

THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

Efficient Modeling and Control of Crushing Processes in Minerals Processing

Marcus Johansson



Department of Industrial and Materials Science
CHALMERS UNIVERSITY OF TECHNOLOGY

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MARCUS JOHANSSON

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Department of Industrial and Materials Science
Chalmers University of Technology
SE-412 96 Göteborg, Sweden
Telephone + 46 (0) 31 – 772 1000

Cover: Figure 1.2, overview of a crushing plant.

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"Destroying solids by cracking them is a fascinating subject which may be considered good or bad depending on one's point of view." - K. Schönert

Abstract

Modeling and simulation is a tool to explore and increase the understanding of a phenomenon. This thesis focuses on developing models of crushers and equipment used in the mining industry. Specifically, the focus is on a branch of modeling called time dynamic modeling which is a model that gives an output as a function of time.

The work is divided into three areas: physical modeling, control modeling, and circuit modeling. Physical modeling deals with how to develop high fidelity unit models of equipment, in this thesis, a model of a jaw crusher and of an HPGR are presented. These models are aimed to be predictive and should predict the process variables under a specific set of operating conditions. The models are developed with the process parameters that are used in the physical unit, in the case of the HPGR, roller speed, and hydraulic pressure. The parameters within the models are parameters with units and have real physical meaning; for example, a dimension of the machine.

The topic of control modeling focuses on how to apply the knowledge from modeling in the control domain to improve operations. An example of setting up a model predictive controller and using it to control a crushing circuit simulation is demonstrated. Model predictive control is an optimal control strategy that can be used to drive the circuit towards a specific goal. As the demand is increased on the mining companies to perform better these types of controllers and operation improving actions are important. This thesis aims to target some of the challenges involved in improving plant operation and control.

Within circuit modeling, a broader perspective is taken to study the operations of an entire circuit or plant. The study presented in this thesis focuses on how sensitive a plant is to variations and how the plant design itself will affect the plant's ability to cope with variations. The approach has been to simulate faster and to use less complex models many times to determine limits and ranges. The method shows potential to understand a circuit better before it is built.

The outcome of the research is a better understanding of how to model machinery, such as the HPGR and the jaw crusher. By developing high fidelity models, insights are gained on how to move between the different modeling domains. The knowledge is useful for studies of circuits, and how to set up optimal controllers. Especially controllers that require models of a specific type or models that have to be fast to simulate.

Keywords: Dynamic modeling, HPGR, Jaw crusher, Minerals processing, Process control, Modeling, Comminution

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List of Publications

Appended papers:

Paper A. Marcus Johansson, Magnus Bengtsson, Magnus Evertsson, Erik Hulthén *A fundamental model of an industrial-scale jaw crusher*, Published in Minerals Engineering, vol 105(69-78), 2017.

Paper B. Marcus Johansson, Magnus Evertsson *A time dynamic model of a high pressure grinding rolls crusher*, Published in Minerals Engineering, vol 132 (27-38), 2019.

Paper C. Marcus Johansson, Magnus Evertsson *Applying linear model predictive control to crushing circuit simulations*, Published in proceedings of IMPC 2018 - 29th International Mineral Processing Congress, p. 3423-3432, Moscow, September 2018.

Paper D. Marcus Johansson, Magnus Evertsson, Erik Hulthén *Analysis of Dynamic Process Characteristics in Crushing Plants from a Robustness Point of View*, Published in proceeding for the 11th International Comminution Symposium, Cape Town, South Africa, April 2018.

Paper E. Marcus Johansson, Magnus Evertsson *Time Dynamic modeling and control of an HPGR circuit*, Published in the proceedings to The Conference in Minerals Engineering, Luleå, February 2018.

Work distribution:

Paper A: Johansson and Bengtsson initiated the idea, Johansson developed the model with support from Bengtsson. Johansson wrote the paper with Bengtsson, Evertsson and Hulthén as reviewers.

Paper B & C: Johansson and Evertsson initiated the idea. Johansson implemented the code. Johansson wrote the papers with Evertsson as a reviewer.

Paper D: Johansson and Evertsson initiated the idea. Johansson implemented the code. Johansson wrote the paper with Evertsson and Hulthén as reviewers.

Paper E: Johansson wrote the paper with Evertsson as a reviewer.

Other relevant publications:

Marcus Johansson, Magnus Evertsson, Gauti Asbjörnsson *Improvement Opportunities using Time Dynamic Simulations*, Published in proceedings of the 15th European Symposium on Comminution and Classification, 2017.

Marcus Johansson, Magnus Evertsson, Erik Hulthén *A Novel Approach to Cone Crusher Feeding Using High frequency Power Draw Measurements*, Published in proceedings of the 15th European Symposium on Comminution and Classification, 2017.

Marcus Johansson, Johannes Quist, Magnus Evertsson, Erik Hulthén *Cone crusher performance evaluation using DEM simulations and laboratory experiments for model validation*, Published in Minerals Engineering, p. 93-101, 2016.

Marcus Johansson, Johannes Quist, Magnus Evertsson *Bonded Particle Model Calibration Using Design of Experiments and Multi-Objective Optimization*, Published in proceedings of the MEI 10th International Comminution Symposium (Comminution '16), 2017.

Marcus Johansson, Johannes Quist, Magnus Evertsson, Erik Hulthén *Investigation of High Speed Cone Crushing Using Laboratory Scale Experiments and DEM*, Published in proceedings of the 14th European Symposium on Comminution and Classification (ESCC 2015), p. 193-199, 2015.

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Part I

Introductory chapters

Chapter 1

Introduction

This chapter will give a brief introduction to the topic of this thesis from a few different points of view.

1.1 Systemic view

The use of tools is one of the distinguishing points that separates humans from other animals. As the evolution of our tools required them to be harder, sharper, and more advanced, metals ended up being the preferred choice of material. Today metals are key to our modern society, and without them the society would not be functioning as we know it today. Metals are used in everything from infrastructure to our cell phones, and a life without them would be very different from today. There are two ways to get a hold of metals, either by mining new ore bodies or by recycling old products. The mining industry is a large industry, responsible for \$683 billion USD [33] in revenue each year as well as consuming non-negligible amounts of resources, such as energy, water, chemicals, and spare parts. The freshly mined metals are sold either at a spot price or as a contract on a global market. The supply and demand for these metals vary and are highly affected by politics, economic cycles, and many other factors. In the process of mining new resources, there is a chain of events that take place before the commodity hits the market. An illustration of the value chain is pictured in Figure 1.1. When a resource is located, mining rights are established, a mine is developed and a concentrator is built, the actual processing can start and it is where this thesis can be applied. As the industry is located in the global system and affected in many ways it is important to keep this in mind when analyzing trends, decisions and responses.

This research focuses on the gray block in Figure 1.1 in general and specifically comminution and classification, the two boxes enclosed by the dashed line. All of the other boxes in the value chain are equally important for the industry, however, it will not be the focus of this thesis.

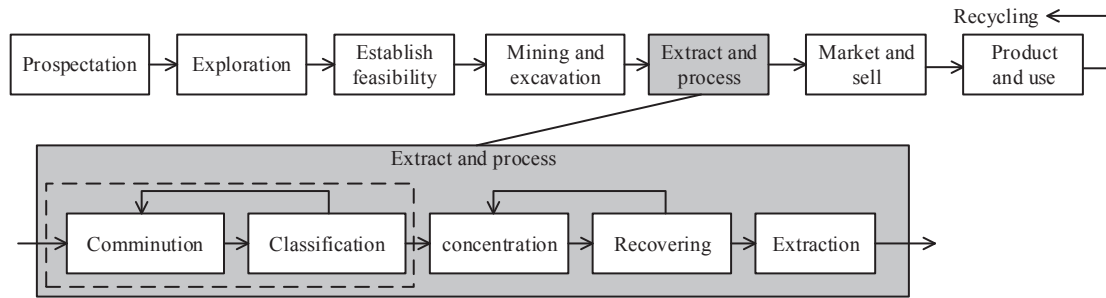


Figure 1.1: A generalized value chain for the mining and metals industry. This thesis focuses on the dashed box on the second row, comminution, and classification

1.2 Physical Plants

Rock is excavated from the ore body by drilling followed blasting, then loaded and handled to be primary crushed as the blasted rock can come in very large pieces and it is not possible to transport on a conveyor. The primary crushed rock is now of adequate size to be placed on a conveyor (0-300 mm). Thereafter the rock is again crushed in a series of crushing stages using, jaw crushers, cone crushers, HPGRs, or in some cases AG- or SAG-mills¹. How many, what type and configuration vary from plant to plant. After crushing, the fine gravel is, in most cases milled to final size and a powder, for example, using tumbling mills. This powder is then transferred to a separation process where the gangue rock and the valuable minerals are separated, in general terms called concentration. A more detailed description of how minerals processing plants are built up is presented by Wills [45]. The concentrators or processing plants are huge installations, as can be seen in Figure 1.2, requiring significant investments and are intended to be used for a long time (several decades). The goal of a plant is to process as much as possible or desired to the highest quality possible. Every hour that a plant is not operating, production volume is lost, and there is no way of getting lost production back.

This research concerns parts of the processing chain, and it is, therefore, vital to see the bigger picture in terms of the processing plant. Two simplified flowsheets are shown in Figure 1.3 and 1.4 to illustrate examples of two different processes. In Figure 1.3, the process contains four stages (1-4) of comminution with classification. This could be a typical setup for a minerals processing concentrator. The product from the pictured flowsheet continues to a separation process and possible further regrinding if needed. In Figure 1.4 a flowsheet for a SAG circuit is drawn. This is an alternative to the more crusher focused flowsheet in Figure 1.3. Pebbles crushing is a popular addition to traditional SAG circuits in the way it is illustrated in Figure 1.4. Especially since the general trend is that ores are getting harder and it, therefore, becomes nearly impossible to grind down the pebbles.

¹S/AG, semi/autogenous grinding



Figure 1.2: A minerals processing plant and an overview of the conveying system between crushers, screens and storage units.

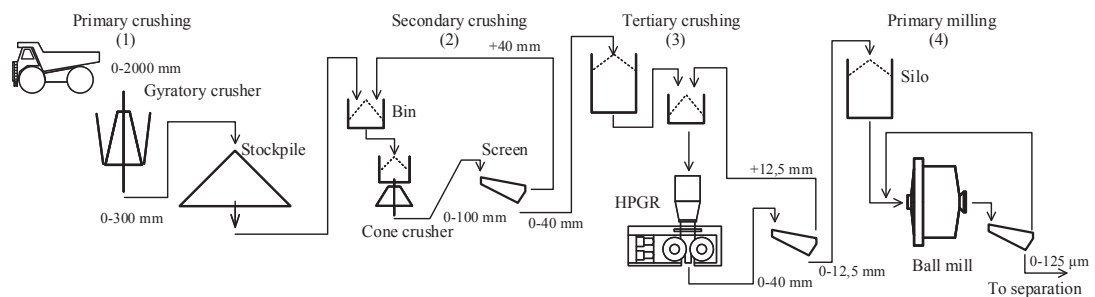


Figure 1.3: A simplified and ideal flowsheet with four stages of comminution and three classification stages.

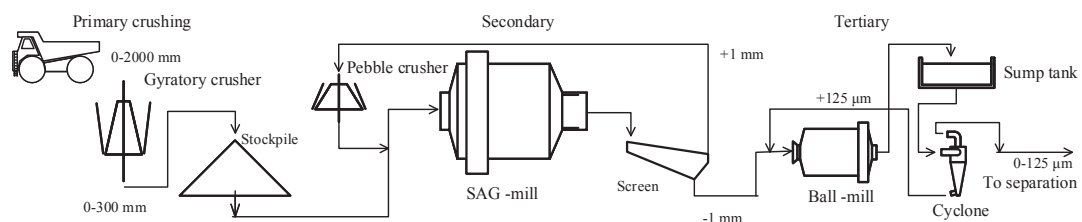


Figure 1.4: A simplified and ideal flowsheet for a SAG-mill circuit with regrinding (ball mill) and pebbles crushing.

