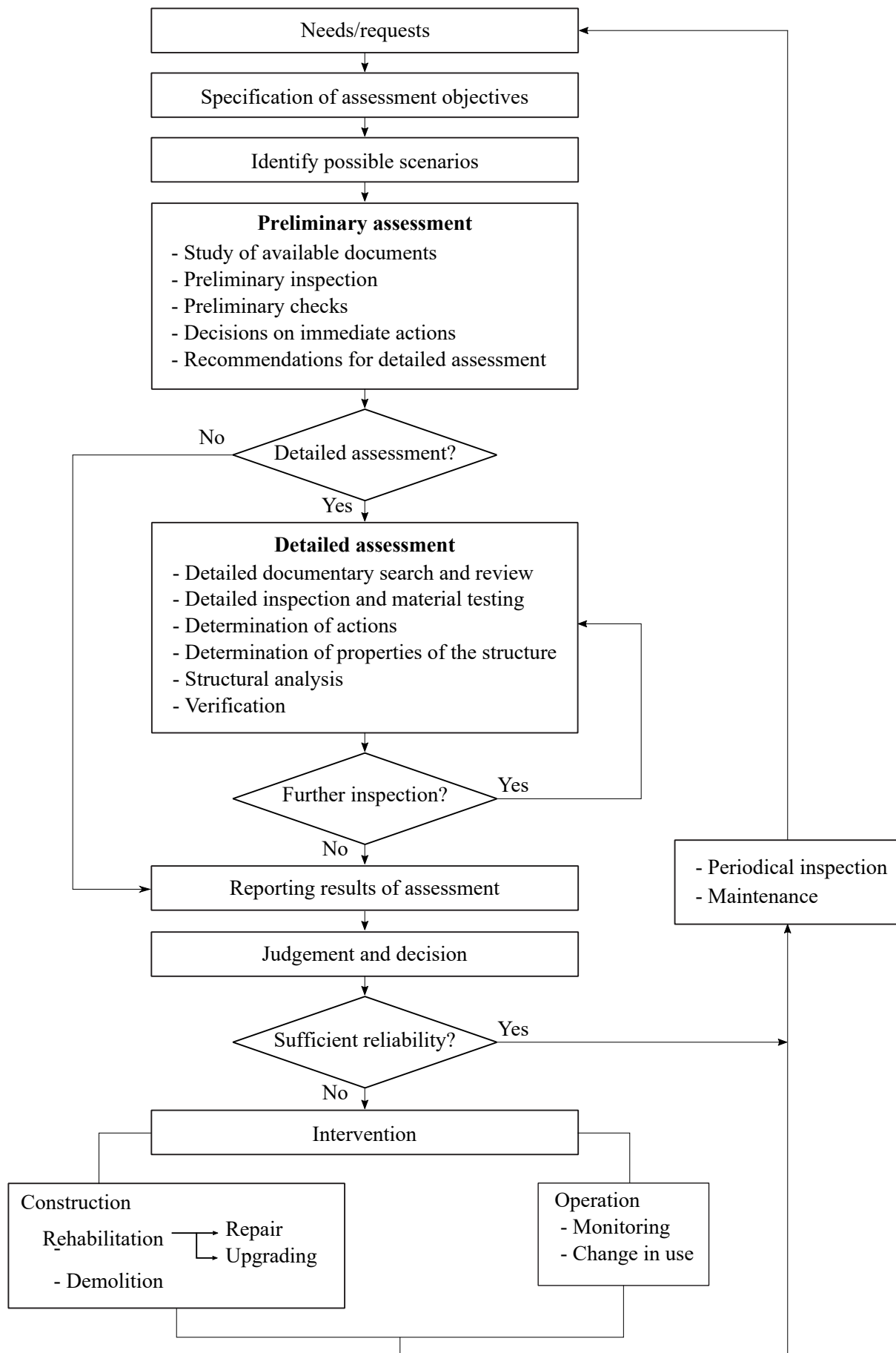


Figure 2.2. Flow chart for structural safety assessments, adopted from [14].

3 Structural analysis of damaged structures

Section 2 described how assessments involve structural analyses of the actual structure. This includes the *as-built* geometry and any damage. Therefore, it is important that the structural analysis methods available to engineers can factor in key damage mechanisms. This work addressed elements of this research area, by proposing both simplified and advanced methods of including the effect of reinforcement corrosion on anchorage capacity (**Papers II and IV**). Furthermore, a structural analysis of RC with cracks from previous loading was also addressed (**Paper V**). Since a finite element analysis of reinforced concrete was used in several of the appended papers, an introduction to this topic is given here. There then follows an overview of corrosion damage and cracking in RC and, to conclude, methods for including corrosion damage and cracks from previous loading in structural models.

3.1 Finite element analysis of reinforced concrete

The finite element (FE) method is a technique for solving the field problems used in many areas of engineering and research [38]. These fields typically describe stresses and displacement in the context of structural engineering. However, the FE method is generally applicable in solving various differential equations.

The modelled structure is divided into small pieces (*finite elements*) which are connected together at node points. The arrangement of elements and nodes is called an “FE mesh”. Figure 3.1 shows an example of mesh discretisation. The method approximates the field over the finite elements (often by a linear or polynomial distribution), allowing the weak form of the differential equations to be solved. Although FE analysis does not provide an exact solution, the FE mesh may be arranged to make the approximated solution’s accuracy sufficient.

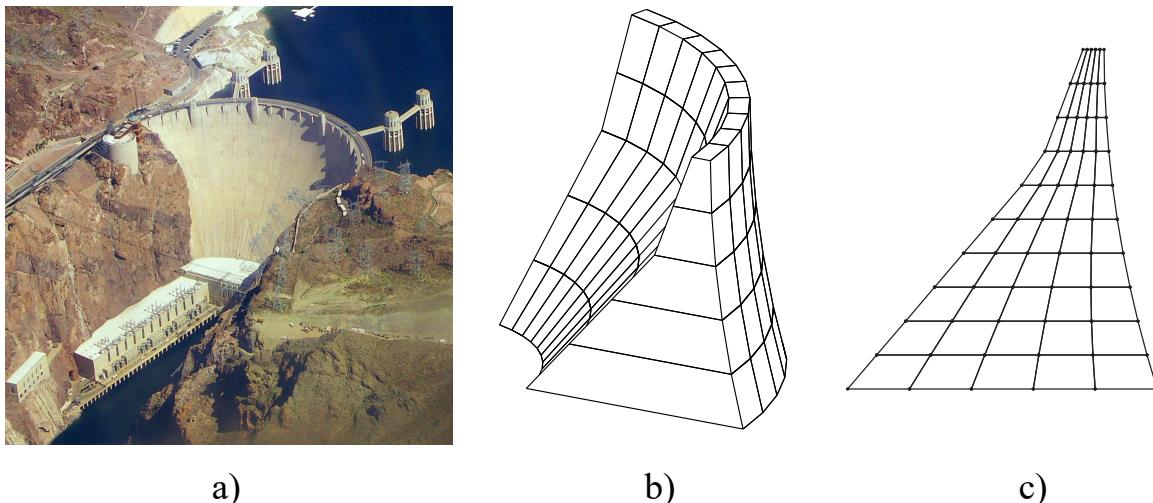


Figure 3.1. Example of FE mesh discretisation: a) picture of the Hoover dam [39], b) simplified 3D FE mesh of the dam and c) mesh for an axisymmetric FE model.

Due to cracking, material behaviour and the like, reinforced concrete shows a nonlinear response when subjected to increasing load. Therefore, a nonlinear finite element (NLFE) analysis should be used when a realistic response is sought from an RC structure. However, linear FE (LFE) analyses are commonly used in designing concrete structures. For instructional reasons and for completeness, both linear and nonlinear FE analysis are presented below.

3.1.1 Linear finite element analysis

In LFE analysis, the materials are described by linear elastic relationships and small deformations are assumed [40]. The calculation may be done in one step, as the applied load does not influence the material properties or the geometrical or boundary conditions. A traditional RC design (based on FE modelling) uses global LFE analyses to determine the load effects and nonlinear sectional or regional models to determine the resistance. One valued aspect of engineering practice is the superposition of loads, whereby the load effect from a combination of loads may be found by addition (or superposition) of the individual load effects. This allows for the efficient treatment of load combinations and is especially important in the case of large LFE models checked against multiple load combinations. An appropriate LFE analysis provides a solution in equilibrium with the applied loads and the resulting forces may be used to design reinforcement (given that sufficient ability to redistribute forces is provided). To this end, design guidelines have been proposed, cf. [41].

3.1.2 Nonlinear finite element analysis

To represent the behaviour of an RC structure more realistically, an FE analysis needs to be conducted that includes nonlinear aspects [40]. The dominant sources of nonlinearity in RC structures are the material behaviour plus the geometry and boundary conditions. For example, cracking of the concrete plus yielding and hardening of the reinforcement are examples of nonlinear material behaviour that may be represented by NLFE analysis. Geometric nonlinearities are characterised by deformation of the structure due to the applied loading; the changed geometry then influences the structural resistance (as with second-order moment effects in slender columns, for example). Moreover, a slab supported on the ground may be deemed to have nonlinear boundary conditions. This because the available horizontal reaction forces depend on the magnitude of the vertical load (due to friction).

3.1.3 Strategy for conducting finite element analysis

Several choices need to be made to successfully conduct an FE analysis and evaluate the results. The description here is from [40] but is complemented by additional information concerning the solution strategy from [17]. An overview of a typical strategy for FE analysis is shown in Figure 3.2 and the rubrics are addressed in the sections below. For specific recommendations, the reader is referred to [20]. These recommendations were also largely followed in the FE analyses conducted within this thesis.

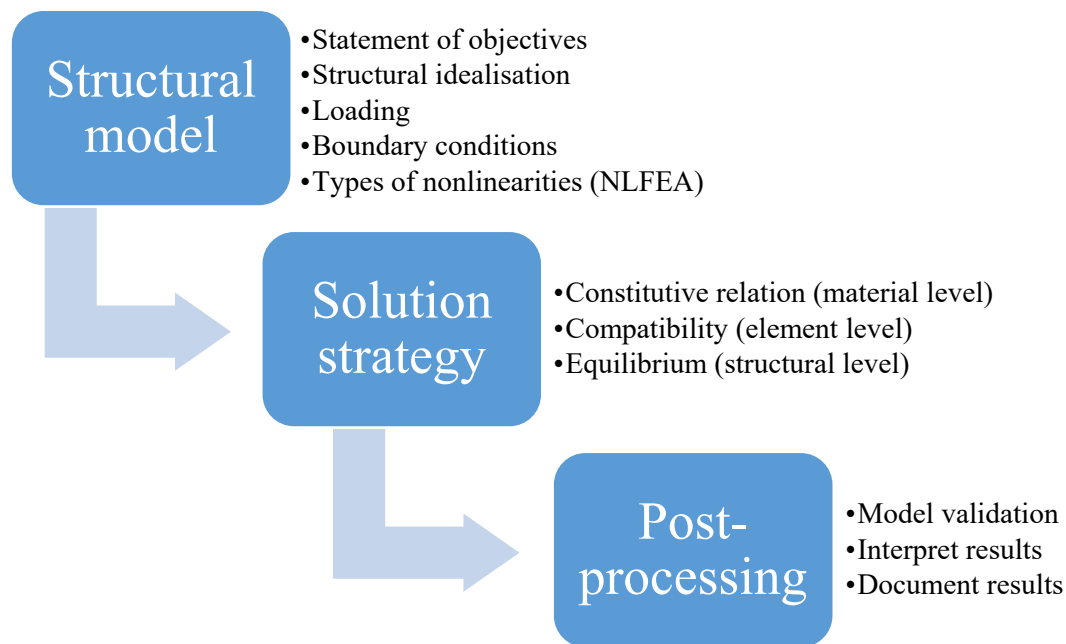


Figure 3.2. Overview of a strategy for conducting a finite element analysis, modified from [40].

Structural model

The starting point for choosing a structural model in general, and an FE model in particular, is to establish the desired outcome from an analysis. Clarifying this aids in idealising the structure. For example, if shear failure should be included in the FE analysis of a beam, this will influence the choice of FEs used in discretisation because the elements should be able to reflect shear failure. Based on the desired outcome, the FE model may be idealised using 1D (a beam or truss), 2D (a plane stress/strain element or shell) or 3D (a brick or tetrahedron) FEs, or a combination of these. The model's level of detail and whether certain regions need to be modelled more accurately due to their importance are also guided by well-thought-out analytical goals.

The choices of loading and boundary conditions for the FE model should also be carefully considered; they are simplifications of reality but should still represent important features of the actual structure. In many cases, it is possible to take advantage of the symmetry conditions of the load and geometry and thus reduce the size of the structural model. However, the analyst should be aware of the influence on the global response and remember that the failure mode is often asymmetric in reality (due to, say, local variations in material strength). Additionally, for NLFE analysis, a determination should be made as to which sources of nonlinearity to include (relating to, say, material behaviour, geometry or boundary conditions).

Solution strategy

A solution strategy for FE analysis comprises choices that may be sorted into three groups concerning: i) constitutive relations on the material level, ii) kinematic compatibility on the element level and iii) force equilibrium on the structural level [17]. In a linear elastic model, the constitutive relations are linear, meaning that the stiffness of the material is independent of the stress level. However, for nonlinear material model of, say, concrete, the stiffness response changes markedly with changes in stress level.

Establishing the kinematic compatibility conditions in the model means discretising it into a number of FEs, with associated degrees of freedom. Many FE formulations treat the displacements as unknown, although alternatives exist. Once the FE mesh is constructed, the element stiffness matrices may be derived using the corresponding material stiffness matrices. The global stiffness matrices are obtained by assembling element stiffnesses according to their interconnections.

Resolving the unknowns (here displacements are used as a typical example) using force equilibrium may be done in one step for a linear model. However, this is not possible for a nonlinear analysis, where the stiffness changes with changes in the stress state. An iterative procedure must therefore be used to solve the equilibrium equations. These are typically based on gradually increasing the load or changing nodal positions (often referred to as “load steps”). For each load step, the equilibrium equations are solved iteratively, using the Newton-Raphson method or other related methods [42]. The system of equations is updated based on the sources of nonlinearity (such as nonlinear material behaviour) and solved for a new displacement field. This iterative process is stopped when the pre-set convergence criterion/criteria are reached. These may be based on such things as normalised values related to energy, out-of-balance force or displacement. Furthermore, the analysis should be stopped if there is a diverging solution. Accordingly, upper bound values may be applied to the same criteria used in checking convergence. If these values are exceeded, the analysis is aborted.

Finally, for some NLFE models, saving all the information generated during the analysis is not feasible due to the large amount of data. For others, it is unnecessary. The outputs of interest should be specified and be in logical agreement with the goal of the analysis.

Post-processing

If the results from an FE model are to be trusted, they need validating. The extent of this validation often depends on the analyst’s knowledge of conducting comparable analyses. A simple type of validation may involve doing hand-calculations for some load cases to check that the output from the FE analysis is as expected. A more involved method of validation involves running a benchmark study, with the response from the FE model compared to experimental results or other trusted data. It is possible (plausible, even) that previously defined parameters such as element type, mesh characteristics, convergence criteria and so on, need modification to pass the validation. Once the model has been sufficiently validated, the load combination(s) of interest may be analysed. The results should be scrutinised using engineering acumen before being thoroughly documented (complemented by plots of the deformed mesh, principal stresses, load-displacement, crack pattern (for NLFEA) and the like).

3.2 Damage caused by corrosion

As stated earlier, corrosion of reinforcement bars is the most common damage mechanism for RC structures. This section therefore gives a brief background on the corrosion process itself and how it affects the structural behaviour. The section then goes on to describe how these effects may be accounted for in structural models.

3.2.1 Corrosion of reinforcement in concrete

Corrosion is an electrochemical process [43] involving oxidation of iron as the anodic reaction and reduction of oxygen as the cathodic reaction. This may be represented by the following half-cell reactions [44]:



Fe^{2+} , in turn, reacts with constituents of the pore solution, resulting in other corrosion products. Given the same mass, the corrosion products occupy a volume around two to six times greater than steel [45]. An internal pressure then arises, due to the volumetric expansion as the steel becomes rust. Eventually, the surrounding concrete fails to carry the tensile stresses and splitting cracks develop along the reinforcement, resulting in an increased corrosion rate [46].

In sound concrete, this reaction is prevented by the high alkalinity of the concrete's pore solution and a passivating layer of iron-oxide is formed on the surface of the reinforcement bar [44]. However, this passivation may be broken. Atmospheric carbon dioxide may react with the cement matrix and lower the pH of the pore solutions within the concrete (carbonation), resulting in depassivation of the reinforcement bars. Another common corrosion initiation mechanism is the ingress of chloride ions through the concrete cover (in, say, a marine environment or from de-icing salts). The types of corrosion typically distinguished for reinforcement bars are uniform (or general) and localised (or pitting) corrosion [47]. As the names imply, a uniformly corroded reinforcement bar shows regular loss of material, while localised corrosion occurs at discrete places along the reinforcement bar. Schematically, a high chloride concentration (or carbonation of the concrete) is associated with general corrosion, while lower chloride concentrations are linked to localised corrosion.

Determining the corrosion level of reinforcement in field conditions is a challenging task [48]. Visual inspection may provide information on cracks and delamination but is limited in its ability to evaluate the actual corrosion level. However, based on the results of visual inspection, other methods (such as potential mapping) may be deployed to investigate whether corrosion has started in other parts of the structure but not progressed enough to show visual signs. An estimate of the corrosion level may be obtained non-destructively by measuring the corrosion rate, half-cell potential and resistivity of concrete [49, 50]. However, the associated uncertainties are great due to, among other things, the fact that the start of the propagation period must be known or estimated. Furthermore, research efforts have also been directed towards characterising the corrosion based on measured surface crack widths. See [51] for a comparison of various methods. The model uncertainty is great due to several factors, including environmental exposure and type of corrosion (general or localised) that is forming. The lack of convincing methods to determine the corrosion level of existing structures makes assessment methods which do not need this information attractive. **Paper IV** investigated the possibility of assessing the structural capacity (concerning anchorage failure in a beam with corrosion-induced cracks along the reinforcement) without information on the corrosion level.

3.2.2 Structural effects of corrosion damage

The structural effects of corrosion for RC members are numerous and concern both the serviceability limit state (SLS) and ultimate limit state (ULS). Figure 3.3 shows an overview of the most important effects [4].

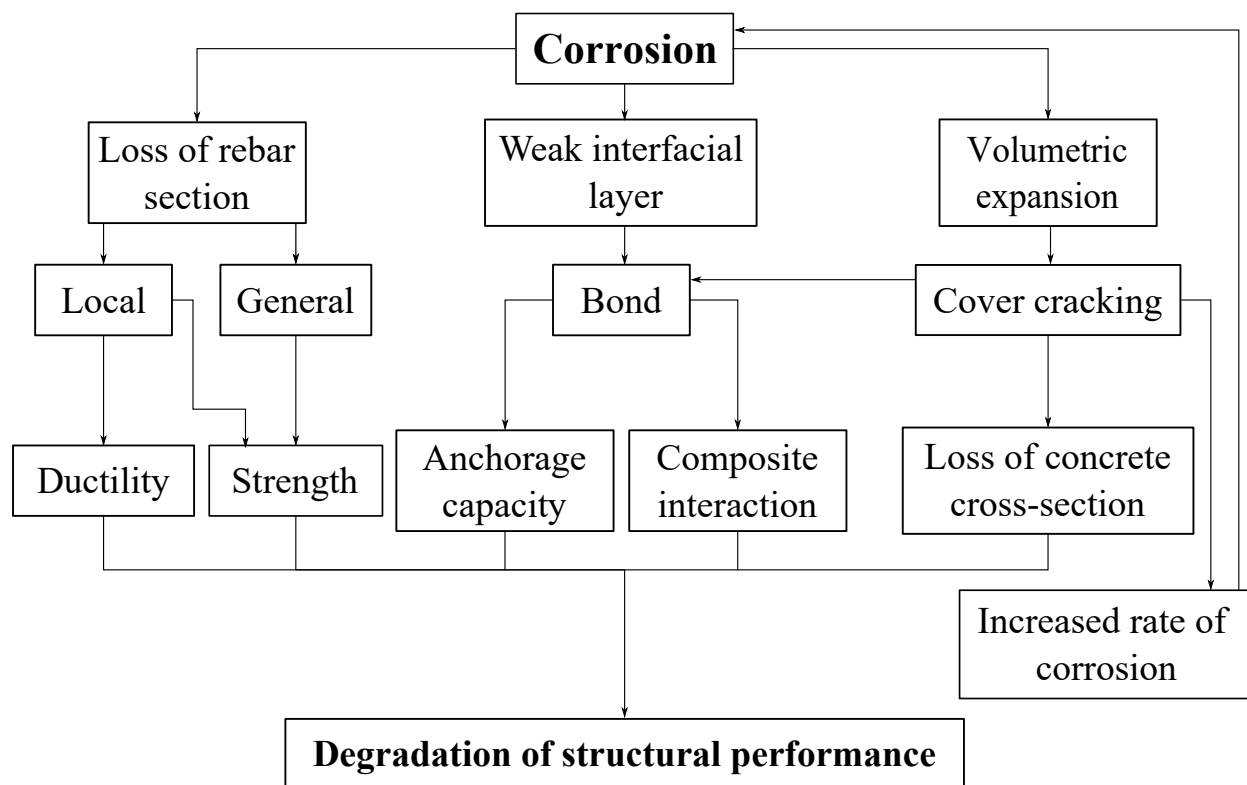


Figure 3.3. An overview of structural effects of reinforcement corrosion, modified from [4].

Corrosion causes a loss of rebar cross-section and a volumetric expansion as steel becomes rust. Furthermore, the corrosion products may create a weak interfacial layer between concrete and reinforcement. Corrosion damage may influence both the ultimate moment and shear capacity, due to several reasons [52]: i) reduction of cross-sectional area of rebar, ii) loss of concrete cross-section and iii) reduction of concrete compressive strength and ductility due to corrosion-induced cracking. Furthermore, the ductility of the reinforcement bars may be impaired due to localised corrosion [53, 54] and the tension stiffening may be affected by the degradation of bond and cracking of the concrete cover. These aspects jointly influence the ultimate deflection, but also the SLS condition in terms of deflection and crack width for service loads. By extension, the plastic rotation capacity may be impaired, which influences the moment redistribution for indeterminate structures, as well as robustness and seismic resistance [4].

The effect of reinforcement corrosion on the bond and anchorage capacity was one of the focal areas of this thesis (addressed in **Paper II** and **IV**) and this structural effect is therefore thoroughly addressed below. Initially, when corrosion of the reinforcement bars propagates, the bond capacity may increase, assuming that the confinement from surrounding concrete is sufficient. With increased corrosion, the tensile hoop stresses in the concrete grow, until they finally crack the concrete cover and longitudinal splitting cracks form. The confinement (and thereby the bond capacity) decreases [6, 55 and 56]. Moreover, the bond may also be decreased by the layer of corrosion products between concrete and rebar. Upon further corrosion, a marked decrease in capacity is expected in case of low levels of transverse reinforcement, while a small increase may be observed in the case of high levels [8, 57–59]. As previously stated, the cross-sectional area is obviously influenced by corrosion and, in some cases, the tensile capacity of the rebar may be the limiting factor on the anchored force. Furthermore, one common method of manufacturing reinforcement bars results in non-uniform strength distribution over the bar's

cross-section, with higher-strength steel close to the surface (as compared to the centre of the bar). Therefore, corrosion may also reduce the average tensile strength of the material [60].

Anchorage failure is typically brittle, whereas a ductile failure mode is preferred since it increases the likelihood of users avoiding imminent danger. Thus, the bond of reinforcement is a particular concern when assessing deteriorated structures and needs to be captured realistically in structural analyses.

3.3 Modelling of corrosion-damaged structures

This section presents important aspects for modelling corrosion-damaged structures. A general overview is given, followed by a description of the bond and anchorage modelling conducted in **Papers II** and **IV**.

Many choices regarding structural modelling relate to the scope of the investigation, cf. the preliminary, detailed assessment in Section 2. Hand calculations with certain simplifying assumptions may be acceptable in some situations; in others, a detailed FE analysis is more appropriate. Various aspects need consideration when comprehensively modelling the influence of corrosion on structural capacity, specifically [52]:

- reduction of cross-sectional area for longitudinal and transversal reinforcement bars;
- reduced ductility of reinforcement bars, due to localised corrosion;
- reduction of cross-sectional area of concrete, due to spalling;
- changes in the constitutive relations (strength and ductility) of concrete, due to cracking caused by expansive corrosion products;
- changed tension stiffening behaviour, due to cover cracking and bond deterioration;
- changed bond behaviour, depending on the corrosion level.

For general corrosion, the cross-sectional reduction of reinforcement bars is simple to implement by changing the geometry. However, quantifying the corrosion level is difficult in practice, as mentioned in Section 3.2.1. Localised corrosion is more complicated to implement (especially since the pit locations and features are typically unknown) and is often simplified in analysis. One of the main influences of pitting corrosion is a reduction in ductility of the reinforcement bars. This effect may be modelled by changing the material properties (cf. [52, 61]). The change in cross-sectional area of the concrete may be considered explicitly in the models. Furthermore, the reduction in compressive strength and ductility of the concrete (due to cracks parallel to the principal compressive direction) may be considered by a modified stress-strain relationship. For example, the relationship proposed by Vecchio and Collins [62], is readily available for use in analyses, at different levels of detail. In more detailed analyses, the changed tension stiffening behaviour may be represented by adjusting the bond stress-slip relationship for the reinforcement-to-concrete interaction. If the ULS is a concern when making the calculations, the effect of tension stiffening is not influential and may be omitted from, say, hand calculations.

The modelling of bond and anchorage has been addressed by several researchers. See [63–65] for examples of simplified models and [11, 12 and 66] for more advanced models. In contrast to the proposal in **Paper II**, these simplified models do not include a bond stress-slip relationship. Rather, numerical expressions (calibrated to experimental data) are used to obtain the reduction in capacity as a function of the corrosion level. The advanced models are implemented in 3D FE modelling and are capable of producing more realistic results for the

bond behaviour. However, they require major effort to implement. Furthermore, input parameters (such as the corrosion level) or the parameters needed to simulate it (relating to such things as material properties, geometry and exposure), are required for all these models. In this thesis, modelling bond behaviour and anchorage capacity were the topics of **Papers II** and **IV**, with **Paper II** presenting a simplified model and **Paper IV** an advanced one. The benefit of using these models is that the full bond stress-slip relationship is obtained in **Paper II** (for use in, say, FE analyses) and, in **Paper IV**, it was possible to estimate the anchorage capacity without knowing the corrosion level.

The simplified model in **Paper II** assesses the anchorage capacity by solving the equilibrium conditions along the reinforcement bar, as described by the differential equation:

$$\frac{\pi \cdot \phi_m^2}{4} \cdot \frac{d\sigma_s}{dx} - \pi \cdot \phi_m \cdot \tau_b = 0 \quad (3.3)$$

where ϕ_m is the reinforcement diameter, σ_s is the stress in the reinforcement, τ_b is the local bond stress and x denotes the longitudinal direction of the bar. The local bond stress-slip relationship constitutes the core of the model and was based on the relationship in *fib* Model Code 2010 [16] but with some modification and additions, namely:

- introduction of equivalent slip to account for bond degradation due to corrosion;
- change of failure mode due to corrosion-induced cracking of the concrete cover;
- modification of the residual bond stress in case of low stirrup content.

The features bulleted above are illustrated in Figure 3.4. The equivalent slip, s_{eq} , was calibrated against a large database of bond tests and the residual bond stress for low stirrup content was modified to reflect the experimental data for beams found in the literature. For an exhaustive explanation of the model, see **Paper II**.

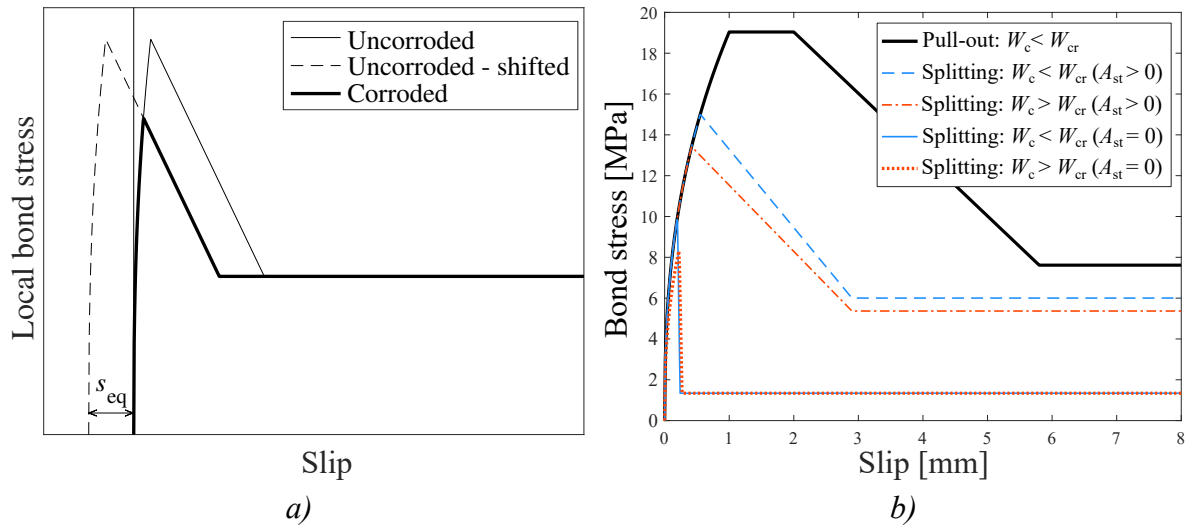


Figure 3.4. a) Concept of equivalent slip illustrated on bond stress-slip relationship, b) change of failure mode due to corrosion-induced cracking. For a corrosion level W_c above the cracking limit W_{cr} , the splitting strength was reduced. Note also that the residual bond stress is greater than zero for cases without stirrups. Adopted from **Paper II**.