1.3 Modeling

A model is a way of describing something real in a simplified way. Models are developed in the discipline of modeling. In this research, mathematics is used as a modeling tool to describe reality. A model is never perfect as it comes with assumptions, as part of the simplified description of the real phenomena. The modeling exercise can go on forever trying to develop a refined model that is perfect. Traditionally modeling and simulation of comminution systems have been carried out

in steady-state. This includes simulations software such as, JKsimMet², AggFlow³ and ModSim⁴. This development of simulation platforms for communiton circuits started before computers were as powerful as they are today. This is one of the reasons for using steady state simulations rather than time varying, dynamic simulations. Whiten [44] presented simulations of a closed crushing circuit and Ford and King [15, 16] presented solutions to simulate entire plants, from crushing to separation. The techniques presented in these papers among others have contributed to developing the above mentioned steady state simulation software.

A generic block model is shown in Figure 1.5, including notation for, Feed (F), model parameters (p), model inputs (u), internal variables (x), Product (P) and model outputs (y).

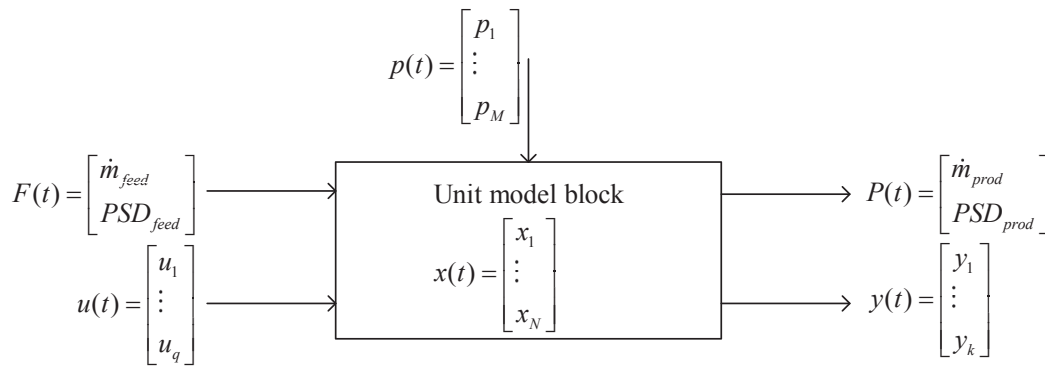


Figure 1.5: An example of a unit model with the different variables and parameters associated with a model.

For research purposes, it is practical to develop models in a stand-alone platform, where there is full access to the source code of the models and the setup. The modeling work within this research builds mainly upon the work of Evertsson, Asbjörnsson and Bengtsson [12, 1, 7]. The environment used for model development and simulation is MATLAB and Simulink, mainly for historical and practical reasons. The end goal with the models is to make as accurate models as possible. The models are aimed to be used in time dynamic simulations, implying that they are stepped one time step at the time and with changing inputs. This is very different compared to a steady-state simulation, where the simulation model is iterated until mass balance is found. Steady-state is an equilibrium state that can be found with dynamic simulations as well, however with all the variations present in real-world plant, the actual existence of steady-state for a longer time period is unlikely.

This research focuses on process models, mostly time dynamic, the requirements on those are as follows:

²<https://jktech.com.au/jksimmet>

³<https://www.aggflow.com/aggflow-design>

⁴<http://www.mineraltech.com/MODSIM/>

- Run faster than real-time when simulated.
- Respond to changes in the process, both machine settings, and operating conditions.
- Be predictive and possible to operate in a specified range of conditions.

1.4 Industrial process control

Industrial sites consisting of large scale equipment, both valuable and powerful, should have proper control installed for many reasons. The different systems that control an industrial plant are pictured in a hierarchical fashion in the triangle in Figure 1.6. The triangle is an interpretation of the structure introduced by Tatjewski [43]. Where the lowest functions are the ones protecting the plant, it is employees and the equipment integrity. These are typically in the form of interlocks, and protective functions that inhibit things to go wrong. The next level above is the Single Input Single Output (SISO) layer of control loops, which are present at most sites within minerals processing. SISO loops are, for example, control of a level or a flow in the process. On top of SISO-loops there could be Multi Input Multi Output (MIMO) controllers. These controllers handle multiple inputs to control multiple outputs towards an objective under a given set of constraints. A MIMO controller could be in the form of a linear quadratic regulator (LQR) or a model predictive controller (MPC). These controllers are in many cases optimizing the different settings to steer the process to be operating at the most beneficial operating point, this is determined by the objective of the controller. On the very top level there is management, who decided, for example production rates and operating hours. The management, for example plant manager, CEO's and principals in the mining industry are influenced by factors such as political stability, metal prices, resource availability and more.

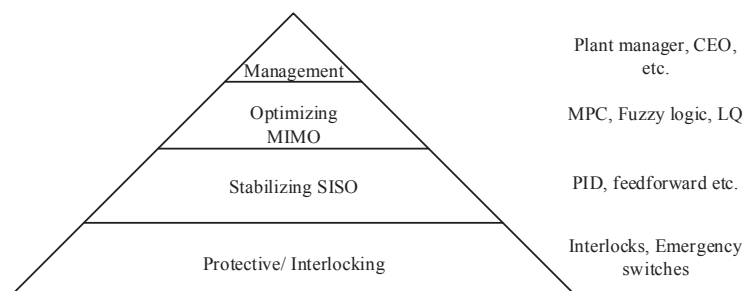


Figure 1.6: Hierarchical construction of the different means of control used on a plant.

Traditionally, especially crushing plants did not have much automatic control before 1980's. Protective systems were there, but in terms of automatic control, most plants were operated manually to large extent. This was very people intensive, and humans tend to get tired of tasks, while computers do not.

Chapter 2

Background

In this chapter, the motivation behind this research will be explained, both in terms underlying drivers and the research questions.

2.1 Drivers

Minerals processing is a process industry, and in order to be profitable it is all about processing large quantities of ore because of the low grade in the remaining ore bodies. Enablers that allows a plant to increase production or produce a better quality product could be the difference between being profitable or not. The mining industry is a global industry found on all continents around the world except in the Arctic and Antarctica. A successful business involves understanding and dealing with an ever-changing operational environment. Industry actors are affected three main ways:

- Economical
- Environmental
- Legal and political

For all companies in mining, it is a delicate balance to make sure all the above aspects are fulfilled. The research presented in this thesis is directly coupled with the economic aspect. However, it can also affect the environmental and legal aspects. Efficiency and performance will determine how profitable an operation is. These two measures are directly linked to operation and how well the operation is executed. Performance is the combined result of quality and quantity of produced products. In minerals processing it is about maximizing the amount of product containing the minerals that can be separated from the gangue ore. Accurate control and understanding of the processes involved in this chain of events will inevitable increase the chances of getting in a better position on the cost curve. To operate profitably is needed to survive and one way of doing that is making sure that the operation of

the process is as efficient as possible. This research aims to answer questions about how to be as efficient as possible.

2.2 Research questions

The following research questions have been formulated. They are divided into three main categories; equipment modeling (RQ1-RQ2) , process robustness (RQ3-RQ4) and process control (RQ5-RQ6). The text underneath the question is to further explain the question and to clarify what the target is.

RQ1: How can high fidelity fundamental models of minerals processing equipment be developed in order to handle machinery of force conditioned type?

A fundamental model is based on physical laws and constitutive relations and should capture a wide operating range of the machine being modeled. A force conditioned crusher is a crusher that will exert the particles to a specific force, compared to machines that exert the particles to a fixed compression (form conditioned) or give them a velocity (energy conditioned). The two later modes of crushing have been explained by Evertsson and Lee [12, 22].

RQ2: How can fundamental models of minerals processing equipment be developed in order to handle machinery with fast dynamic behavior?

Fundamental models are aimed to describe the inner workings of the crusher, the crushing process is fast in many cases and to accurately describe the process fast dynamic behavior needs to be resolved.

RQ3: How can a minerals processing plant's degree of robustness be studied?

A robust plant is insensitive to variations up to a certain degree. This question targets how the robustness property can be studied.

RQ4: What consequences do robustness studies have on plant design?

What is it that makes certain plants robust and how can plants be designed in order to be robust?

RQ5: How can models based on fundamental principles be used to improve plant control?

RQ6: What methods are used for moving between the high fidelity modeling domain and the control modeling domain?

What methods can be used to develop control models, given that information about a high fidelity fundamental model is available for the specific unit?

Summarized answers to the research questions can be found in Section 7.1.

Chapter 3

Scientific approach

The scientific approach used in this research is explained in this chapter.

The research carried out at the Chalmers Rock Processing Systems (CRPS) research group is problem oriented. The group has its roots in product development and specifically machine elements, with strong connections to the germanic school around machine design. The research approach is problem oriented, and for this work the approach previously described by Evertsson [12] and Hultén [20] have been adopted to the topic. A graphical description of the approach is illustrated in figure 3.1.

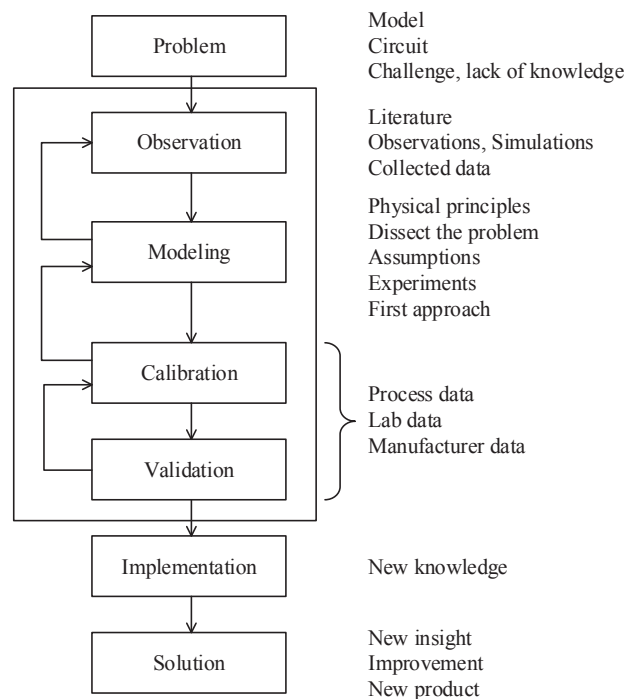


Figure 3.1: A graphical illustration of the research approach used in this research.

A problem oriented approach is rooted in the need to solve an identified problem and focuses largely on the solution to the problem rather than the method to arrive at the result. The specific details of the method varies with the problem type. The general approach assumes that more knowledge about the problem, process or unit will help in solving the problem at hand. In other words, more understanding on a required adequate level is something positive. To describe the process illustrated in Figure 3.1, most of the time it will start with a problem or a knowledge gap of some sort. It is important to study the problem thoroughly, in order to be sure the understood problem is the problem that needs to be solved. Thereafter the research loop starts, including activities such as, literature review, observations and simulations to build a knowledge base. This base is further used to establish what physical principles could be used, how to split up the problem in suitable sizes. By creating sub-problems or functions each part can be tested separately and verified, this especially helpful in debugging and development. After the initial model or solution is developed the calibration and validation can start. Important with the calibration and validation procedure is to separate the data sets, i.e. in tuning on a data set and then always using a different set for validation. In some cases there is lack of data and then the model can not be claimed to be validated, just calibrated. The entire research method is iterative in nature and models and solutions are updated and improved over time.