Paper IV investigated various methods of assessing the capacity of reinforcement bars anchored in concrete with corrosion-induced cracks. The most promising, advanced method is presented here. It is based on an NLFE analysis, in which both the concrete and reinforcement

are 3D-modelled. A friction model was used for the interface between concrete and reinforcement, to explicitly model the confinement from the surrounding concrete. The effect of corrosion-induced cracks was represented by locally weakening the concrete's tensile properties for the FEs at the crack position, thereby reducing the confinement of the reinforcement bar. The tensile properties were weakened based on the measured crack width, using the bi-linear softening relationship assumed for the concrete in tension. For further information, see Section 3.5. A graphical representation of the weakened FEs for a beam is shown in Figure 3.5.

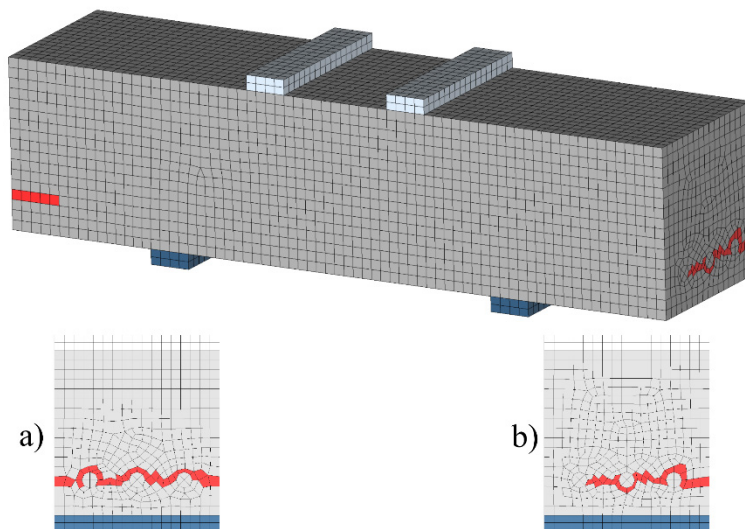


Figure 3.5. Elements assigned weakened elements on a) the left-hand and b) the right-hand side of a beam specimen. Adopted from **Paper IV**. Note that the reinforcement bars were debonded, except for the outer 100 mm on each side, similar to the experiment being modelled.

3.4 Cracking in reinforced concrete

As mentioned in the introduction, cracking in RC structures is common and not necessarily detrimental. Many structures are designed to crack under service loads; this is necessary if mild reinforcement is to carry any significant tensile force. Current structural design codes specify crack width limitations and minimum reinforcement levels, to control cracking (cf. [67]) and meet durability and aesthetic requirements. In practice, cracks may arise for numerous reasons at many stages of a structure's life. **Papers IV** and **V** investigate how cracks formed due to corrosion and external loading (respectively) may be included in structural analyses. An overview in this section provides background and context for cracking in concrete, as illustrated in Figure 3.6.

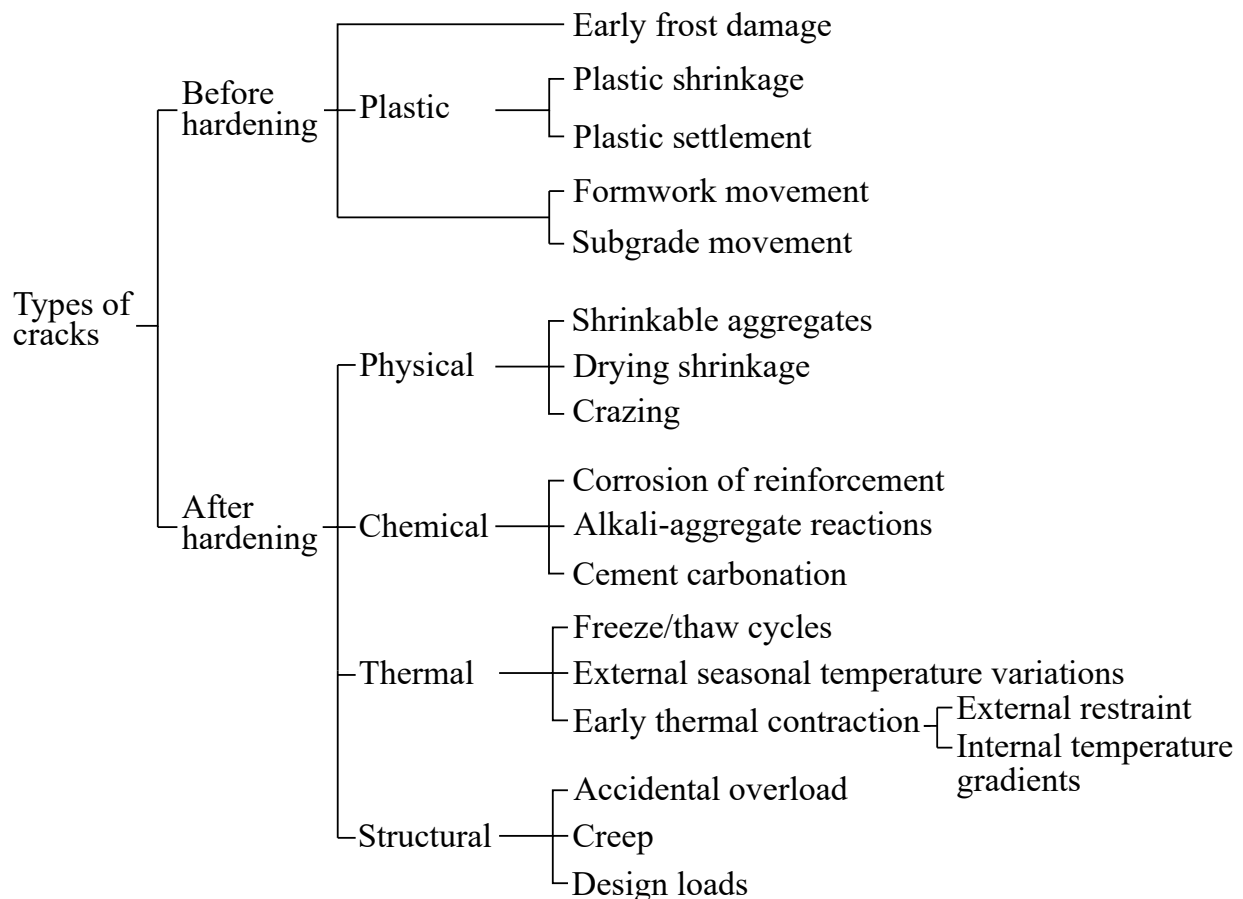


Figure 3.6. Overview of different types of cracks in concrete, adopted from [68].

The different types of cracks may be sorted into those occurring before and after hardening. Before hardening, cracks may occur due to plastic shrinkage or settlement, or due to movement of the subgrade. Early frost damage may also induce cracks before the concrete has hardened. After hardening of the concrete, several causes of cracking may be identified, including physical, chemical, thermal and structural. Physical cracks may be observed as crazing on the concrete surface, or drying shrinkage when contraction of the concrete is prevented by internal or external restraints. Moreover, certain types of aggregates may shrink and give rise to cracking. Cracks may also form due to chemical reactions such as corrosion of the reinforcement (addressed in Section 3.2), alkali-aggregate reactions or carbonation of the concrete. Thermal causes of cracking also include freeze-thaw cycles and external seasonal temperature variations as well as early-age thermal contraction (externally or internally restrained). Lastly, cracking due to structural causes may arise from loading to design levels, accidental overloading and stress-dependent strain due to creep.

Cracks may be sorted into two main categories; structural cracks and non-structural cracks [68]. The label “structural” is used to describe cracks developed due to the load-carrying mechanisms of the concrete structure. For example, external loading of a member will cause a certain stress distribution within the structure. When these stresses exceed the tensile strength of concrete, structural cracks will form. This category of cracks is also the most likely to influence structural behaviour. Non-structural cracks, on the other hand, are not directly related to the load-carrying mechanisms of the structure and include cracks formed due to restrained shrinkage and thermal movement. Most non-structural cracks are unlikely to influence the short-term structural behaviour, although they may do in the long term (in the case of, say, internal frost attacks).

However, it should be noted that cracks due to reinforcement corrosion might affect the load-carrying capacity if anchorage of the bars is a limiting factor. Moreover, determining the exact causes of cracking very often requires thorough investigation. With knowledge of the load-carrying system, it is sometimes possible to distinguish between different types of cracks based on their location, appearance and the time of their formation. This information may be complemented with concrete core samples from the structure. These may be used to prepare “thin sections” which provide further information on the mechanism of cracking, visually or via microscopy [69].

In the experiments analysed in **Papers IV** and **V**, different methods were applied to induce cracks in the test specimens. In **Paper IV**, reinforcement bar segments in both anchorage regions of a simply supported beam were artificially corroded, to induce splitting cracks along the reinforcement. By contrast, the cracks in **Paper V** originated from restraint shrinkage with subsequent tensile loading: this resulted in crack planes with normal directions coinciding with the length direction of the beam. The next section presents methods for including cracks such as these in structural models.

3.5 Incorporating pre-existing cracks in FE modelling

At the time of assessment, the structure may be cracked due to one or more of the mechanisms presented in Section 3.4. If structural behaviour is to be realistically modelled, it may be important to include cracks in the analysis. Within this work, the term “pre-existing cracks” was chosen to describe cracks present in the structure at the start of the FE analysis, in contrast to the simulated cracks developing during the analysis. It is worth noting that, in practice, including all cracks in the analysis will often be unfeasible. Based on an understanding of the structural system and the location and characteristics of the cracks, engineering judgement should be used when identifying critical cracks for inclusion in the analysis.

In a standard NLFE analysis of reinforced concrete, the two most common methods of modelling cracks are the smeared and discrete concepts [70]. The smeared crack approach treats a cracked solid as a continuum, while the discrete-crack approach treats a crack as a geometrical discontinuity. While smeared crack models often employ a total-strain-based approach with either fixed or rotating cracks, properties of the discrete cracks are defined based on their relative displacements in the normal and transverse directions. Discrete cracks are commonly used to incorporate pre-existing cracks in assessments of dams [69], where they are assigned frictional properties. In such large structures, it is feasible to omit the aggregate interlock in the shear transfer and only consider frictional mechanisms. However, this is not generally acceptable in other structures, such as concrete beams with low levels of shear reinforcement [71]. Another example of how discrete cracks are used may be found in [72]; in this instance, they were used to model through-cracks in RC slabs tested by bending, for fatigue assessment. In this case, the treatment of aggregate interlock was tailored to represent the fatigue response of the slab and was not generally applicable.

Moreover, examples in which the smeared crack approach has been used to incorporate pre-existing cracks are scarce in the literature, with only a few works found. In an assessment of concrete columns with pre-existing cracks [73], a tailored load arrangement was applied in analyses to mimic the pre-existing cracks before the loading of interest was subsequently applied. Although this approach represented the pre-existing cracks and the maximum loading reasonably well, the implementation was deemed cumbersome and the approach

computationally expensive for the treatment of multiple cracks. Furthermore, a methodology for implementing cracks also appears in [74]. There, the constitutive relationship for the cracks was derived from the uncracked state, with a scalar damage parameter to reduce the stiffness and strength. The influence of different crack widths was discussed. However, this was not implemented in the analyses, where the same damage parameters were used for all cracks. The proposed methodology in [74] has similarities to the models presented in **Papers IV and V**, in that the constitutive relations of the continuum elements are weakened to resemble pre-existing cracks. However, the derivation of damage parameters is different. The work included in this thesis develops a modelling methodology to incorporate pre-existing cracks in FE analysis and addresses weak points identified in the literature, by making the damage dependent on the individual crack width and including aggregate interlock as shear retention.

Papers IV and V investigated two advanced approaches for including pre-existing cracks in FE analysis. These were: a) weakening the continuum elements at the position of the crack, and b) introducing discrete crack elements with weakened properties. Approach a) was deemed the most promising in both papers and is presented here. The reader is referred to **Papers IV and V** for a presentation of approach b).

In the weakened elements approach, the FEs at the positions of the pre-existing cracks are assigned weakened tensile properties, as compared to the intact concrete. A bilinear, mode-I, stress-to-crack width relationship for undamaged concrete served as a basis for deriving the weakened tensile properties. Based on a measured crack width, the weakened properties (in terms of tensile strength) and remaining crack opening until stress-free crack, were derived for the specific crack from the bilinear relationship. The procedure is shown in Figure 3.7. By using the measured crack width (w_c) in the bilinear tensile stress-to-crack opening relationship for undamaged concrete, the tensile stress ($f_{ct,c}$) and residual fracture energy (G_{F,w_c}) were determined. Furthermore, the corresponding stress-strain relationship, shown in the middle of Figure 3.7, was derived using the modulus of elasticity (E_{cm}) and an assumed crack bandwidth (h). In determining the crack bandwidth, the cracks were assumed to be localised within one weakened element row. The ultimate strain ($\varepsilon_{w_c,ult}$) in the stress-strain relationship (used as

input for the weakened elements) was determined so that the area under the stress-strain relationship equalled the fracture energy divided by the crack bandwidth ($G_{F,w_c}/h$), meaning:

$$\varepsilon_{w_c,ult} = \frac{w_{ult} - w_c}{h} \text{ for } w_c > w_s \quad (3.4)$$

Moreover, to prevent interaction of the strains in different directions, a Poisson's ratio of zero was used for the weakened elements. In some cases, the measured cracks widths were also larger than w_{ult} . In such cases, low tensile properties were assigned by using a crack width of $0.99w_{ult}$ in the calculations.

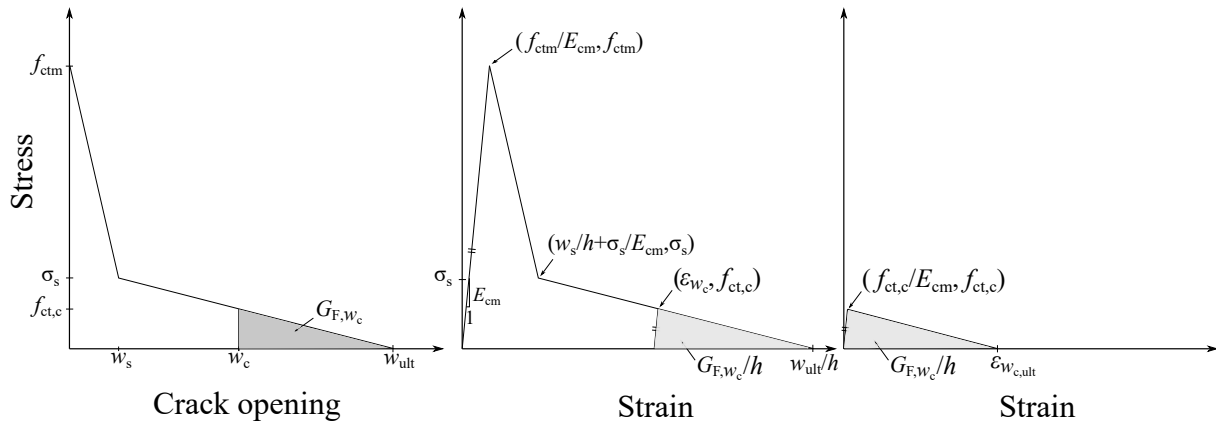


Figure 3.7. Principal idea for weakening tensile properties of FEs representing cracks, from **Paper V**.