Once a new model or solution is developed, it is implemented to evaluate if it can solve the posed problem or gap. For research the purpose is now insights and better understanding, the commercialization or productification is a later step, however equally important. The final step is where the utilization comes in and the research is value adding for the greater good.

For the modeling contained within this thesis the approach is not limited in how to structure the models and what tools to use. However as a general rule, the simpler tools that are used the better. This has many advantages, for example simulation speed, debugging and possible translation into other coding languages. The mindset of using the simplest possible approach is also beneficial when it comes to the control modeling. For the process control parts of this work, the aim is to apply known techniques rather than inventing new schemes. In control theory there are many techniques available for linear problems and in general terms easier to apply than the non-linear methods.

A systematic way of thinking is applied on top of the problem oriented approach presented above. The systematic way includes a top down perspective, always keeping in mind the bigger picture. This approach is especially useful for dividing the problems into sub-problems that can be solved separately, adding in especially in the modeling part where problems are dissected in Figure 3.1. The systems approach also have to be applied as there are humans involved in the operation of the real world plants. Humans are part of bigger system, both around the asset and the software and structures around the plants.

Chapter 4

Theory

This chapter introduces topics the reader need to be familiar with in order to understand the work presented in this thesis.

4.1 Modeling of comminution and classification systems

As mentioned in Section 1.3 comminution modeling have historically almost exclusively been done with steady state simulators. This thesis focuses mainly on dynamic modeling and in this section some background to both types of modeling approaches will be given.

Steady state modeling takes a flowsheet and looks at the nodes, balances the mass flow and iterates until all streams have reached convergence. Steady state models do not contain elements that cause delays, accumulation or have process control related functionality. The simulation will yield one answer for the massflow in each stream and one particle size distribution. In Figure 4.1 the same circuit flowsheet is drawn for both a steady state simulation and a dynamic simulation. A major difference in the results from the simulation of the two flowsheets is the type of data that will be generated. From the dynamic flow sheet, the data will be a time series of values over time, while for the steady state flowsheet there will only be one value. A full description of the difference is done by Asbjörnsson [1]. Further, since dynamic simulations include more elements of the actual circuit than the steady state simulation, for example, materials handling, controllers, interlocks, variations, machine wear to mention a few. The effort needed to develop, simulate and interpret these models is more time consuming than steady state simulations. There is a trade-off between the required information and amount of time available that needs to be investigated before initiating a simulation effort.

As this thesis focuses on time dynamic simulations and the topic of steady state will not be further discussed. Dynamic simulations can be used in many applications,

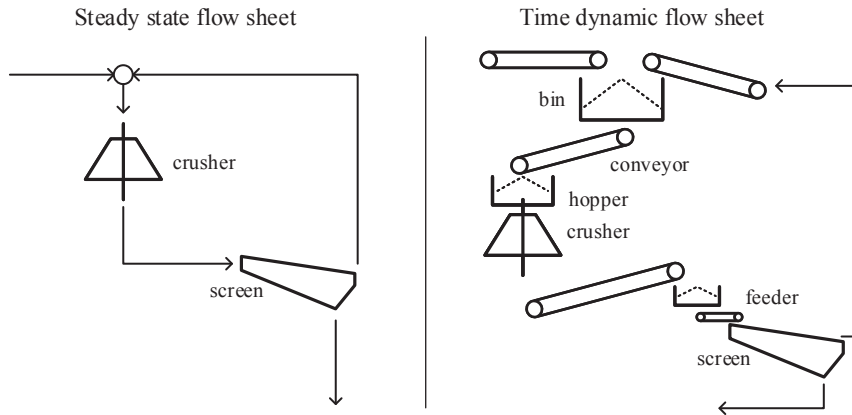


Figure 4.1: Two identical circuits, the left one is the steady state version and the right is the time dynamic version.

Asbjörnsson have for example described and worked with, debottlenecking and operator training [1]. Steyn and Brown used dynamic simulations explore new operational strategies and saved time by using the dynamic model to generate step responses for the advanced control system instead of the actual plant [35]. Legare [23] have presented a framework similar to Sbarbaro's work, concluded in [38]. These frameworks and simulation platforms have been implemented in Matlab Simulink, as this is today one of the accessible choices for researchers. All implementations so far within this thesis have also been completed in Matlab Simulink.

4.2 Crushing machines

There are many different types of crushers used in the industry. In this research two new models are presented, one for a jaw crusher and one for a high pressure grinding rolls crusher, the basics of these crushers will be explained in this section.

4.2.1 Jaw crushers

A jaw crusher is a type of primary crusher handling rocks with top size of 1500mm down to 500mm for a full sized crusher. The capacity range of a industrial sized jaw crushers is between 30 and 1200 tons per hour [45]. The jaw crusher is a work horse and can commonly be seen as a mobile crushing unit. A principal illustration of the crusher is shown in Figure 4.2. There are two plates, one fixed and one moving. The moving one is driven by an eccentricity on the shaft, which the fly wheel sits on. The rock material is fed in between the two plates and is compressed repeated times as it falls further down into the chamber. The size control is implemented by setting the gap between the two plates at the bottom of the crusher. Jaw crushers come

in two different type, the single toggle crusher and the double toggle crusher [45]. The first mathematical models of jaw crushers were developed in the 1950's, by for

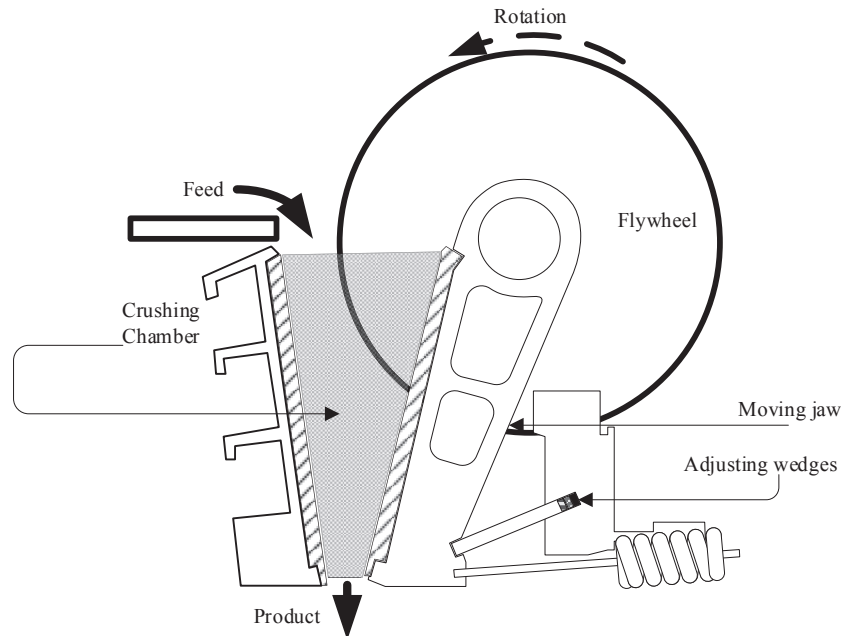


Figure 4.2: Schematic illustration of a jaw crusher by J. Quist.

example Gauldie [18]. The focus in these early model attempts were mainly capacity predictions. Later models have also focused on energy consumption[24], linear wear [25] and motion [31].

4.2.2 HPGR

The High Pressure grinding rolls (HPGR) crusher originates from the roller crusher and was developed in Germany during the 1980's by Prof. Klaus Schönert [41]. The reasoning behind the development started with Schönert classifying different means of comminution, concluding that the most efficient mode of breaking rock is single particle breakage (one rock at the time, 2 contact points), and thereafter inter particle breakage (many rocks at once, many more contact points) or bed comminution [39]. A explanation of single particle and inter particle comminution is given by Evertsson [12]. A schematic illustration of an HPGR is shown in Figure 4.3. The HPGR machine is a roller crusher equipped with hydraulic cylinders pushing on the floating roller (left side in Figure 4.3) towards the fixed side, creating a compressed bed of rock. Both rollers are rotating with the same speed but in opposite directions. The rock is fed in between the two roller and compressed. The high stress causes the rock to break and if pushed very hard it is known to initiate micro cracks in the rock, which have shown to be beneficial in down stream comminution and recovery processes [27]. Due to the very high pressures the roller wear is significant and most HPGR rollers have tungsten carbide studs on the surface to protect the actual roller

body, this is different from the older traditional smooth roller crusher. The HPGR is a crusher operating on material smaller than 50 mm top size for large machines. The capacity can be tuned by setting the roller speed and the maximum capacity is above 3000 [tons/hour] (tph) for large units. For a unit with installed variable frequency drives (VFD) for the motors, the roller speed can be changed online, which is also the case for the pressure setting in the hydraulics. Roller speed and pressure are the two main manipulated variables that can be used to control the HPGR.

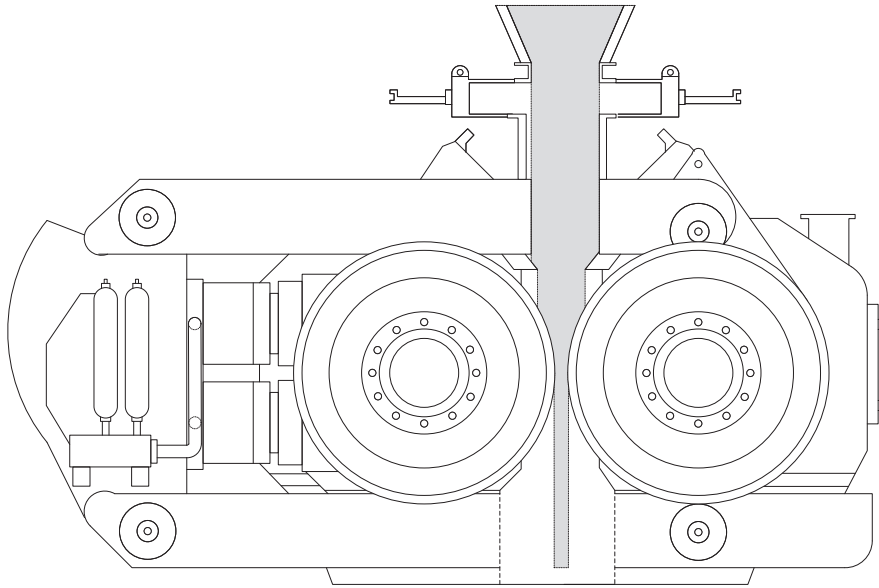


Figure 4.3: Schematic illustration of a FLSmith HPGR, by J. Quist

The HPGR have been subject to extensive modeling, the background models were developed by Austin for roller crushers with soft feed materials, such as coal [3, 2]. After the appearance of the high pressure grinding rolls crusher Schönert and Feurstenau published multiple papers on modeling attempts [41, 17, 40]. Later steady state models appeared from for example Benzer and Ardogan [8, 4] and Morrell [29]. Modeling of the HPGR have also been attempted with DEM by for example Quist [34] and Barrios [6, 5]. Research on industrial HPGR applications have been demonstrated by Powell [32], Daniel [10], Rule [37] and Herbst [19].

4.3 Model based control algorithms

In this research the advanced control algorithms applied have covered linear model predictive control. The topic of linear Model Predictive Control (MPC) will be briefly explained here to help the reader better understand the later application of it. The predecessor to MPC, Dynamic Matrix Control (DMC) was developed by Cutler [9] at the Shell company, in Houston, Texas, to advance the control of

petrochemical plants. The concept was brought to light as computers got more and more powerful during the second part of the 20th century. The idea with the control strategy was to be able to handle large MIMO control problems effectively, especially with systems that have long lagtime. Previously with PID-control the only way to control slow systems was to make the integral part of the PID-controller small. DMC was based on a least squares problem to try to minimize the error over time for a MIMO system. As well as being influenced by the theory and idea behind Receding Horizon Control (RHC) [28]. DMC later evolved along with a similar method called Generalized Predictive Control (GPC), where DMC utilized step response models in the controller and GPC transfer functions. The history of DMC, GPC and MPC is described by Morari [28]. DMC was very successful and is implemented in all most all new petrochemical plants today.

MPC is a control strategy that calculates future inputs and simulates the system to find the optimal inputs given a certain objective. A graphical representation of this is shown in Figure 4.4. The reference trajectory is the target the controller wants to

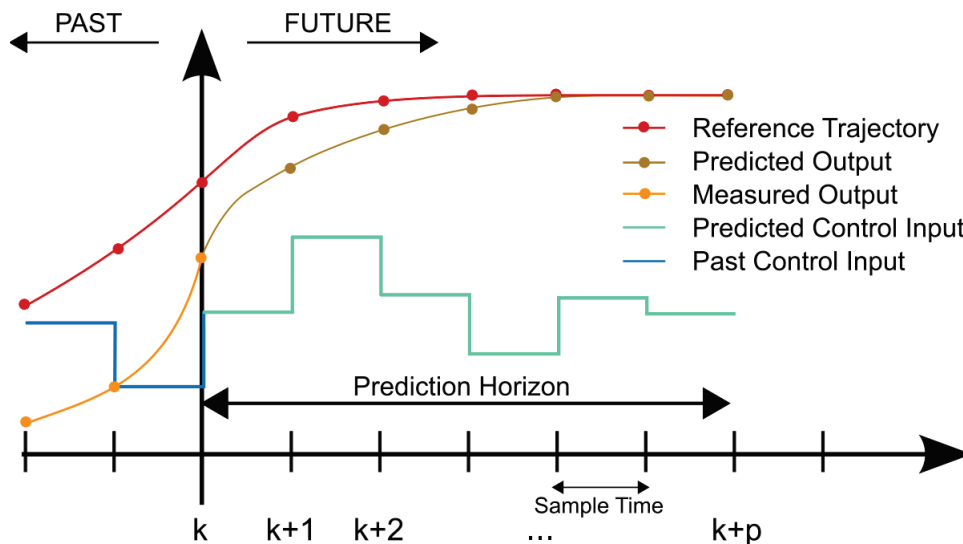


Figure 4.4: A graphical representation of MPC.

hit, this could be static or moving depending on the problem. The predicted output is the output of the model predictions within the controller. The measured output is the measurement from the plant or the physical unit being controlled. The predicted control input is the sequence of optimally calculated controller inputs based on the controller model and measurements at time k . The prediction horizon is the number of steps (p) into the future the controller is making predictions for. One of the properties that makes the MPC scheme successful is that the calculation of optimal control inputs happens every sampling instance, but only the first input is applied to the system. In other words, only the input at time k is applied to the system, the rest of the values are discarded, and when time $k + 1$ happens the same optimization problem is solved again for p steps into the future. In some cases the control horizon and the prediction horizon could be different, in that case the prediction horizon should be similar to the settling time of the system and the control horizon the rise

time of the system.

MPC in this thesis refers to the academic version based on the state space formulation of the control model. A linear discrete state space model is formulated as shown in Equation 4.1, where A is the model for the states and B is how the inputs affect the states. Using the state space formulation allows for mathematical analysis with linear algebra of the properties of the problem, including the possibility to utilize the vast availability of solvers for solving sets of linear equations.

$$x(i+1) = Ax(i) + Bu(i), \quad x(0) = x_0 \quad (4.1)$$

For the implementations within this thesis the solver ForcesPro [11] was used. The optimization problem is shown in Equation set 4.2, is on the form needed for an implementation in ForcesPro. First inputs are shifted to deviation form and the notation is made more compact, as shown in Equation 4.3. The shift to deviation in the control inputs allows for constraints on the rate of change of the control inputs, which is useful in some applications to avoid large oscillations in the inputs. Equations 4.4 and 4.5 defines the model within the controller, essentially being the same as Equation 4.1 but in the augmented form to fit with the z vector used in the solver. The matrices D_i and C_i can be different for different i as the dynamics of the system change for different operating points. The solver requires z_1 to be known as an initial condition.

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^N \frac{1}{2} z_i^T H_i z_i + f^T z_i \\ \text{subject to} \quad & D_1 z_1 = c_1 \\ & C_{i-1} z_{i-1} + D_i z_i = c_i \\ & z_{i,min} \leq z_i \leq z_{i,max} \end{aligned} \quad (4.2)$$

$$z_i = \begin{bmatrix} \Delta \mathbf{u}_i \\ \mathbf{x}_i \\ \mathbf{u}_i \end{bmatrix} \quad (4.3)$$

$$C_{i-1} = \begin{bmatrix} \mathbf{0} & A & B \\ \mathbf{0} & \dots & I \end{bmatrix} \quad (4.4)$$

$$D_i = \begin{bmatrix} B & -I & 0 \\ I & 0 & -I \end{bmatrix} \quad (4.5)$$

The MPC formulation allows for constraints on states, inputs and rate of change of the inputs. The control objective is defined by matrix H_i and the vector f_i , where H have to be positive definite. The objective can be quadratic or linear. The type of objective is an important property in order for the problem, such that there exists one minimum, which is the global minimum. The complete set of requirements are given by the solver manual provided by Embotech for the ForcesPro solver [11].

Chapter 5

Method

In this chapter the methods used in developing this research are explained.

From a context point of view, the modeling is assumed to be the basis in this research, and from a top-down approach, the system looks as pictured in Figure 5.1. A plant consists of multiple sub-circuits; in turn, these circuits consist of multiple units. In every unit there are different physical events taking place when the unit is operated or used. The modeling approach is rooted in the belief that it is important to understand the physical function or event occurring therein. The modeling methodology in this work is to build up a model with as simple blocks as possible. The less complex a phenomenon can be described, the better, both for model understanding, simulation speed, and complexity. Ljung classifies mathematical models in two groups, physical models and identification models [26]. In this thesis most models are of the physical type, however in instances where available data fit well with a certain model approach, identification models are utilized. This is along the lines of finding the least complex approach to describe something. This idea trails throughout Paper A, B, C and D. This is especially important when it comes to control modeling, where certain methods limit the choice of models.

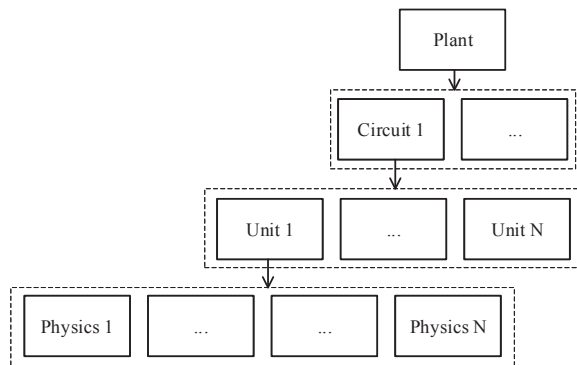


Figure 5.1: The different system levels in modeling of minerals processing plants.

5.1 Physical modeling

In Paper A and Paper B two new models were developed. The jaw crusher model in Paper A was first aimed to be a steady state model. However, it can be adapted to be dynamic. On the other hand in Paper B, the HPGR model is time dynamic. The main distinction between these two machines is the working principles of the crushers. A jaw crusher is a stiff form conditioned crushing machine, while the HPGR is force conditioned due to the design with gas accumulators. Evertsson and Lee [12, 22] have described the difference between form conditioned and energy conditioned crushing. The stiff form conditioned crushing that takes place in the jaw crusher will always exert the rock to a specific compression, while the force conditioned crushing will exert the particle to a force rather than a compression ratio. If the rock is too hard in the jaw crusher, the crusher will stop. In an HPGR the floating roller will back off, and the gap will open. In both cases the particles will be exposed to a compression at the end, wherein the HPGR this compression needs to be solved for. The two different model structures for both the jaw crusher and the HPGR are shown in 5.2. The main difference between these two models is that in order

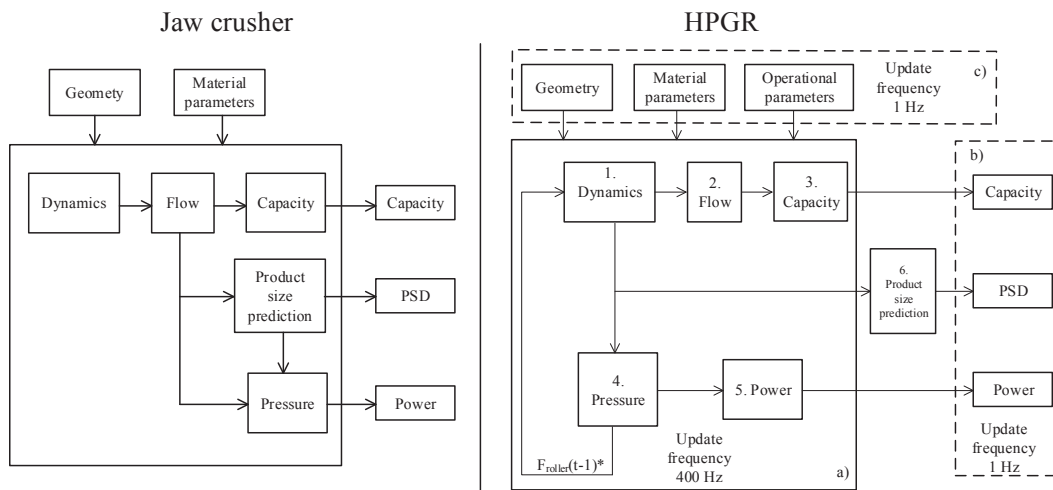


Figure 5.2: Side by side comparison of the Jaw crusher and HPGR crusher model structures.

to resolve the unknown force for the HPGR, the predicted force from the previous iteration is fed into the next iteration of the model. The predicted force is then used to solve the equation for the force balance of the roller. The force balance is solved within the dynamics block of the model. The dynamics block keeps track of where the floating roller is positioned and its movement in the horizontal direction. The equation of motion for the roller in one dimension can be derived from Figure 5.3-b). The resulting equation is shown in Equation 5.1. By solving the differential Equation 5.1 in time and for x , which is the relative position of the floating roller to the fixed roller, the compression of the material can be solved for. This was demonstrated in Paper B.

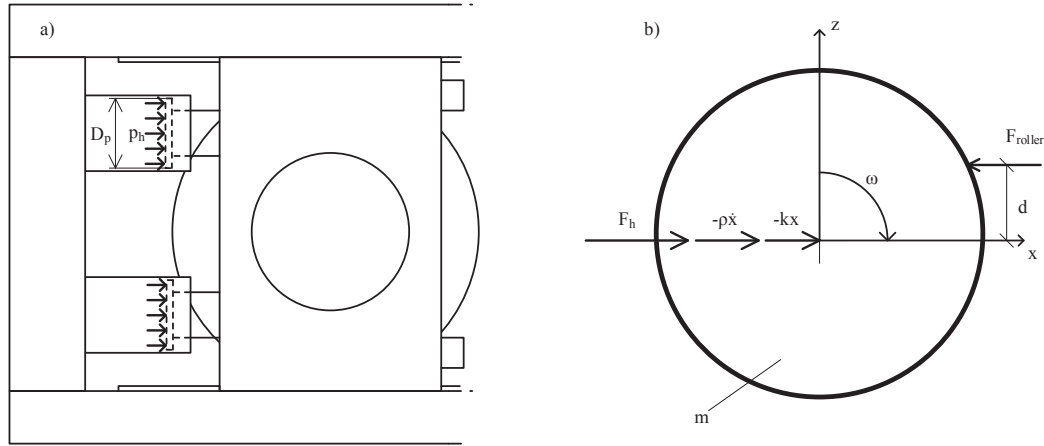


Figure 5.3: a): A Standard frame HPGR with the hydraulic cylinder illustrated, two cylinders on each side of diameter D_p and the pressure p_h . In b) the free body diagram of the floating roller is shown.

$$m\ddot{x} = \sum F = F_h + (-\rho\dot{x}) + (-kx) - F_{roller} \quad (5.1)$$

The difference between the forces F_h and F_{roller} is the force that gives rise to movement of the roller. The stiffness (k) and damping (ρ) terms are present to stabilize the model. This was needed since the available data resolution for the validation data was too low and created large step changes. The model can be executed with process data, in this case the hydraulic pressure (F_h) and roller speed (ω), or it can be operated as a stand alone model.