The fixed-crack approach, in which a “shear-retention factor” specifies the remainder of the initial shear modulus, was used for the weakened elements. The shear-retention is influenced by several factors, such as the type of aggregates used. The choice of suitable shear-retention for analysis is not trivial. Guidelines for NLFE analysis [20] stipulate variable (“damage-based”) shear retention which decreases after cracking, based on the increase in normal strain. This type of shear retention was used for the sound concrete material in **Paper V**. However, a fixed shear-retention factor of 0.01 was used for the weakened elements, to compare the results from analyses of weakened elements and discrete cracks. This is because the analyses were conducted in the commercial FE software DIANA 10.3 [42], in which variable shear retention for discrete cracks is not available. The choice of shear retention is also thoroughly discussed in **Paper V**.

An example of how this method may appear in the FE model is shown in Figure 3.8. This shows a surface crack pattern for side 1 of specimen PC1 in **Paper V** in a), with blue indicating cracks formed in the restrained shrinkage phase and red indicating cracks formed during tensile loading. The widths of the cracks may be found in **Paper V** using the notations indicated in the figure. The cracks on each surface were extruded towards the centerline of the beam, with one-element imbrication at the mid-section. The FEs coinciding with the extruded cracks were assigned weakened tensile properties using an automated selection procedure. Furthermore, additional elements were manually weakened to allow the path of the weakened elements to open without cracking sound concrete elements. The FE mesh with weakened (cracked) elements in red for side 1 of PC1 is shown in b) and the top side is shown in c). The manually selected elements in b) and overlapping elements in c) are indicated in orange.

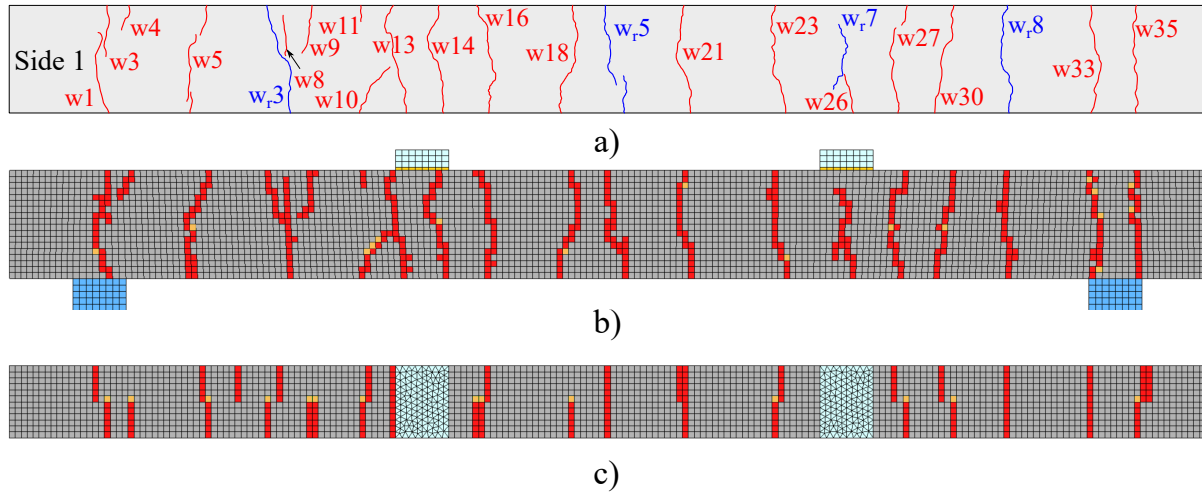


Figure 3.8. Example of incorporating pre-existing cracks in an FE mesh, modified from **Paper V**. a) shows the surface crack pattern for side 1 of specimen PC1, b) shows side 1 of the FE mesh with weakened elements in red and orange and c) shows the top of specimen PC1. The manually selected elements in b) and the overlapping elements in c) are indicated in orange.

It should be noted that some types of cracks most likely necessitate further modifications in the analysis. For example, representing frost damage requires modification of the compressive strength [75] and (in the case of corrosion) the reduction of reinforcement cross-sections must be considered for the bending and shear failure modes. This thesis has studied corrosion-induced cracks in the anchorage region and cracks from restrained shrinkage and tensile loading. It was therefore deemed reasonable to modify only the tensile properties of the elements representing the cracks.

4 Background on structural reliability assessment

Within this thesis, **Paper I** evaluated safety formats and **Paper III** derived partial factors. To set this work in the context of structural reliability assessment, this section provides a brief background of the subject and explains some important concepts. Specific presentations will be given of limit state design, uncertainties in basic variables, reliability level and deterministic and probabilistic means of ensuring the target reliability level is reached. These procedures are related to the verification stage shown in Figure 2.1.

4.1 Principles of limit state design

Internal stresses will arise in a structure when subjected to applied loading. The response, in terms of stress distribution and magnitude, depends on the characteristics of the load as well as the stiffness, strength and shape of the structure. “Limit states” are defined, to determine whether the response from the structure is acceptable. The two main limit states are presented more thoroughly below; namely, the ultimate limit state (ULS) and serviceability limit state (SLS).

ULS concerns the maximum load-carrying capacity of a structure but also includes the maximum deformation capacity. Its nature is typically irreversible, resulting in failure at the first violation. According to Eurocode 1990 [15] and the Joint Committee of Structural Safety (JCSS) probabilistic model code [76], the ULS may be exemplified by the following situations:

- excessive deformation, transformation of the structure or any part of it into a mechanism, or rupture;
- loss of equilibrium in the entire structure or part thereof, causing rigid body movement;
- sudden change of structural system, such as snap-through behaviour;
- time-dependent failure, such as rupture of connections or members due to fatigue.

SLS reflects the service requirements put on a structure. According to JCSS [76], a few examples are:

- local damage, possibly affecting the durability or appearance of an element in the structure. For example, overly wide cracks in concrete;
- excessive deflections;
- observable damage due to time-dependent effects, such as fatigue;
- disallowed deformations, unacceptably impairing the functionality or appearance;
- excessive vibrations, affecting people’s comfort and the function of equipment.

In contrast to ULS, an SLS violation is quite often reversible. For example, excessive deflections caused by external loading may decrease to allowable levels once the loading is reduced. Concerning SLS, the specific failure limit values should be stipulated based on utility considerations [76].

Assessment of structural reliability aims to find the probability of limit-state violation at any point in a structure’s life [77]. The more specific term *structural safety* assessments concerns ULS in particular (a concern addressed by the work of **Papers I** and **III**). In practice, it is common to use a combination of frequency observations and subjective estimates of structural elements and properties to determine the probability of a limit state being violated.

4.2 Uncertainty of basic variables

Basic variables describe the performance and safety of a structure and include physical properties related to such things as materials, loads and dimensions. [77]. A broad definition also includes parameters characterising the model itself, with the basic variables assumed to contain all the necessary information for input into the calculation model [76].

A basic variable may be described as a random variable with a suitable probability distribution. A particular distribution is often chosen based on observed data and experience, or taken as normally distributed based on central limit theorem [77]. Random processes (random variables evolving) and random fields (spatial differences) may also be used to describe basic variables. Examples of standardised probabilistic representations of common basic variables (such as the material properties of concrete) may be found in [76].

The representation of basic variables must include uncertainties from all key sources. Three main types of uncertainty may be distinguished [76]:

- inherent physical or mechanical uncertainty;
- statistical uncertainty;
- model uncertainty.

Inherent physical or mechanical uncertainty may be represented by the variation in material strength and variation in geometrical properties. This uncertainty is considered by the probability distribution representing the basic variable. Moreover, statistical uncertainty arises when using statistical estimators (such as sample mean and standard deviation) to determine a suitable probability density function and related parameters. This is because the observations rarely represent the variable perfectly and because different statistical estimators are typically obtained based on the choice of sample data set [77]. Furthermore, the modelling uncertainty reflects the fact that the results obtained from a model will be more or less incomplete and inexact (due to such things as simplifications or lack of knowledge). Note that several types of uncertainties may be (and typically are) present for the same basic variable. For example, consider load models, in which the probability density functions describing the load magnitudes are based on statistical data and therefore marked with statistical uncertainty. However, these are also simplifications of reality and carry modelling uncertainties. To illustrate the meaning of modelling uncertainty, consider the following calculation model for general random variables [76]:

$$Y = f(X_1, X_2, \dots, X_n) \quad (4.1)$$

where Y is the model output, $f(\cdot)$ is the model function and $X_1, X_2, \dots, X_n$ are the basic variables. The model function, $f(\cdot)$, is typically not complete and exact. Therefore, the output Y cannot be perfectly predicted, given a set of basic variables X_i in an experimental setting. The model may instead be expressed:

$$Y = f(X_1 \dots X_n, \theta_1 \dots \theta_m) \quad (4.2)$$

where $\theta_1 \dots \theta_m$ are random variables containing information about the model's uncertainty. Their statistical properties are typically determined from observations or experiments and the mean values should be set so that, on average, Y is predicted correctly. However, it should be noted that the model's uncertainty may be difficult to assess in some cases. For example, when the correct failure mode is difficult to capture in the numerical analysis, as shown by [78].

4.3 Target reliability level

When conducting a safety assessment (**Paper I**) or deriving partial safety factors (**Paper III**), the intended reliability of the structure needs to be expressed. A widely used measure of reliability is the generalised reliability index β [76]. This is based on the standard normal (or Gaussian) distribution, in which the mean value is zero and the standard deviation is one. The expression for β may be written:

$$\beta = -\Phi^{-1}(p_f) \quad (4.3)$$

where p_f is the probability of failure and Φ^{-1} is the inverse of the normal cumulative distribution function. Consider the limit-state function $Z = R - S$, where resistance R and load effect S are normally distributed. β may then be understood as the number of standard deviations away from the mean value, resulting in the area p_f under the curve in the unsafe domain ($Z \leq 0$), as shown in Figure 4.1.

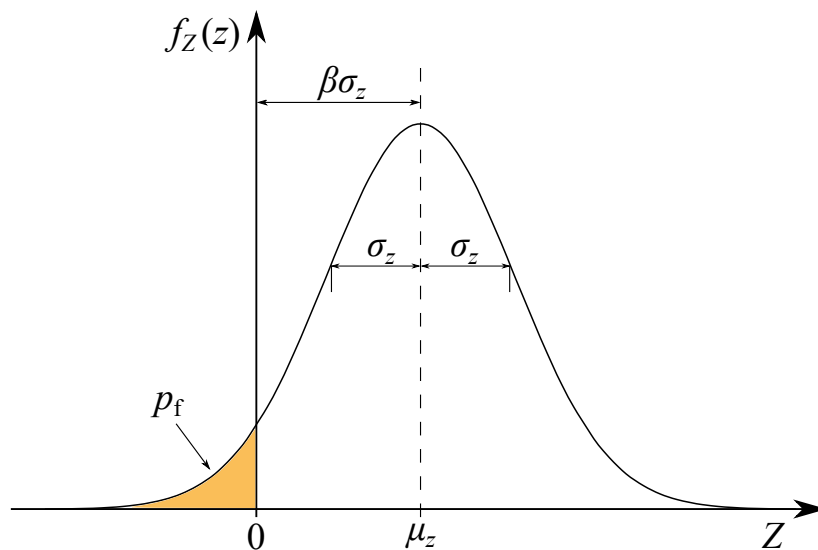


Figure 4.1. Illustration of β in relation to p_f for $Z = R - S$, where resistance R and load effect S are normally distributed (modified from [77]).

A structure is designed to meet the ULS and SLS requirements. However, since uncertainties are present in the basic variables and calculations models, the probability of failure cannot be zero. Rather, the design should comply with a target reliability index (directly equivalent to a probability of failure). In one sense, this may be viewed as an optimisation problem where the optimal β may be found, based on the failure cost (related to the consequences of failure) and the cost of increased safety. However, such an approach may be ethically questionable in cases when aspects relating to fatalities, injuries and culture take precedence over economic loss. Therefore, risk-benefit analyses are often used to relate the reliability of the structure to the cost per life saved. This cost may be chosen by comparison with other, similar structures; it forms the basis for determining the target reliability. There are various recommendations for target reliabilities, such as the ones by Eurocode [15], the International Organisation for Standardization (ISO) [34] and the Joint Committee on Structural Safety (JCCS). The latter's recommendations for target reliability in ULS [76] are shown in Table 4.1. Allegedly, their basis was public-facing cost-benefit analyses of simple but representable sample structures.

Table 4.1. Tentative target reliability indices β (probability of failure in parentheses) related to the ULS for a one-year reference period, adopted from [76].

Relative cost of safety measure	Consequences of failure		
	Minor	Moderate	Large
Large (A)	3.1 (10^{-3})	3.3 ($5 \cdot 10^{-4}$)	3.7 (10^{-4})
Normal (B)	3.7 (10^{-4})	4.2 (10^{-5})	4.4 ($5 \cdot 10^{-6}$)
Small (C)	4.2 (10^{-5})	4.4 ($5 \cdot 10^{-6}$)	4.7 (10^{-6})

For a complete explanation of the table, the reader is referred to the probabilistic model code [76]. In simple terms, the consequences of failure include the risk of casualties in case of failure as well as the economic consequences, while the “relative cost of safety measure” considers the cost of increasing the safety. For **Paper I**, a target reliability of 3.8 for a 50-year reference period was used, as stipulated in Eurocode [15] for the design of new, class RC2 structures. However, a target reliability of 3.7 for a one-year reference period was used in **Paper III**. This corresponds to a case which had large consequences of failure and a large relative cost of safety measure in Table 4.1. This was considered reasonable for an existing structure at the end of its service life.