In both Paper A and B the power draw is calculated by splitting up the crushing zone in discrete sections. For the jaw crusher, the material sees repeated compressions, while in the HPGR it is one continuous compression. In Equations 5.2 and 5.3 the calculation of the nominal power draw is stated for both crushers.

$$P_{JAW} = f \sum_{i=1}^n \frac{P_i Area_i S_i}{2} \quad (5.2)$$

$$P_{HPGR} = \omega T = \omega \sum_{j=1}^{25} \sum_{i=1}^{n_{zones}} c_{scale,j} \frac{F_{comp,i,j}}{\cos(\alpha_i)} R_{roller} \mathbf{sign}(\alpha_i) \mu \quad (5.3)$$

The power draw predictions share an idea of that the forces associated with the crushing should correlate to the power draw of the machine. The implication of this is that the power draw predictions are nominal, and addition for no-load and losses needs to be included to get the actual draw. For the jaw crusher in Equation 5.2 f is the frequency of the shaft [Hz], n the number of crushing zones, P_i the maximum pressure from the rock in each crushing zone [Pa], S_i the compression distance in each zone [m]. Assuming the force is linear with the compression distance, the work

for each crushing zone is calculated with Equation 5.4, and the total work is the sum of all crushing zones. The power draw can be calculated by multiplication of the work per cycle and the number of cycles per unit time (f).

$$\frac{P_i Area_i S_i}{2} \quad (5.4)$$

For the HPGR in Equation 5.3, the required power draw to rotate the rollers is the total torque on the roller around the axis of rotation times the rotational speed of the roller (ω). The total torque can be calculated by taking the tangential force component times the radius of the roller (R_{roller}). The tangential force can be calculated by taking the normal force onto the roller divided by the cosine of the angle from the normal plane. The factor $c_{scale,j}$ is there to deal with if the crusher has cheek plates or not. The cosine function is positive for both negative and positive angles; therefore, the sign function is used to model the extrusion effect of the particle bed. The bed that has passed the horizontal will be pretensioned and partly elastically relax, this effect creates a torque on the roller with opposite sign from the one needed to compress the bed. By comparing process data and the predictions from the model, the μ which is utilized can be calculated.

Important in comminution is the prediction of particle size distributions (PSD), both models in Paper A and B utilize the framework developed by Evertsson [12]. The framework is illustrated in Figure 5.4.

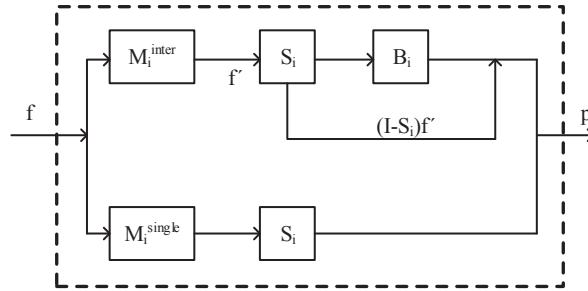


Figure 5.4: Schematic block diagram for how a feed size distribution f , is transformed to a product p

In order for the PSD framework to be compatible with the model for the jaw crusher and the HPGR, the machine function needed to be translated into discrete compressions and s/b for each crushing event. In paper A the material sees multiple compressions as it passes through the chamber, making it more similar to the cone crusher, while in Paper B the compression is one single continuous compression. The particle size prediction is done according to Equation 5.5. The breakage parameters can be retrieved by doing compression tests with a piston and die as described by Evertsson and Lee [12, 13, 22]. The construction of the matrices, M^{inter} , M^{single} ,

S , B^{inter} and B^{single} have been described by Evertsson, Lee and others [12, 13, 14, 22].

$$p = (M^{inter}[SB^{inter} + (I - S)] + M^{single}B^{single})f \quad (5.5)$$

In Equation 5.5, f is the feed vector, in the frequency of the different size fractions and p the product after a pass through the crusher, also in the frequency of the different size fractions. The breakage matrix B and selection matrix S are updated depending on the current compression ratio the feed is exposed to. M^{inter} and M^{single} are the mode matrices, which decides if the particles are exposed to inter-particle breakage or single particle breakage. This decision depends on if the particles in the feed are larger than the gap. All those sizes which are larger than the gap are exposed to single particle breakage and the rest to inter-particle breakage.

5.2 Process modeling

High fidelity process models can be used for most tasks regarding process and control modeling, however, for a first analysis and more complicated setups, simpler models could be useful. The high fidelity models in Paper A and B are between 10-100 times faster than real-time. If there is a need to run hundreds of simulations of different cases a high fidelity circuit model might be unfeasible. In that case the types of models used in Paper D could be useful. In Paper D it is of interest to find out how robust certain combinations of circuits are with installed control and variations present. In a comminution process or any industrial process that handles flow, it is most important to keep track of the mass, if there is no mass there is nothing to simulate. In Paper D a comminution circuit is stripped down to only simulate the mass flow of material. The investigated circuit is shown in Figure 5.5, where a) is the more traditionally drawn flow sheet, and b) is the simulated model. In part b) of Figure 5.5 the model setup can be studied. The circuit is made up of a combination of two connected closed crushing circuits, typically operating the cone crushers in choke fed conditions at a constant CSS. In order to build the simplified model, the approach is to look at the elements in the circuit and their behavior, to determine what describes them the best. In this specific case, the crushers are modeled as throttle valves, bins as tanks, and the screens as splitters. Conveyors are modeled as pure time delays. The control logic is implemented, including PI-control of the flow between the sub-circuits as well as interlocks and startup delays when interlocks are activated. The full setup and all parameters can be found in Paper D. The model in b) can be simulated at 1000-10 000 times faster than real time. In Paper D, the capacity of the crusher, the bin sizes, and the inflow were varied in different ways. The performance of the complete circuit was measured by studying the available inflow relative to what made it through during an eight hours simulation. The amplitude of the variations were changed to different levels to see the effect on the circuit performance.

The motivation for doing the analysis in Paper D was to approach the modeling and simulation problem from the opposite side compared to the commonly used

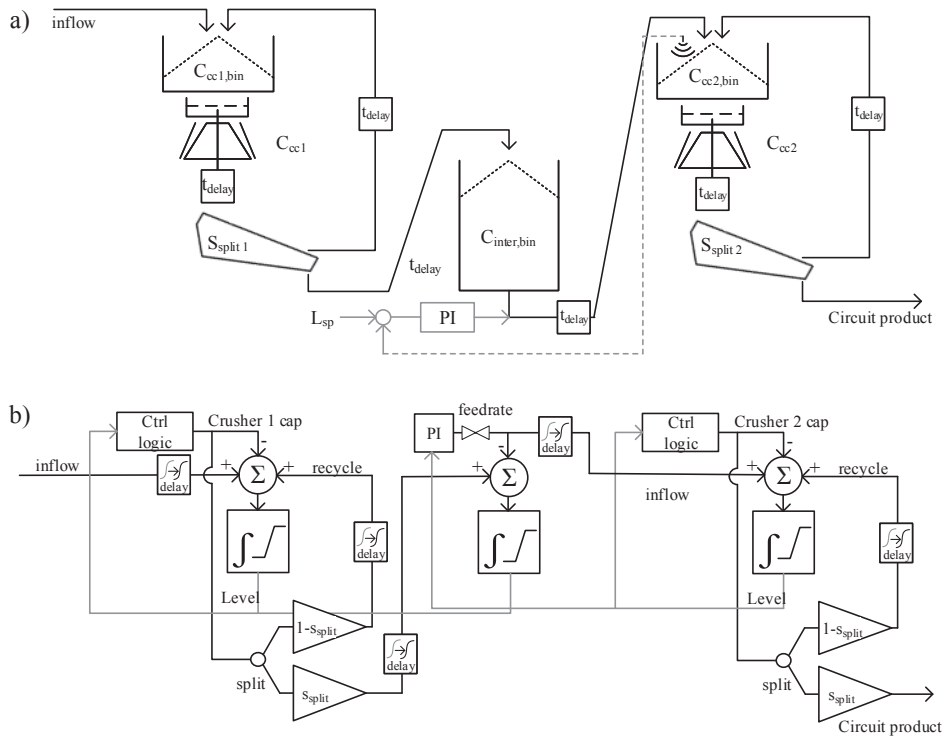


Figure 5.5: a) Flowsheet of the originally modeled plant, and b) the flowsheet implemented for the simulation of the plant

approach of very high fidelity models based on survey data and lab tests. In the work presented in Paper D, the focus was rather on the variations introduced than the model accuracy. The approach can be used to see the effect of varying bin-sizes, ill-designed circuits and to what degree a design can handle material or incoming variations.

In Paper E, a high fidelity circuit model was developed, assembled and used for simulations. The general method for developing simulation models of existing plants consists of the following steps:

1. Find documentations and draw a flowsheet of the plant that should be modeled
2. Find or develop all unit models included in the flowsheet
3. Test and tune all unit models separately
4. Assemble all unit models into the circuit, testing can here be done in manual mode
5. Apply existing control functionality, such as PI-loops and interlocks
6. Run the circuit model with inputs for the physical plant and record the simulation model outputs
7. Compare the output from the simulation model with the plant data for the same measurement points.

The list above consists of the method used in Paper E but can be applied to other circuits or problems as well. The list appears linear, but in practice it is an iterative exercise. Developing models that should correspond well both in phase and amplitude over time is difficult. The modeling of a complete plant is a process that can take many months or years. This framework can be utilized to develop models for digital twin applications.

5.3 Control modeling

In Paper C an application of MPC was presented. The circuit used for the study is drawn in Figure 5.6. It is a tertiary crushing circuit that produces a -10mm ball mill feed. The controller was used to control the circuit model developed by Johansson [21] and summarized in Paper E. The simulations ran in the computer, controlling a copy of a physical plant, which was realized as a time dynamic model. The method to set up the controller will touch on the controller itself and how to interface it with the simulation model. The simulation model is assumed to be available as it was in the case with Paper C.