4.4 Measures of structural reliability

Measures of structural reliability are used to safeguard the target reliability within a structural design or assessment. The overview presented here is based on [77]. In structural engineering practice, a simplified measure of limit-state violation is typically deployed. The reliability is checked by using a deterministic representation of the basic variables, derived from their probabilistic distributions, directly in the calculations. These deterministic methods have developed over time and the most common current procedure is to use partial safety factors. This measure of safety was developed by combining two others; the *factor of safety* and the *load factor* measure. The factor of safety approach involves reducing the material capacity on the sectional level, while the load factor approach entails upscaling the loads on the structural level. The resulting equation for limit state i may be written:

$$\frac{R}{\gamma_{R,i}} \leq \gamma_{D,i} \cdot S_{D,i} + \gamma_{L,i} \cdot S_{L,i} + \dots \quad (4.4)$$

where R represents the member resistance and γ_R the corresponding partial safety factor. The load effects from dead and live loads are represented by S_D and S_L , respectively and the corresponding partial safety factors are γ_D and γ_S . The partial safety factor values are calibrated to result in a design with the intended safety level. The simple nature of this safety measure, (while still allowing the different uncertainties related to different basic variables to be represented) has made it widely used within structural engineering. In **Paper III**, partial safety factors were derived for the model that was used to assess the anchorage of corroded reinforcement (**Paper II**).

The partial safety factors developed in **Paper III** were verified, to confirm that they led to a design with the intended probability of failure. This was done by using a probabilistic measure of limit-state violation, to determine the probability of failure for several designs made using the partial safety factors. Again, as a simple example, consider a structure with resistance R , subjected to a load effect S (both independent random variables). The probability of failure may then be expressed:

$$p_f = P(R \leq S) = P(R - S \leq 0) = P[G(R, S) \leq 0] \quad (4.5)$$

where p_f is the probability of failure, $P()$ simply means the probability and $G()$ is a general expression of the “limit-state function”. To calculate the probability of failure, the bivariate (joint) probability density function (*pdf*) of R and S may be used:

$$p_f = P(R - S \leq 0) = \iint_D f_{RS}(r, s) dr ds \quad (4.6)$$

where D denotes the failure domain. Since R and S were assumed to be independent, this may be written:

$$p_f = \int_{-\infty}^{\infty} \int_{-\infty}^{S \geq r} f_R(r) f_S(s) dr ds \quad (4.7)$$

remembering that the cumulative distribution function (*cdf*) is defined as:

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(y) dy \quad (4.8)$$

Combining Eqs. (4.7-4.8), the probability of failure may be written as a single integral:

$$p_f = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx \quad (4.9)$$

Eq. (4.9) basically expresses the probability of the resistance being lower than the load effect and integrates over the entire domain to obtain the probability of failure. A visual representation of the load and resistance, and the probability of failure, is shown in Figure 4.2.

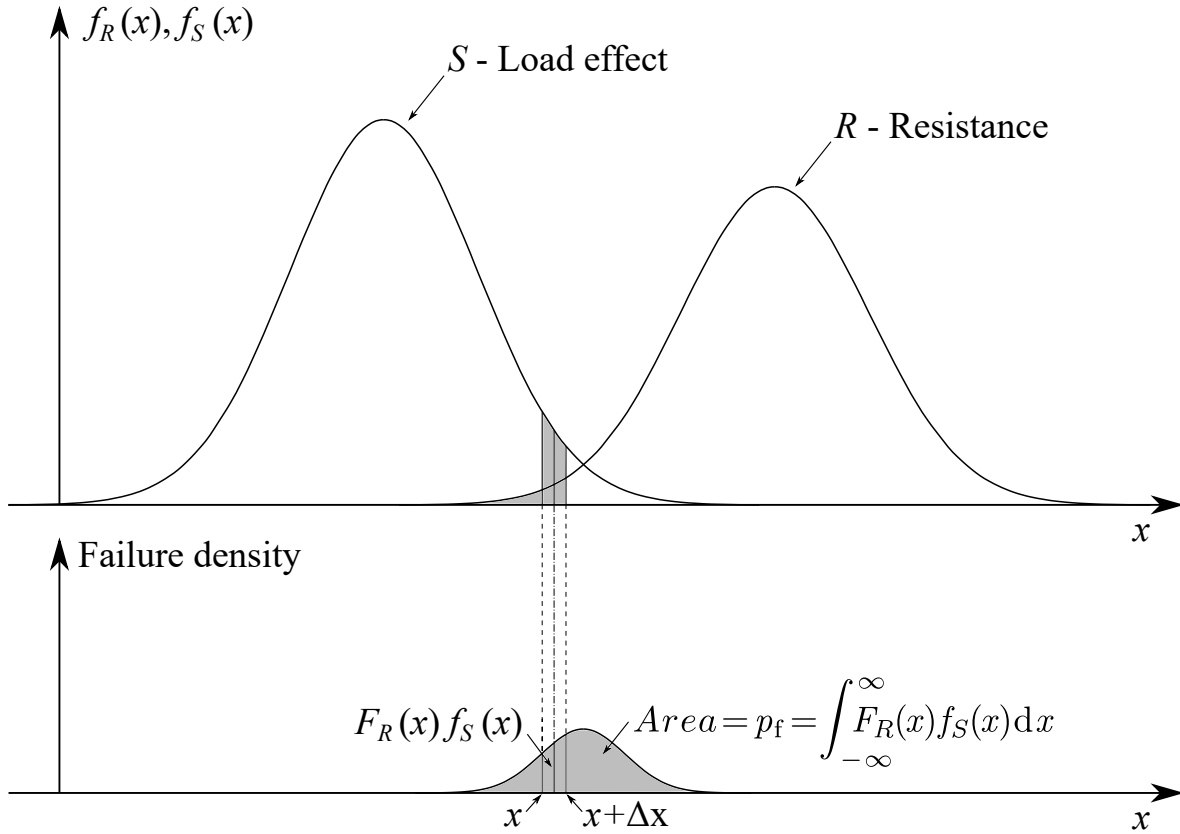


Figure 4.2. Probability density functions of load and resistance, with visual interpretation of probability of failure. Adopted from [77].

Although a simple case is suitable for instructional reasons, to be useful in practical applications it must be generalised. This because, in many cases, both the resistance and load effect are functions of several random variables. They may also be dependent, meaning that the load influences the resistance (as with a beam subjected to bending and normal forces, for example). If the limit-state function is described by $G(\mathbf{X})$, the probability of failure may be written:

$$p_f = P[G(\mathbf{X}) \leq 0] = \int \dots \int_{G(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (4.10)$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint *pdf* of the n -dimensional vector $\mathbf{X}$ containing all the basic variables. Note that the terms R and S are not explicitly included, but rather are implicit in $\mathbf{X}$. In general, the probability of failure in Eq. (4.10) cannot be obtained by integrating analytically over the failure domain $G(\mathbf{X}) \leq 0$. The next section presents approximate methods for evaluating the probability of failure.

4.5 Structural reliability methods

To evaluate the probability of failure, Eq. (4.10) must be solved. However, no analytical solution is available in practice since both integrand and failure domain may be quite complex. Two main approaches have been developed to overcome this challenge; approximation methods and simulation methods [79]. In approximation methods, a local approximation of the limit-state function is found at a point of interest. Simulation (or Monte Carlo) methods, on the other hand, numerically estimate the multidimensional integral based on input sampled from the basic variables. In **Paper I**, the probability of failure was estimated by approximation methods, with the first-order reliability method (FORM) applied to response surfaces established by

evaluating the limit-state function. **Paper III** used a combination of approximation and simulation methods; a FORM analysis was conducted, with subsequent “importance-sampling” simulations in *UQLab: A framework for Uncertainty Quantification in MATLAB* [80].

A brief introduction to the methods used in **Papers I** and **III** is given below. For a detailed overview of common structural reliability methods, the reader is referred to [77, 81 and 82], among others.

4.5.1 First-order reliability method

A common approximation approach is the first-order reliability method (FORM). This involves three main steps, to approximate the probability of failure [79]:

- transformation of the input vector of basic variables $\mathbf{X} \sim f_{\mathbf{X}}(\mathbf{x})$ into a standard normal vector $\mathbf{U} \sim N(\mathbf{0}, \mathbf{I}_M)$, where $\mathbf{I}_M$ contains the ones pertaining to the standard deviation in an M -dimensional case;
- a search algorithm is used to find the design point $\mathbf{U}^*$, corresponding to the point in standard normal space where failure is most likely to occur;
- linear approximation of the limit-state function at $\mathbf{U}^*$ and subsequent calculation of the resulting estimated probability of failure p_f .

In the first step, all the variables are transformed to standard normal distributions (that is, with zero mean and unit standard deviation). If the basic variables are uncorrelated and normally distributed, the transformation is simply [77]:

$$U_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (4.11)$$

where μ_{X_i} and σ_{X_i} are the mean value and standard deviation of the basic variable X_i , respectively. If the basic variables are correlated, or if they follow distributions other than the normal ones, the transformation becomes more involved and is not addressed here. Rather, the reader is referred to [77]. With the basic variables transformed to the standard normal space, the joint probability density function $f_{\mathbf{U}}(\mathbf{u})$ is the standard multivariate normal distribution. Logically, the limit-state function should also be transformed, thus leading to the failure definition $g(\mathbf{u}) \leq 0$, see Figure 4.3.

The point on the failure (hyper)surface closest to the origin in standard normal space is referred to as the design point (or checking point) $\mathbf{U}^*$. Since the basic variables and limit-state function were transformed to standard normal space, this is also the most probable point of failure. In mathematical terms, the design point may be defined [79]:

$$\mathbf{U}^* = \underset{\mathbf{u} \in \mathbb{R}^M}{\operatorname{argmin}}(\|\mathbf{u}\|, g(\mathbf{u}) \leq 0) \quad (4.12)$$

In many practical cases, the limit-state function is non-linear and $\mathbf{U}^*$ needs to be solved in an iterative procedure. There are various algorithms to this end, such as the Hasofer-Lind-Rackwitz-Fiessler algorithm and improvements thereto [83–85]; the reader is referred there and to [77] for an explanation of the iterative procedure. Note that the limit-state function is approximated by a first-order Taylor series expansion in the iterative procedure to find $\mathbf{U}^*$; it is therefore called the first-order reliability method.

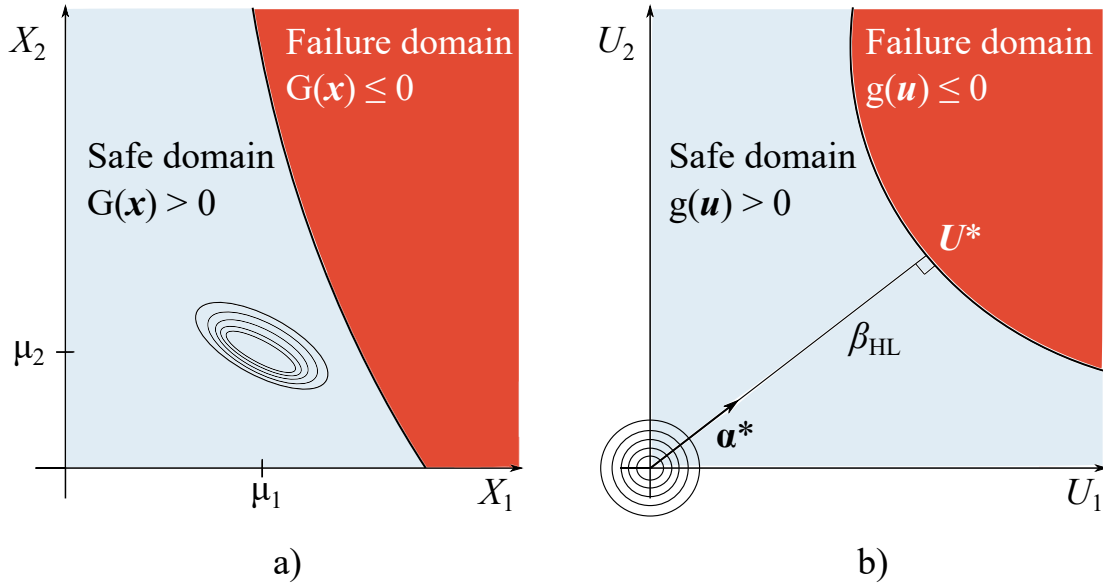


Figure 4.3. a) Basic variables and limit-state function in physical space, b) basic variables and limit-state function transformed to standard normal space with the design point $\mathbf{U}^*$ and the Hasofer-Lind safety index β_{HL} indicated (modified from [79]).

The Hasofer-Lind safety index β_{HL} [83] may be calculated based on $\mathbf{U}^*$ and $\boldsymbol{\alpha}^*$, where the asterisk indicates the converged design point, as follows:

$$\beta_{HL} = \boldsymbol{\alpha}^* \cdot \mathbf{U}^* \quad (4.13)$$

This may be used to find the probability of failure through Eq. (4.3). See Figure 4.3 b) for a graphical representation of β_{HL} and $\mathbf{U}^*$. The direction cosines (collected in $\boldsymbol{\alpha}^*$) represent the sensitivity of the limit-state function to changes of the basic variables [77]. The direction cosines squared (α_i^2 for the i^{th} basic variable) are called “sensitivity factors” and total to unity. These may have important practical application since basic variables associated with high sensitivity levels need to be accurately described. Those associated with low sensitivity may not need the same level of accuracy and, to reduce the dimensionality of the problem, may even sometimes be deemed deterministic.

4.5.2 Monte Carlo simulations

The other type of main approach to estimating Eq. (4.10) is called “Monte Carlo simulations” or just “simulations”. It involves random (or rather pseudo-random) sampling of values from the probabilistic distributions of the basic variables, so as to conduct numerous evaluations of the limit-state function. The probability of failure may then be calculated as:

$$p_f \approx \frac{n(G(\mathbf{x}_i) \leq 0)}{N} \quad (4.14)$$

where $n(G(\mathbf{x}_i) \leq 0)$ denotes the number of limit-state violations and N is the total number of evaluations. Logically, N governs the accuracy of the estimate of p_f and an increase in the number of evaluations increases the accuracy.

Sampling basic variables

The most straightforward way to sample from the basic variables is to generate “random variates” from their probabilistic distributions. This may be achieved by such means as the

inverse transform method, in which a random value between zero and one is generated and the inverse of the *cdf* is used to obtain the random variate, see Figure 4.4.

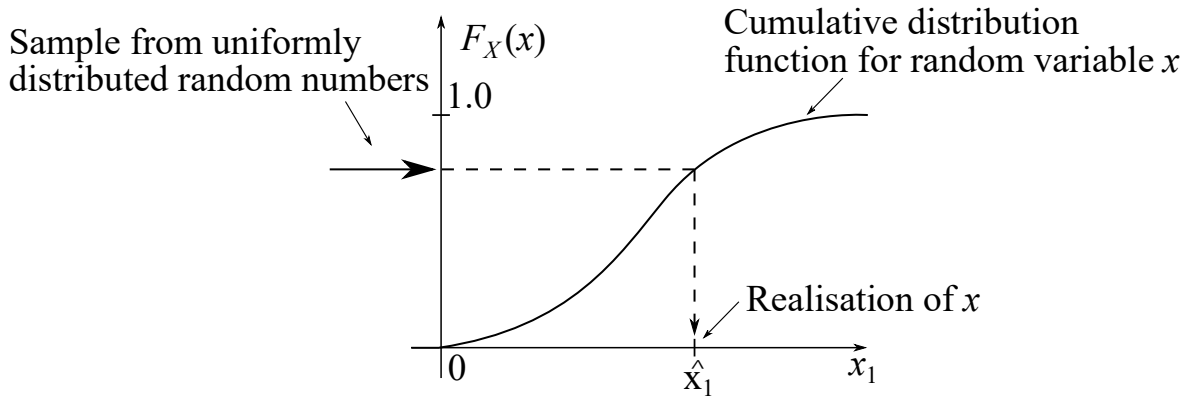


Figure 4.4. Generation of random variates by the inverse transform method, modified from [77].

This type of sampling is referred to as “crude Monte Carlo” sampling. However, since the probability of failure is typically very small (cf. Table 4.1), evaluating the limit-state function using this sampling procedure seldom results in failure. Therefore, random variates of the basic variables need to be generated many times (with subsequent evaluation of the limit-state function) to obtain a good representation of the probability of failure. This makes the procedure very expensive in terms of computations and time [86]. As a rule of thumb, about 10,000-20,000 simulations are required for a confidence interval of approximately 95%, depending on the function being evaluated [87].

Variance reduction by importance sampling

As described, crude Monte Carlo requires many simulations to obtain an accurate estimate of p_f (that is, with low variance). This is a substantial drawback and hinders practical application [77]. However, techniques for reducing the variance of the probability of failure estimate have been developed; these are known as “variance reduction” techniques [88]. Given the same desired accuracy of p_f , such techniques may reduce N .

Information about the problem at hand may be used to reduce the variance [77]. Consider the probability density functions (representing R and S) in Figure 4.2. In crude Monte Carlo sampling, random variates are based on their probability density, which makes values in the centre more likely than values in the tail regions. This makes limit-state violations unlikely. However, if sampling is conducted from the overlapping region between R and S , the likelihood of interesting results increases. This is the basic principle behind importance sampling; the samples are generated from probabilistic distributions other than those of the basic variables. This may be described by:

$$p_f = \int \dots \int I[G(\mathbf{x}) \leq 0] \frac{f_x(\mathbf{x})}{h_v(\mathbf{x})} h_v(\mathbf{x}) d\mathbf{x} \quad (4.15)$$

where $I[G(\mathbf{x}) \leq 0]$ indicates limit-state violation, $h_v(\mathbf{x})$ is the “importance sampling” density function. Choosing a suitable sampling function may be difficult in some cases. However, an efficient function may be found, based on results from a previous FORM analysis [77]. For further treatment of the sampling function, please see [77, 79].

4.5.3 Response surfaces

When structural reliability methods are to be used for implicit limit-state functions, such as FE analyses, the response surface method may be useful. A response surface is constructed based on multiple evaluations of the implicit limit-state function [77]. In practice, this is done by pointwise exploration of the safe and unsafe domains of the implicit limit-state function, using deterministic input values for the basic variables. Thereafter, a polynomial or other type of function is fitted to the set of results.

A tractable simplification of the implicit limit-state function is thus created and may be used for both approximation and simulation approaches. The accuracy of the response surface should be highest in the proximity of the design point, as it is the most probable failure point; a lower level of accuracy may be accepted elsewhere. For a thorough explanation of the procedure please refer to [77, 89 and 90].

5 Summary of appended papers

This section summarises the appended **Papers I to V** to provide their most central content in condensed form. The focus is on the key features of the methodology and the most important results.

Paper I. Evaluation of safety formats for non-linear finite element analyses of statically indeterminate concrete structures subjected to different load paths.

In **Paper I**, the safety formats available in *fib* Model Code 2010 [16] for non-linear finite element analyses (NLFEA) were evaluated for a frame structure subjected to two different load histories. The loading consisted of a vertical point load in the middle of the span and a horizontal point load on the top left-hand corner. The two load histories were defined as application of the vertical load followed by the horizontal load and vice versa, see Figure 5.1.

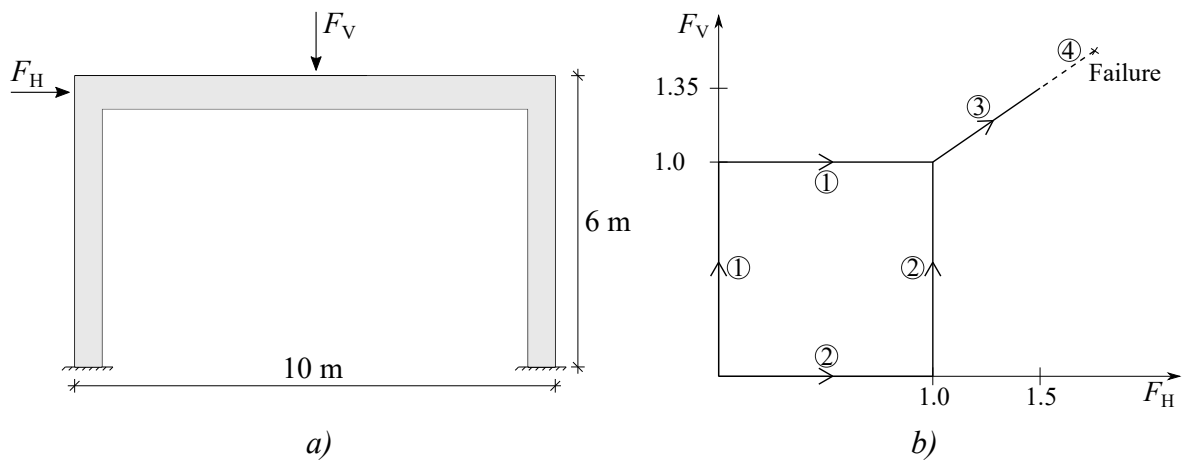


Figure 5.1. a) frame and load application, b) load histories, in which the main load history follows 1, 3, 4 and the inverse follows 2, 3, 4.

The resistance was defined as the total load at failure, while the safety assessment was based on the limit-state function:

$$g(\mathbf{X}) = R(\mathbf{X}) - R_{SF} \quad (5.1)$$

where $R(\mathbf{X})$ was the resistance function, obtained by fitting response surfaces to multiple NLFEA results and R_{SF} was the design resistance, according to the studied safety format. In the present study, the basic variable $\mathbf{X}$ included the concrete compressive strength and reinforcement yield strength. The first-order reliability method (FORM) was used to evaluate the probability of failure of the limit-state functions constructed for each load history and safety format. A 3D representation of the limit-state function in case of the main load history and the ECOV safety format is shown in Figure 5.3, plus a contour plot of the limit-state function and the FORM design point.

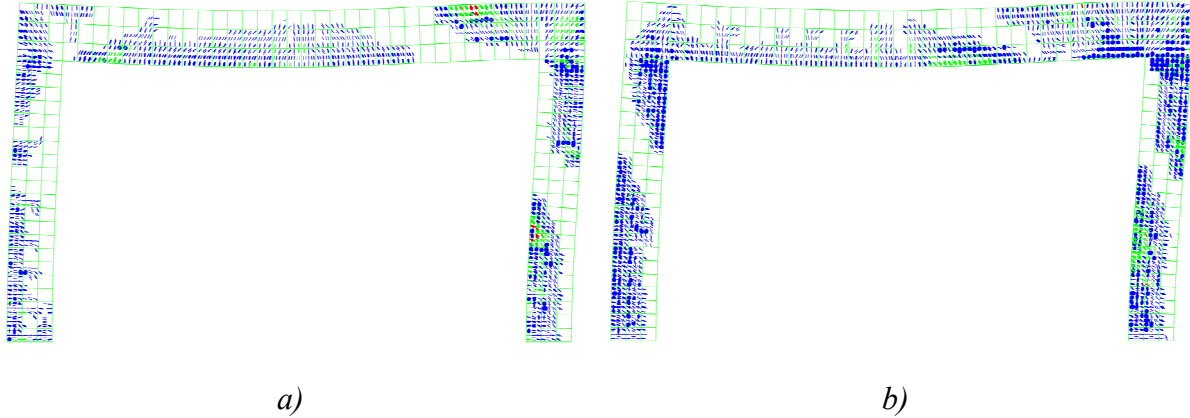


Figure 5.2. Deformed shape and crack pattern for the frame, loaded with both loads to their characteristic level, adopted from **Paper I**. The figure shows a) the main load history, and b) the inverse load history.

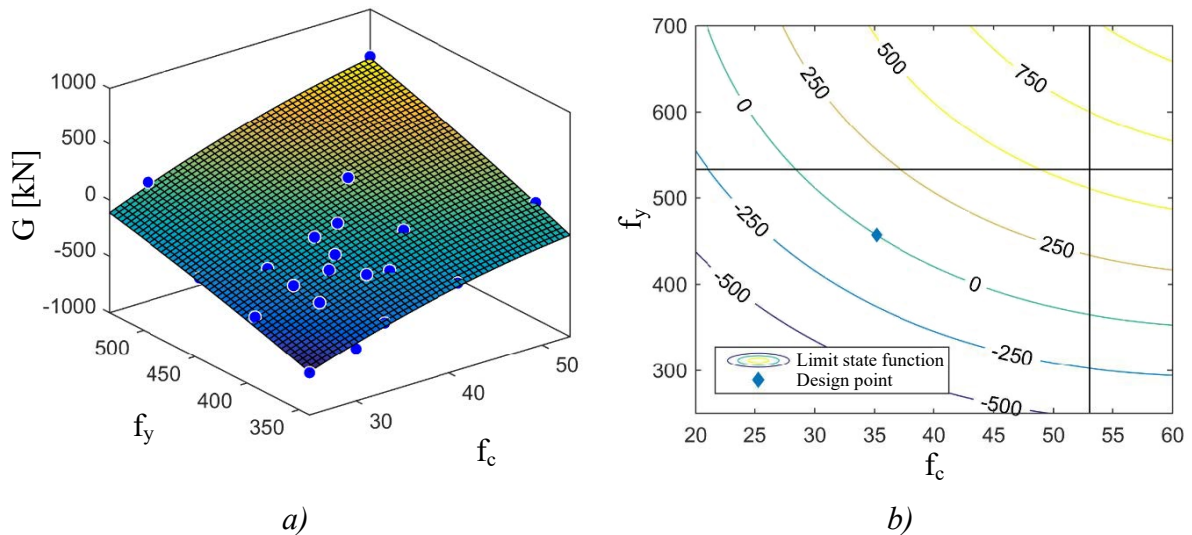


Figure 5.3. a) 3D plot of the limit-state function for the main load history and ECOV safety format, b) contour plot of the stated limit-state function, with the FORM design point indicated by a diamond. The two solid lines indicate the mean values of concrete compressive strength and yield strength of reinforcement. Adopted from **Paper I**.

The main findings from **Paper I** may be summarised as follows:

- the load history was shown to be influential for the load-carrying capacity as well as the crack pattern. The latter may be observed in Figure 5.2;
- response surfaces and FORM were applied to study the reliability of the three safety formats (ECOV, GRF and PSF) for the two load histories, see Figure 5.3;
- the ECOV safety format did not reach the intended safety level for the main load history;
- a more detailed definition of the structural resistance was requested, for use with the safety formats.

Paper II. Engineering bond model for corroded reinforcement.

In **Paper II**, a bond model (named ARC2010) for assessing the anchorage capacity of corroded reinforcement bars was further developed and validated against a large test database. The bond model determined the anchorage capacity by solving the equilibrium conditions along the reinforcement bar, as described by the differential equation:

$$\frac{\pi \cdot \phi_m^2}{4} \cdot \frac{d\sigma_s}{dx} - \pi \cdot \phi_m \cdot \tau_b = 0 \quad (5.2)$$

where ϕ_m is the reinforcement diameter, σ_s is the stress in the reinforcement, τ_b is the local bond stress and x denotes the longitudinal direction of the bar. The local bond stress (as a function of slip between reinforcement and concrete) constituted the core of the model. This was based on the relationship in *fib* Model Code 2010 [16] but some elements were added and modified, specifically:

- introduction of equivalent slip, to account for bond degradation due to corrosion;
- change of failure mode due to corrosion-induced cracking of the concrete cover;
- modification of the residual bond stress, in case of low stirrup content.

See Figure 3.4. for an illustration of these bulleted features.

The database used to calibrate the bond model has been depicted in terms of its size and distribution, as well as the variation of relative bond strength among subsets of test results, see Figure 5.4. In Figure 5.5, the relative average bond strength obtained from the ARC2010 model (in other words, the deteriorated bond strength normalised by the non-corroded bond strength) is plotted against the corresponding database values for cases without and with stirrups. For ARC2010, increases in relative bond strength were deliberately not allowed. However, a number of tests in the database show increased capacity. The scatter among the bond tests was visibly quite large. However, it may also be noted that the results are distributed equally along the diagonal line. This represents the same result in the database and ARC2010 but with a slight skew towards the safe side.

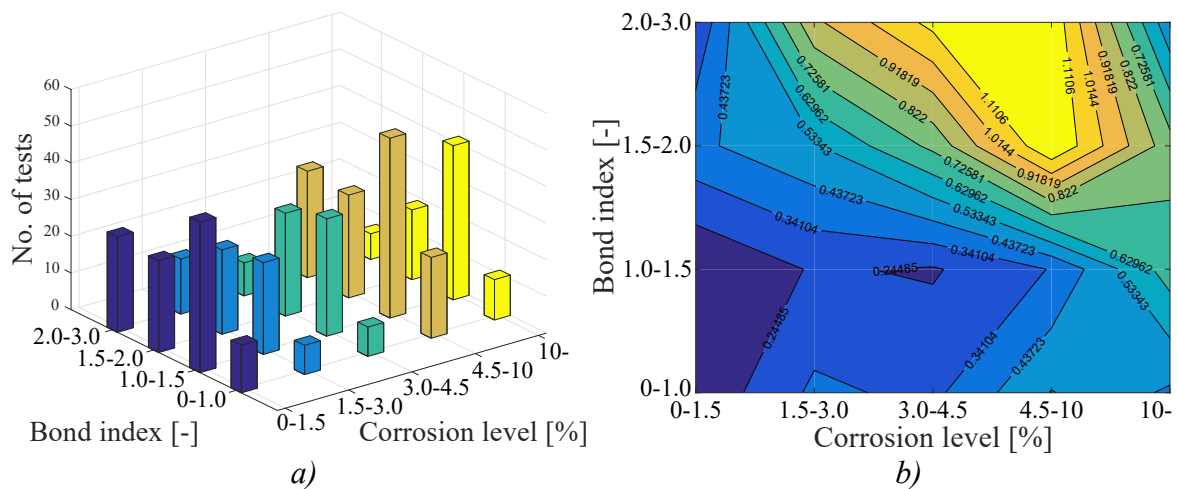


Figure 5.4. a) Four bond index (confinement) intervals and five corrosion level intervals on the horizontal axes and number of test results in each group on the vertical axes and b) contour plot of the coefficient of variation for relative bond strength in the database for varying bond index and corrosion level. Adopted from **Paper II**.