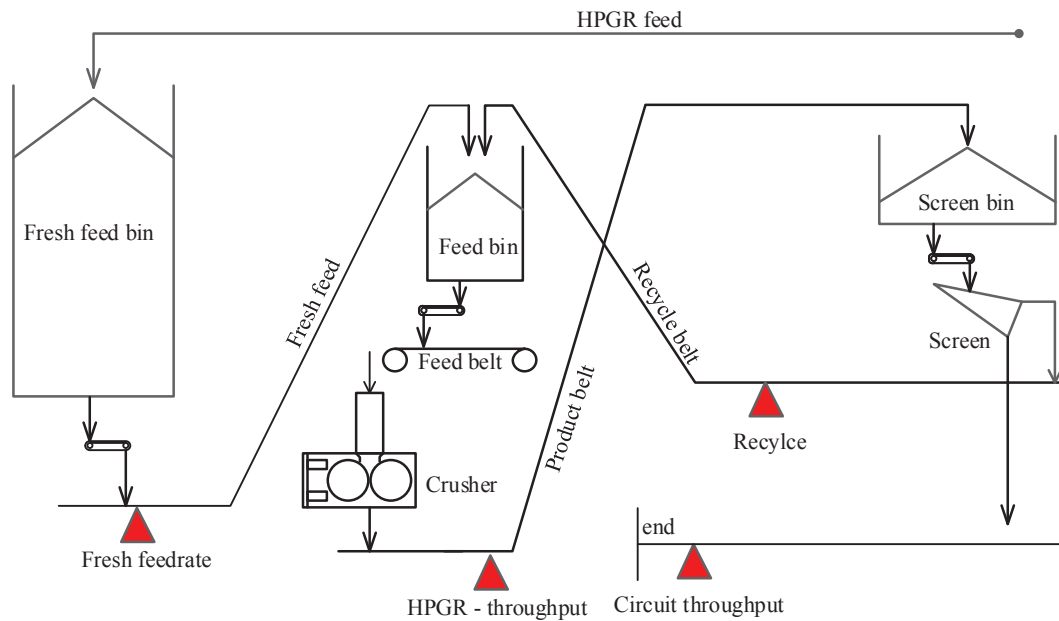


Figure 5.6: The flowsheet that the controller will be applied to,

In order to set up an MPC for the circuit, a similar understanding to that needed for Paper D is needed. From Figure 5.6, the following needs to be identified:

- Controlled variables
- Manipulated variables
- Elements to be considered in the controller model
- Elements that are not considered in the controller model

Visually this work is to move from the flowsheet in Figure 5.6 to the flowsheet in Figure 5.8. As described in Paper C, the conveyors are translated to delays, the bins to integrators, and the screens to splitters. The controller model only considers the flow of material, however it can be extended to cover size and other properties if needed. In Figure 5.7 the ideas behind the models in the controller are described. For conveyor in a) the material enters from the left, it moves one slot per unit time. The material is a packet of a certain mass per unit time. The conveyor needs to run at constant speed for this approach to work, and the length of the conveyor divided by the belt speed determines the delay. The delay in time can then be translated to how many states that need to be allocated for the conveyor. For example, as with the circuit in Paper C and E the sampling time of the controller is 10 seconds. To represent a 180 seconds long conveyor it is required to allocate 18 states in the controller model. In Figure 5.7 b) for the screen the material flow is split by a constant factor, in other words multiplied by a constant, corresponding to approximately what portion goes to the oversize and undersize respectively. Finally, in Figure 5.7 c), the bins are modeled as a tank or a pure integrator, summing the difference between inflow and outflow. All these types of models can be implemented in a state space system. Furthermore, a sampling time of the controller needs to be chosen, in Paper C this was 10 seconds, implying the controller would solve the optimal control problem once every 10 seconds. Also, the length of the prediction and control horizon needs to be determined, in this case 70 steps were used. The prediction and control horizon were equal for this implementation.

For visual purposes the states are marked in the flowsheet in Figure 5.8 and indicated where they are located. The manipulated variables are marked as u_1 , u_2 and u_3 . The manipulated variables were circuit inflow, HPGR capacity, and screen feed rate. Measurements of the flow of material at the actual plant are indicated by the red triangles, which are belt scales.

The main reason for developing a controller is to control some objective, and this objective is defined in the matrix H and the vector f . In Figure 5.9 on the left are the non-zero entries in the H matrix and f vector shown. Based on the control formulation in Section 4.3, in this case, the first three entries are penalization of movement of the control setting, and the bottom six entries are to define the objective, which is to keep the two intermediate bins, the HPGR-feed, and the screening bin to 50% level and finally to set a target for the output of the circuit. The values in the H matrix needs to be set in such a way that control prioritizes

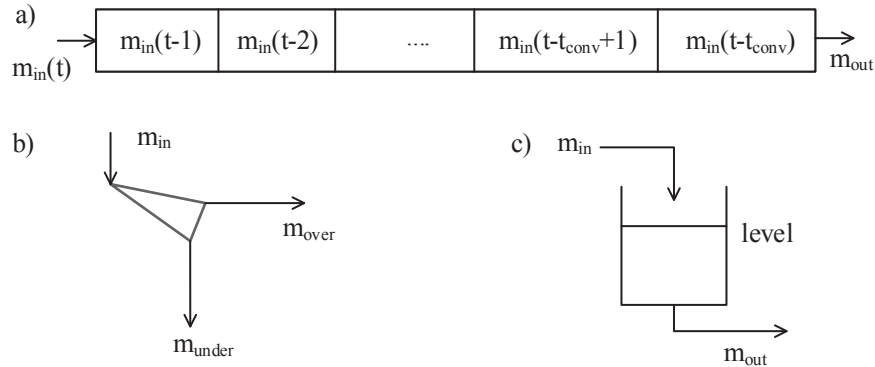


Figure 5.7: An illustration of the different unit models included in the controller model.

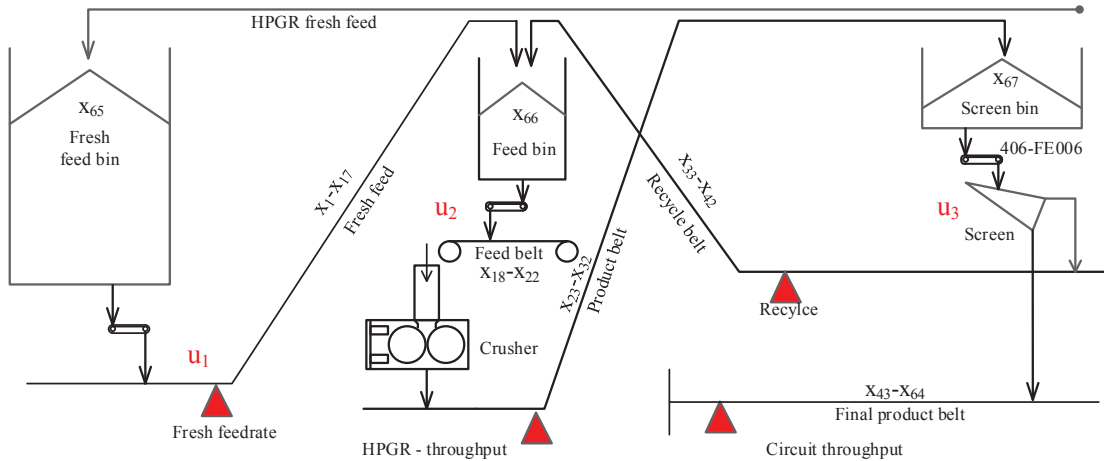


Figure 5.8: The flowsheet of circuit with introduced states and manipulated variables for the controller model

the most important of the targets first. Finding a balance between the numbers is typically done by trial and error methods, it is therefore important to have a model for trials and evaluation before deploying a controller in the field. This difficulty is acknowledged by Rawling and Mayne [36].

From control theory, for functional controllability, it is required to have equally many manipulated variables as controlled variables [42]. In Paper C the circuit has three available manipulated variables, making it possible to control three outputs, which in this case is adequate. This is the reasoning behind how the controller is set up to achieve the intended function.

The approach used in this work to connect the controller to the circuit includes a concept called set point slaving. The MPC will not directly write to the actuators in the model, rather update the set point for the controller. How the application in Paper C was set is explained by Johansson [21]. This approach is more stable if the MPC fails to find a solution to the optimal control problem, as the PI-loop continues

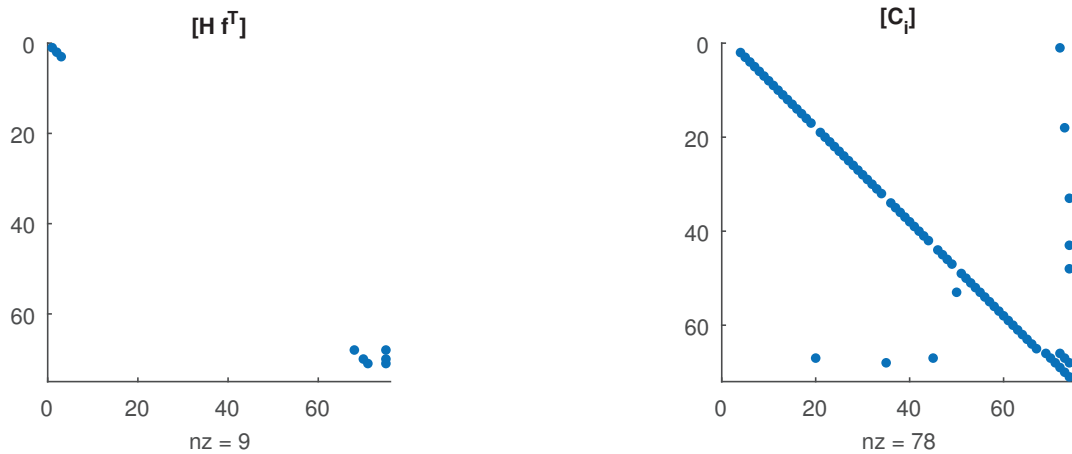


Figure 5.9: Visualization of the non-zero elements in the matrices determining the objective and controller model for the MPC in Paper C.

to run with the previous setpoint. To integrate the controller with the simulation model the following needs to be considered:

- Units of set points passed between the controller and simulation model because of different sampling times.
- Initialization of the controller model.
- Passing updated values to the controller model when things change, for example, the screen split.
- Making sure the constraints on the MPC and the PI-loops limits are the same.
- Allowing the sampling time of the MPC to be slower than the simulation model.

Additionally, not considered for the work presented in Paper C or E, in order to work correctly on an actual plant start-up and shut down routines are needed, where typically the MPC is disengaged, and a sequence of events are executed to safe start or stop the plant.

Chapter 6

Results

In this chapter, the results of the research are presented in summary form. The results are split up in the subcategories, modeling, process control, and circuit modeling

6.1 Equipment modeling

In Paper A and Paper B, new models for crushing equipment were presented, one intended for static simulation and one time dynamic. The model development shares the same structure and fundamental ideas. The outcome in both Paper A and B are two models that can predict machine performance for each machine respectively.

In Paper A multiple scenarios of different settings were simulated with the developed crusher model. As presented in Paper A, the model can handle different settings of CSS, eccentric throw, and eccentric speed. In Figure 6.1 the resulting internal variables from crushing a material are shown, this includes, a) the compression ratio for each compression, b) the position of a material package over time, c) the progression of the particle size distribution as the package of material makes its way through the chamber. Finally d) is the calculation of the bulk density in the crushing chamber. From the graph it can be concluded that for a specific setting, the material is compressed a certain number of times, how long time a material package spends in the chamber, and how tightly loaded the crusher is. If the bulk density goes above the inherent material density [$2600 - 2700 \text{ kg/m}^3$] then packing will occur, and the crusher will stop or stall.

The jaw crusher model predicts power draw, capacity and particle size. Validation data were available for the capacity and the particle size for the modeled crusher. However, the power draw has not been validated. In Figure 6.2 a) the power draw is shown for different CSS settings and flywheel speeds, in Figure 6.2 b) the capacity prediction of the model is shown and compared against data from a manufacturer for the specific crusher size, and in Figure 6.3 the comparison of the particle size is shown for different CSS settings. The model response has not been tuned to the validation data, instead compared to and should not be viewed as a result of

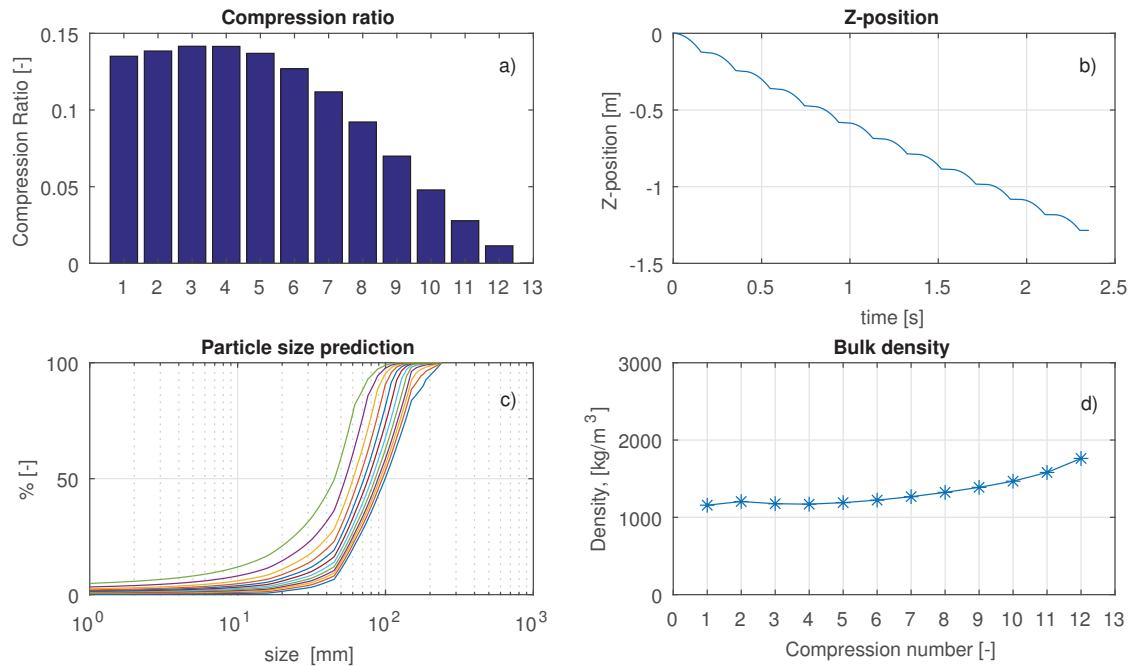


Figure 6.1: Model prediction for the behavior of the jaw crusher set at a 50mm CSS and operated at 300 rpm flywheel speed.

a tuned model. When evaluating a model response it is equally important to look at the response for a relative change as well as the actual value. For example, the PSD prediction in Figure 6.3 the response in the range 10 – 60 mm is off from the validation data, however comparing the results for $CSS = 100$ mm and $CSS = 80$ mm the relative distance between the prediction and the validation data is close. This supports that the model replicates the phenomena that are being modeled, but it needs further tuning for higher accuracy.

In Figure 6.2 (a), the prediction of the power draw is shown, and it peaks at around 300 rpm for all CSS settings, and for higher speeds it decreases. This is the nominal power to crush the rock only, and with increasing speed it is expected to find an increased power draw due to the increased mechanical losses in drives, motors, and transmissions as well as the eccentric bush. In Figure 6.2 (b) the capacity is predicted to increase with increasing CSS, which is expected. The lower values from the manufacturer have a similar slope to the predicted capacity curve, however it is far off in the lower end and within the high and low manufacturer capacity values at the large end. This is an area of improvement for the model in future versions. In Figure 6.3 the particle size prediction for different CSS is plotted and compared against the manufacturer data. The correspondence is relatively good at the large end however is slightly off in the finer end. In Paper A it was unknown what feed were used for the data that came from the producer, the fines of the feed will have an influence on the prediction results. Apart from the difficulty in sampling the feed to a primary crusher it would be a natural step to explore in the development of version two of the model.

For the dynamic HPGR model presented in Paper B, and in Figures 6.4, 6.5 and

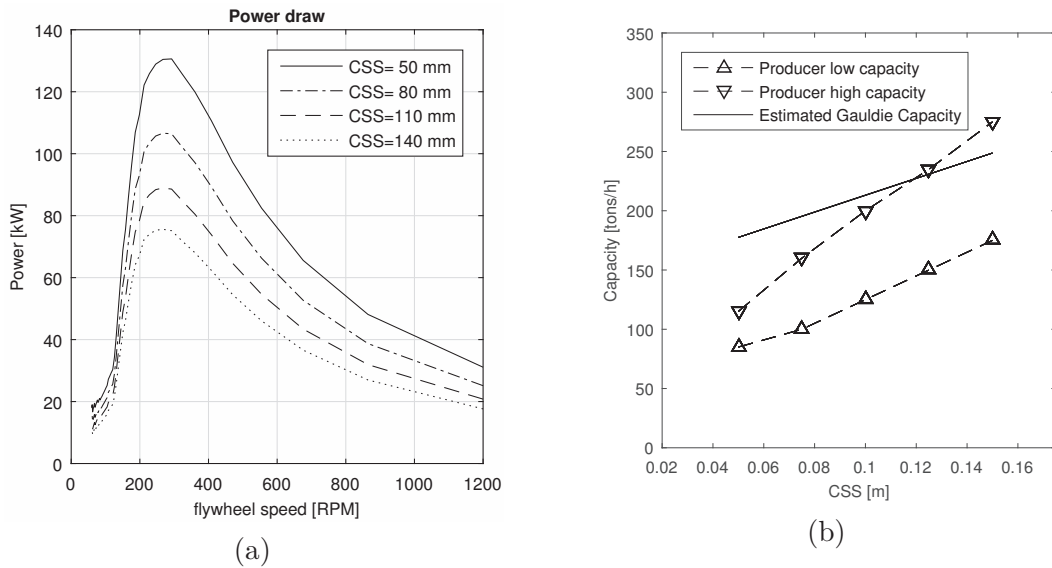


Figure 6.2: (a) Simulated nominal power draw for different CSS and different flywheel speeds, the power draw does not include any mechanical losses. (b) The simulated capacity of a jaw crusher compared to manufacturer data for a crusher of equal size.

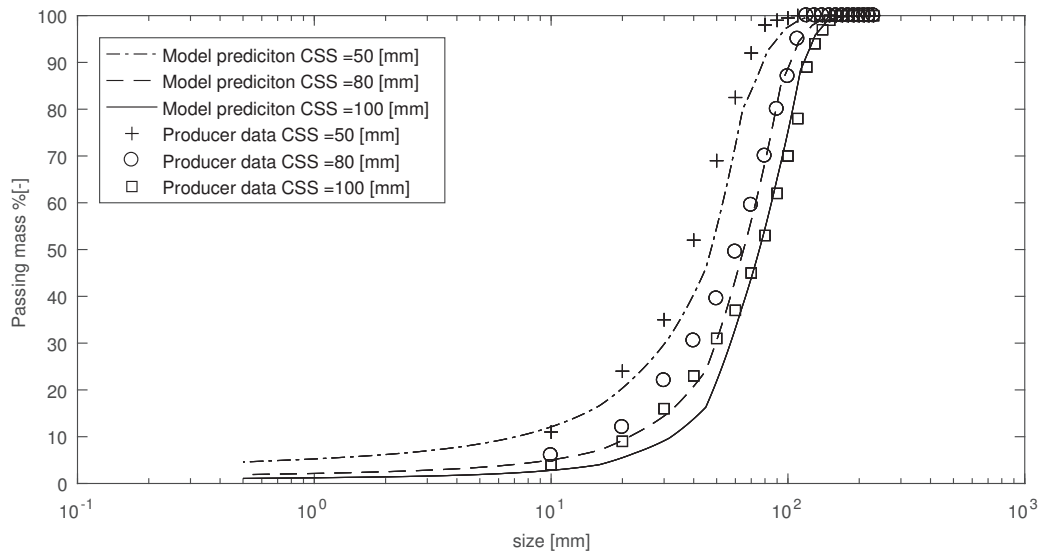


Figure 6.3: Simulated particle size predictions for three different CSS settings.

6.6 are outputs from the model. The model responds to changes in particle size, roller speed and hydraulic pressure as the main inputs that can be changed. The model can also be changed in terms of machine size, and the simulated machine parameters are presented in Paper B.

In Figure 6.4 the power and throughput have been simulated for different pressure settings and roller speeds. The model can even if it is a dynamic model be simulated to a steady value as there are no variations present in feed or control in Figure 6.4. The mapping is a good way to see how the model behaves over a bigger span of

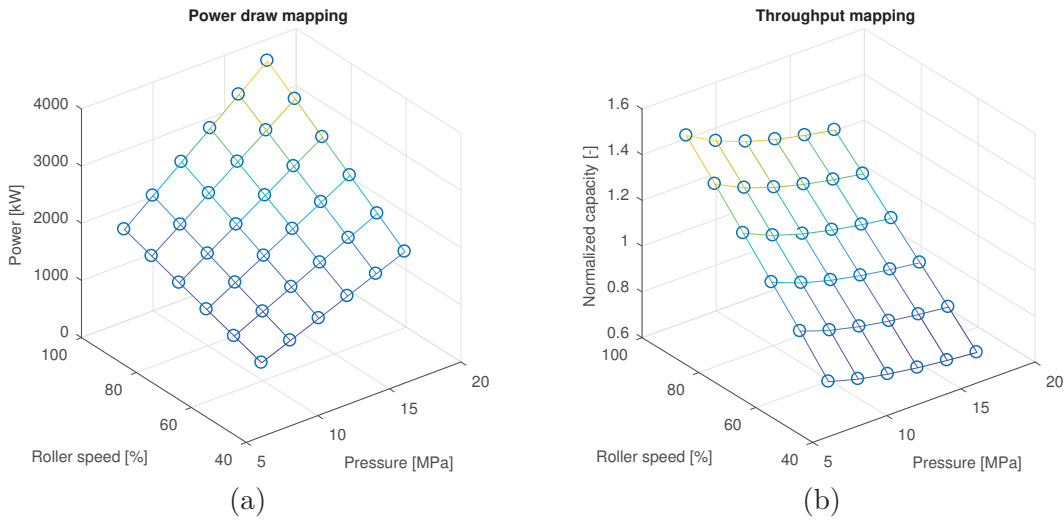


Figure 6.4: Mapping of the performance for the HPGR model. In (a) the power draw is plotted for different pressures and roller speeds. In (b) the throughput is plotted for different pressures and roller speeds.

operating conditions. The results in Figure 6.4 are inline with the simulations showed by Numbi [30].

In Paper B, two data sets were used to validate the model, and again these data sets were not used to tune to, just comparing the model prediction to what was seen in the plant. Validation set one is presented in Figure 6.5 and 6.6. In Figure 6.5 the time dynamic response of the model is plotted against the recorded data for the physical plant operating under the same conditions. The recorded performance numbers were capacity or throughput, nominal power draw, and the gap prediction. The bottom plot in Figure 6.5 are the inputs, roller speed, and hydraulic pressure, that were fed to the model. First for the capacity prediction and the roller speed, there seems to be some sampling or logging error in the signal from the plant, the oscillations in the input may be due to a poorly tuned controller. However, the number of data point is sparse which is unfortunate. For the prediction of the capacity the model predicts the capacity over time to an accuracy of about 5% (normalized root mean squared error) over 2800 seconds. The predicted power draw follows the measurement, and most trends in the measurement can be seen in the prediction, which is a sign that the model picks up the effects which have the greatest impact on the power draw. For the gap prediction the results are acceptable, but in the beginning at time about $t = 350[s]$ there seems to be missing logic in the model that makes it behave differently from the physical plant. This should be worked on in an updated version of the model.