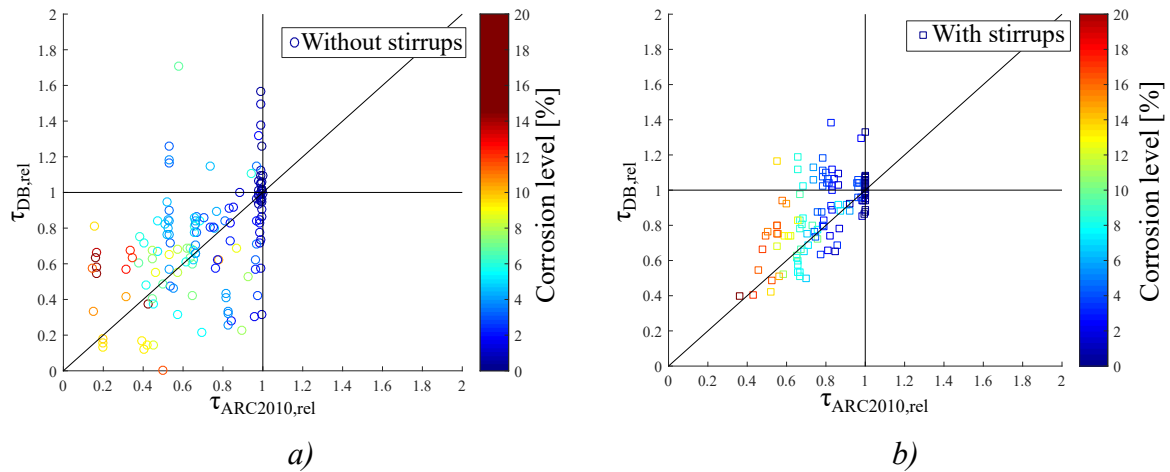


Figure 5.5. Relative average bond strength (from database) plotted against ARC2010 values for cases a) without stirrups and b) with stirrups. Adopted from **Paper II**.

The main findings from **Paper II** may be summarised as follows:

- a model for assessing anchorage of corroded reinforcement was proposed. This included the confinement effects from concrete and transverse reinforcement, plus the changed confinement at the point where corrosion cracks the concrete cover;
- the model was shown to represent the physical behaviour well, with a marked decrease in bond strength at the point where corrosion cracks the concrete cover (in cases of low stirrup contents). For higher stirrup contents, the decrease was less pronounced;
- the full local bond stress-slip relationship is obtained from the model, rather than just the maximum bond strength or reduction in anchorage capacity;
- reasonably good agreement with a large database of bond test results was shown.

Paper III. Partial safety factors for the anchorage capacity of corroded reinforcement bars in concrete.

In **Paper III**, partial safety factors were derived for the bond model established in **Paper II**. First, a probabilistic version of the bond model was set up by including the input parameters (basic variables) as random variables, following appropriate probability distributions. The response of the probabilistic model was studied using Monte Carlo simulations; over 600 design cases were run, with varied input for the basic variables such as concrete strength, embedment length and so on. Thereafter, probability distributions were fitted to the results and these were used to derive partial safety factors. In Figure 5.6 a), the resulting probability distributions (describing the anchorage capacity for three different embedment lengths) are shown for a sample case. Figure 5.6 b) shows the partial safety factors derived for 480 design cases with stirrups, in two different plots sorted by reinforcement bar diameter. It is worth noting that the smaller bar diameter required a higher partial factor for corrosion levels of 15 and 20%.

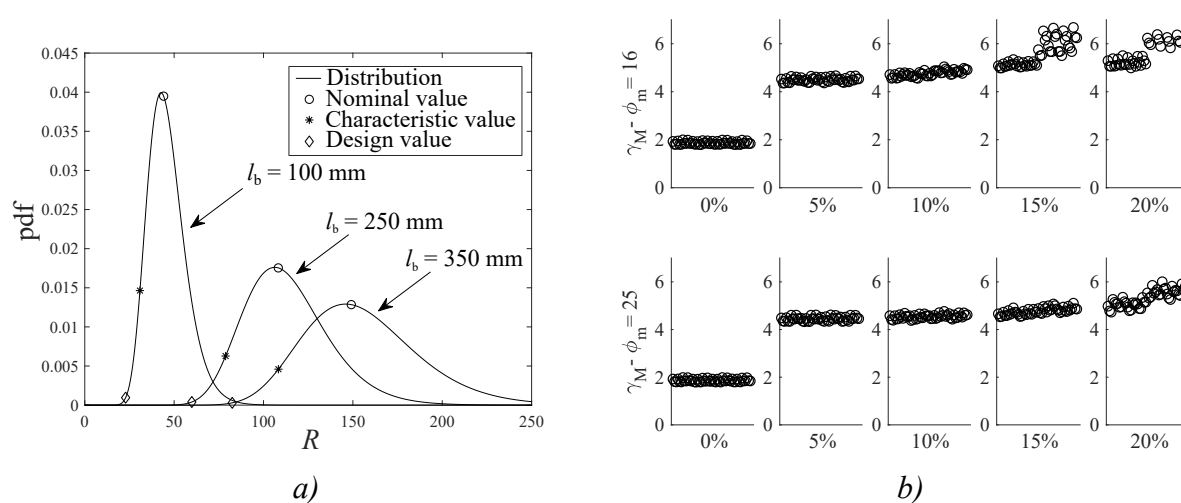


Figure 5.6. a) Probability distributions describing the anchorage capacity for three different embedment lengths for a sample case, plus the nominal, characteristic and design values. b) Comparison of partial factors for two different reinforcement bar diameters for the 480 design cases with stirrups. Adopted from **Paper III**.

The resulting partial safety factors, conservatively derived from the design cases, are assembled in Table 5.1.

Table 5.1. Partial safety factors for cases without and with stirrups.

	Corrosion level	γ_M
Without stirrups	Uncorroded	2.0
	Corroded	3.4
With stirrups	0%	1.9
	5%	4.7
	10%	4.9
	15%	5.2-6.4*
	20%	5.2-7.6*

*Intermediate cases may be interpolated, in cases where n_b is 1-5.

The partial safety factors (shown in Table 5.1) were verified, based on reliability analyses of anchorages designed using the stated factors. The end region of a beam was considered, so that 20 design cases with stirrups might be composed, with varying embedment lengths, number of anchored bars and corrosion levels. Four design cases without stirrups were also included. A design value of the anchorage capacity was calculated for each design case, using the partial safety factors. By setting this value as equal to the design load, it was possible to determine the probability distributions of the loads for each design case. The reliability analyses were conducted by evaluating the limit-state function:

$$g(\mathbf{X}) = R_{\text{ARC2010}}(\mathbf{X}) - L(\mathbf{X}) \quad (5.3)$$

where $R_{\text{ARC2010}}(\mathbf{X})$ is the anchorage capacity and $L(\mathbf{X})$ the load. For each design case, the resulting reliability was quantified using a combination of FORM and importance-sampling simulations. The resulting reliability indices and “sensitivity factors” for FORM are shown in Figure 5.7. Note that all reliability indices met the target reliability, β_t , of 3.7 for a one-year reference period. The sensitivity factors may be viewed as the influence of the basic variables on the reliability. The top four basic variables in the legend relate to the load, while the bottom four relate to the resistance. Note that with increasing corrosion level, the influence of the basic variable related to the load decreases, while the influence of the resistance-related basic variables increases. It is also noteworthy that the major influence of the resistance model uncertainty θ_R (which dominated for corrosion levels of 5%) successively decreases at higher corrosion levels, as the corrosion level itself exerts more influence.

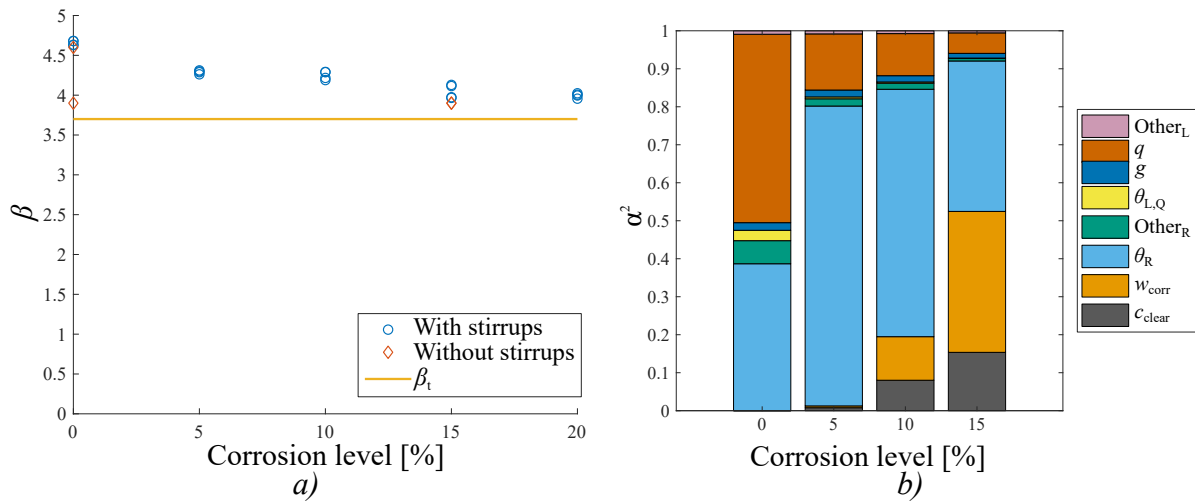


Figure 5.7: a) resulting reliability indices for different corrosion levels, b) sensitivity factors obtained from FORM for a representative case with stirrups. Adopted from **Paper III**.

The main findings from **Paper III** may be summarised as follows:

- partial factors were derived for the anchorage capacity of corroded reinforcement bars in concrete;
- the intended safety level was verified using probabilistic reliability analyses for a number of designs, based on the partial safety factors;
- the modelling uncertainty was shown to be highly influential on the anchorage capacity of corroded reinforcement;
- the corrosion level was shown to influence the sensitivity factors of the basic variables.

Paper IV. Incorporation of pre-existing longitudinal cracks in finite element analyses of corroded reinforced concrete beams failing in anchorage.

Paper IV investigated methods of incorporating pre-existing, corrosion-induced, cracks in anchorage assessment. The methods were based on the four modelling levels proposed by [91], see Figure 5.8. **Paper IV** focused mainly on analyses on levels 3 and 4. 3D FEs were adopted for the concrete, which was modelled using a total-strain-based, smeared-crack approach. The reinforcement was modelled with 1D and 3D FEs for levels 3 and 4 respectively. Analysis on modelling level 3 depends on a bond stress-slip relation capable of representing the loss of confinement due to cracking of the concrete cover, cf. **Paper II**. Meanwhile, on level 4, the analyses use a friction model to explicitly include the confinement. Three methods of accounting for cracks were applied: a) modifying the bond stress-slip relation, b) weakening finite elements at the position of the crack and c) weakened discrete crack elements. All methods were applied on level 3, whereas b) and c) were applied on modelling level 4.

Level description	Level 1	Level 2	Level 3	Level 4
	Residual bond capacity and available anchorage length	1D bond-slip relationship and available anchorage length	FE analysis with 1D bond-slip relationship	3D FE analysis including frictional bond model
Structural model				
Bond model				

Figure 5.8. Overview of modelling levels for assessment of anchorage. Adopted from **Paper IV** as modified from [91].

Experiments on the anchorage capacity of corroded beam specimens from [92] were analysed using the three methods. An FE mesh of the concrete beam, load and support plates and the weakened and discrete crack elements are shown in Figure 5.9. Note that the reinforcement bars were de-bonded, except for the outer 100 mm on each side.

Resulting load-deflection curves for analyses on levels 3 and 4 are shown in Figure 5.10. The weakened elements or discrete cracks exerted no influence by on the results of level 3 analyses. Thus, this analysis level failed to describe the impact of pre-existing cracks. However, for level 4, the weakened elements led to a lower estimate of the ultimate capacity, compared to omitting cracks or implementing them as discrete cracks.

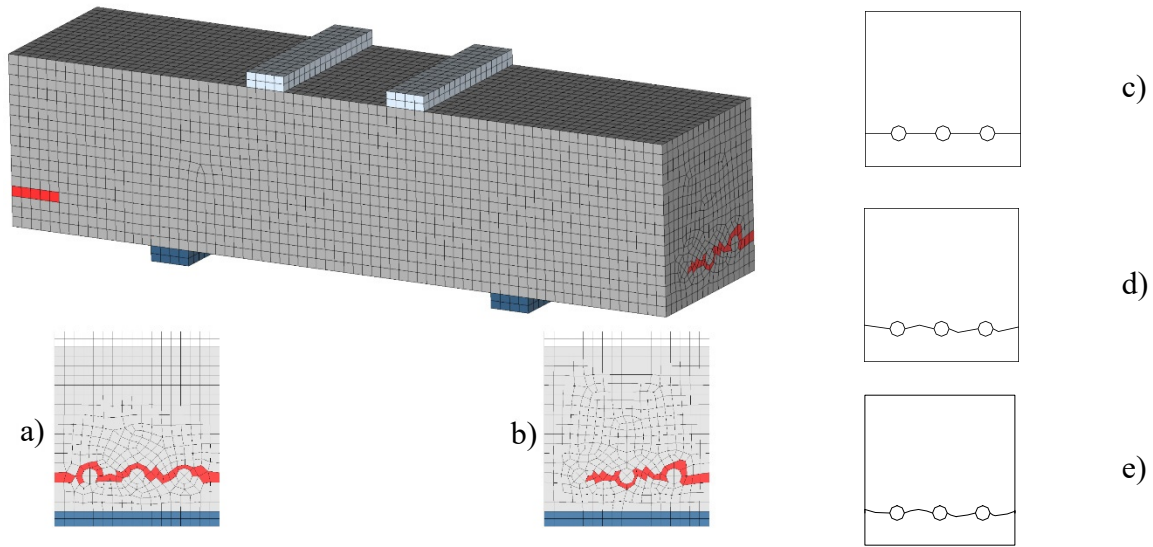


Figure 5.9. Elements assigned weakened elements on a) left-hand and b) right-hand side of specimen 5. c-e) Level of detail for the geometrical representation of discrete cracks for the left-hand side of the same specimen, with low, intermediate and high levels of detail shown in c-e) respectively. Adopted from **Paper IV**.

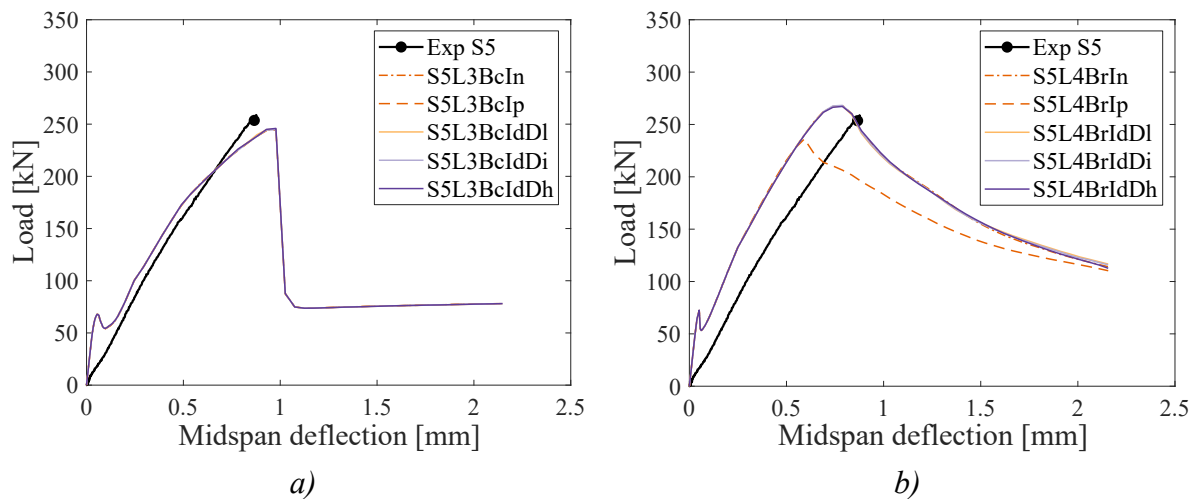


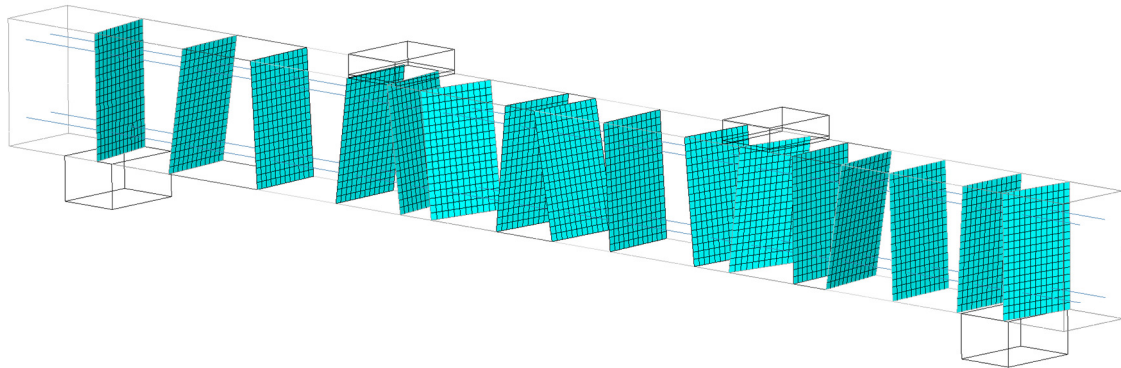
Figure 5.10. Load-deflection curves for specimen 5 analysed on a) level 3 and b) level 4, with crack implemented through the bond stress-slip relation (level 3) and through weakened elements (suffix Ip) and discrete cracks (suffix Id plus three levels DI-h). From **Paper IV**.

The main findings from **Paper IV** may be summarised as follows:

- simple hand calculations might, with little effort, provide a lower bound for the capacity, without knowing the corrosion level;
- an increased modelling level resulted in improved prediction of the ultimate capacity, at the expense of increased analytical and computational efforts;
- pre-existing cracks in FE models with 1D reinforcement did not influence the results;
- the use of discrete cracks had no influence when included in the models with low, intermediate, or high detail, due to the stress state in the anchorage zone;
- 3D FEA with solid reinforcement elements (level 4) with cracks included as weakened elements provided the most complete and accurate results for this case. One advantage of this modelling method is that input regarding the corrosion level is not required.

Paper V. Incorporation of pre-existing cracks in finite element analyses of reinforced concrete beams prone to shear failure.

Paper V applied the method of incorporating pre-existing cracks in FE analyses (outlined for corrosion-induced cracks in **Paper IV**) to represent cracks from previous loading. A comparison was also made with experimental results from pre-cracked beams. Figure 3.7 presented the procedure for weakening the tensile properties of elements representing the cracks. The weakened properties of the discrete cracks were similarly derived (fully described in **Paper V**). Examples of pre-existing cracks incorporated using weakened elements were shown in Figure 3.8 and incorporation via discrete cracks is shown in Figure 5.11.



*Figure 5.11. 3D mesh of PCI with discrete cracks. Adopted from **Paper V**.*

The load-deflection curves from FE analysis using weakened elements and discrete cracks, plus experimental results are shown in Figure 5.12.

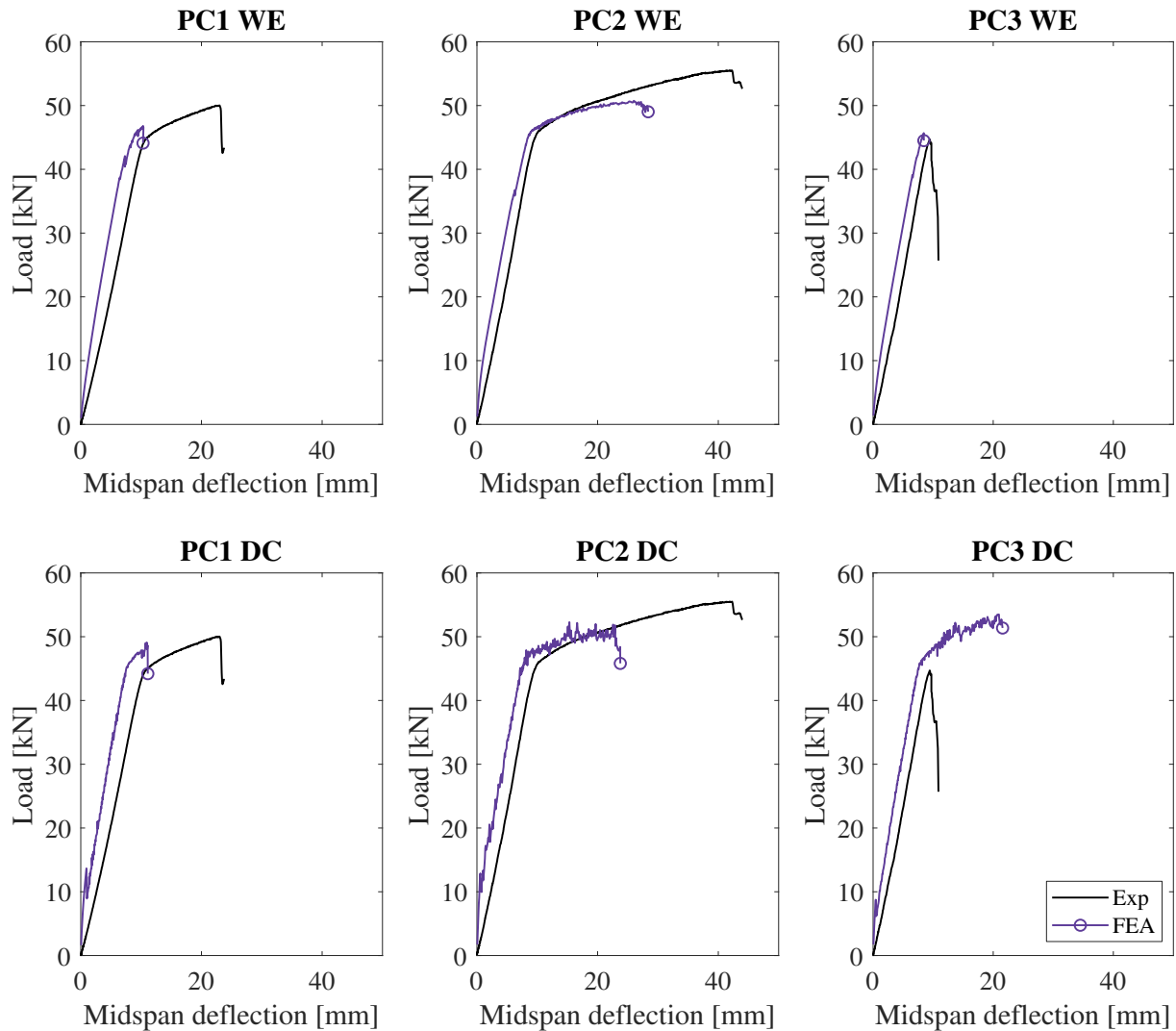


Figure 5.12. Load-deflection curves for specimens PC1 to PC3, from experimental results and FE analyses of weakened elements (top row of plots), plus discrete cracks (bottom row of plots). The end of the analysis (unconverged step) is shown as a circle in each plot.

The main findings from **Paper V** may be summarised as follows:

- pre-existing cracks were shown to influence the failure mode, ultimate capacity and ductility. However, it was noted that the test setup was designed to make the cracks influential;
- incorporating pre-existing cracks in weakened elements led to better agreement with test results, compared to traditional FEA (without pre-existing cracks), particularly for the ductility and failure mode. However, the estimate of the ultimate capacity was also slightly improved;
- incorporation by discrete cracks gave improved prediction of the failure mode. However, traditional FEA provided better estimates of the ultimate capacity and ductility;
- modification of the compressive behaviour of the weakened elements (to reflect closure of cracks in the compressive zone), plus reduced reinforcement bond stiffness and strength (to represent damage from previous loading) was shown to influence the

bending stiffness in the analyses towards better correlation with the experimental observations;

- the choice of shear retention for the weakened elements was shown to influence the results from the analyses. However, its influence was minor for the discrete cracks.

6 Conclusions and future research

This section presents the conclusions drawn from the work within this thesis and offers suggestions for future research.

6.1 Conclusions

The aim of this thesis was to develop methods of reliable structural assessment of concrete structures with corrosion damage as well as pre-existing crack patterns. The work comprised methods of structural analysis and verification of structural safety. After brief introductory paragraphs, the main conclusions are presented in the same order as the research questions (see Section 1.2).

An investigation was conducted to see whether available safety formats for nonlinear analyses could also be used successfully on a more complicated geometry and loading (compared to a beam subjected to vertical load). The safety formats available in *fib* Model Code 2010 [16] were applied to a concrete frame, subjected to a vertical and horizontal (non-proportional) load. Two load histories were studied, a vertical load followed by a horizontal one and vice versa. Based on this investigation, the following conclusions could be drawn:

- the importance of load-history considerations in NLFE analysis was confirmed as, for one of the studied load histories, the ECOV safety format did not meet the intended safety level;
- a framework of engineering guidelines for NLFE analysis, dealing with all important aspects for conducting an accurate analysis, is desirable. Once this investigation was complete, guidelines were published by the Netherland's Ministry of Infrastructure and the Environment [20], which partly addressed this need. However, the author sees a need for clearer recommendations on how to treat the load history and how to define the design resistance for more complicated load cases.

Furthermore, a simplified model for assessing anchorage of corroded reinforcement in concrete was developed and validated (called ARC2010). This means that representative estimates may be obtained, of the anchorage capacity of corroded reinforcement in engineering applications. The following conclusions may be drawn based on the work with ARC2010:

- the model was developed based on understanding created by advanced models and experimental tests. It has the potential to assist practicing engineers in assessments, as it is straightforward to understand and use;
- despite the simplified nature of the model, the physical behaviour is well represented;
- the bond stress-slip relationships obtained from ARC2010 may also be used in NLFE analysis for a more realistic representation of the structural behaviour (regarding shear failure, for example);
- using *fib* Model Code 2010 as the basis for ARC2010 has made it possible to account for the reinforcement bar position in the cross-section and include other effects covered in the code, such as transverse stresses and longitudinal cracking;
- when compared to experimental results, ARC2010 predicts slightly lower normalised average bond strengths than those observed in the tests.

Moreover, verification of structural reliability in design and assessment typically compares the design value of the load effect with the design value of the resistance, cf. Eq. (4.4). Partial safety factors were derived for the proposed anchorage assessment model (ARC2020), to provide a design value for the anchorage capacity with a sufficient safety margin. Based on this derivation, the following conclusions may be drawn:

- a design value for anchorage capacity, consistent with the target reliability level, might be obtained by deriving partial safety factors for the anchorage capacity of corroded reinforcement in concrete;
- for assessing anchorage of corroded reinforcement, the modelling uncertainty is high. This contributes to high values for the partial factors.

There was also an examination of whether the influence of pre-existing cracks could be represented by locally weakening the material properties of the concrete. This was investigated for both continuum elements and discrete crack elements, in two markedly different situations: a) assessing the anchorage capacity, for corrosion-induced splitting cracks along the reinforcement and b) moment and shear capacity, for beams with cracks formed under previous external loading. Based on these investigations, the following conclusions may be drawn:

- an NLFE model with 3D elements for concrete and reinforcement (with splitting cracks due to reinforcement corrosion included as weakened continuum elements) provided a reasonable estimate of the anchorage capacity for the studied case, with no input regarding the corrosion level;
- the stress state in the anchorage zone meant no significant influence could be observed for the analyses of corrosion-induced cracks implemented using discrete cracks;
- when weakened continuum elements were used to incorporate pre-existing cracks in FE models of RC beams (with cracks from previous external load tested in bending), improved estimates (compared to traditional FEA without pre-existing cracks) were obtained, particularly for the ductility and failure mode. However, the ultimate capacity was also slightly improved;
- incorporation of pre-existing cracks by discrete crack elements for the same case did not improve estimates of the ultimate capacity and ductility compared to traditional FEA. However, the failure mode was better estimated;
- compared to discrete cracks, less implementation effort and computational time was required for crack incorporation using weakened continuum elements.

Finally, some overarching conclusions addressing the aim of the work may be drawn from the aggregated work. These may be summarised as follows:

- if improved models for estimating the capacity of damaged structures are applied more frequently in practice, unnecessary strengthening or dismantling of structures may be reduced;
- when simplified models are not sufficient, perhaps because they have been simplified, or because there is a requirement for input data which is not available (such as corrosion level), advanced models for improved capacity estimates may be used but at the cost of greater effort;
- to be practically relevant, simplified and advanced models must be equipped with suitable safety formats, if they are to produce results with a sufficient safety margin.

6.2 Suggestions for future research

Based on the work conducted in this thesis, multiple areas of importance for future work have been identified. The following suggestions may be offered:

- in the evaluation of safety formats for NLFE analysis, it was found that one safety format did not lead to the intended safety level. It would be interesting to further study the applicability of available safety formats for cases with more complex loading and geometry (such as a shell structure subjected to a variety of loading conditions). Different failure modes and their effect on the modelling uncertainty should be included;
- in the calibration of partial safety factors conducted in this work, it was evident that the common assumption for the sensitivity factor for resistance ($\alpha_R = 0.8$) did not correspond well at higher corrosion levels. A study of sensitivity factors for basic variables for deteriorated concrete structures is therefore proposed;
- many of the assessment methods for concrete structures with corroded reinforcement require input regarding the corrosion level. For now, this is often difficult to determine in practice. Development of non or semi-destructive methods to estimate the corrosion level of reinforcement bars in existing concrete structures would therefore be of major practical importance.

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