For the validation dataset on particle size distribution, only one sample was taken on the product belt after the HPGR. It was unclear when during the recorded data. Therefore the decision was in Figure 6.6 to look at the entire period the model predicted PSD and plot the finest and the coarsest distributions. In Figure 6.6 it can be seen that the prediction of the particle size is capturing parts of the distribution, top size and in the fine end. The model was simulated with the same feed as the one

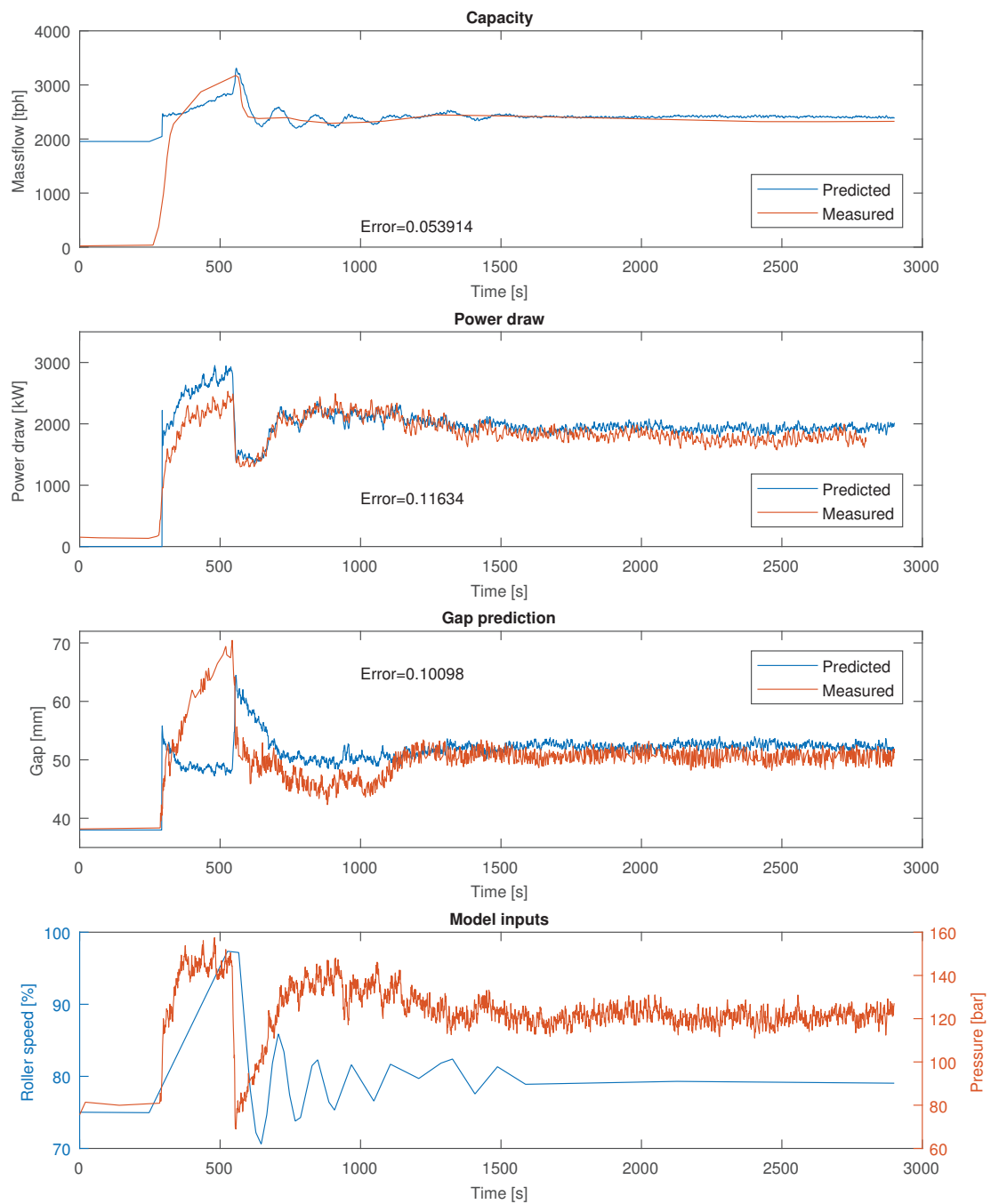


Figure 6.5: Model output predictions for the HPGR model for the survey conditions.

the measured one from the survey. The shape of the distribution from the model is slightly more curved than the test-data. If this discrepancy were further investigated in future studies, it would be recommended to utilize more test data and laboratory test with for the same conditions and try to understand what is happening.

The model presented in Paper B for the HPGR is a high fidelity model capable of predictions for a range of operating conditions. It can also handle different ore

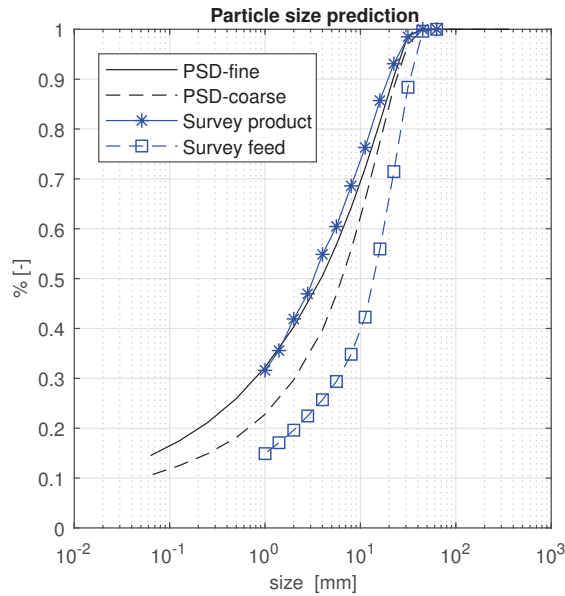


Figure 6.6: Comparison between simulated PSD predictions and the recorded PSD for the survey data used in the study.

hardness and how this affects the performance as it was demonstrated in Paper B. The model shows good potential to be used for the optimization of circuit performance and further strengthen the results in Paper E. The idea with the model is that it should correspond to changes in the operating parameters such that it can be used for control and operations investigations.

6.2 Process control

In this section, the results from Paper C are presented, as this paper is mainly focused on process control. In Figure 6.7 and 6.8 two different objectives of the controller were simulated for an eight hour period. The objective was in both cases to keep the bin levels to 50%, and for Figure 6.7, the controller was free to produce as much as possible on the final product belt. For the case in Figure 6.8 the controller should instead keep the product belt tonnage to a specific rate. As the control problem is functionally controllable, the controller was able to settle into these set points for both the cases with a set point on the circuit production. For the maximized production the controller reaches the constraints on the feeders in the circuit model.

In Figure 6.7, the objective is set to maximize the outflow from the circuit, and the controller will, therefore, drive the HPGR throughput to the limit of the feeders. The green and the red lines in the top graph in Figure 6.7 are the throughputs of the HPGR, and the screening throughput since all the material that comes from the HPGR has to be screened these two should end up at the same level when a steady-state is reached. The inflow and the outflow also have to match once a steady state is reached. This can be seen as the black and the blue lines settle into the same level. The two bin levels plotted in the lower graph of Figure 6.7 also settle into

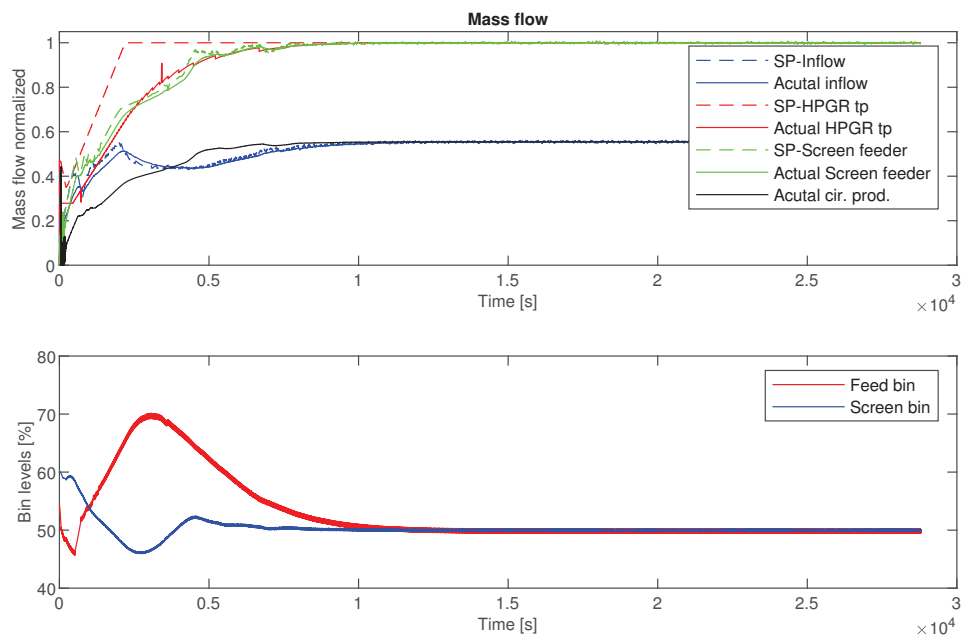


Figure 6.7: Simulation results for a controller which maximizes the output of the circuit (Actual cir. production)

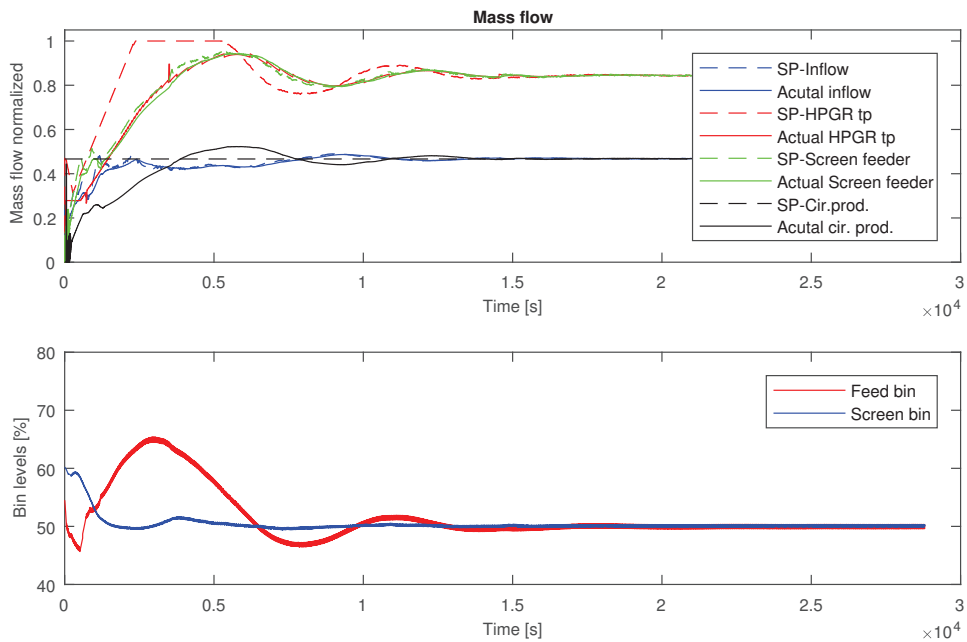


Figure 6.8: Simulation results for a controller with three objectives, bin level control and production rate control.

their set point at 50% after a few hours, the settling time of these levels depends heavily on the prioritizing between the objectives in the controller. When it comes to comminution circuit designs, bins and silos are there as buffers, and one of their functions is to absorb the fluctuation within the circuit. As long as they are kept within the safe operating conditions, neither risking that they will run empty or overfilling, they should be allowed to vary.

For the second case, plotted in Figure 6.8, a setpoint is active on the circuit output, which is shown in the top graph of Figure 6.8 as the dashed black line. In this case the red and the green lines will settle into a slightly lower level as the set point for the production of the circuit is set lower than the maximum one. For the bin levels in the bottom graph, the feed bin in red has a slightly smaller overshoot than in Figure 6.7. When comparing the two simulated cases the second case is preferred, as when the maximization reaches the constraints of other parts in the circuit it will be limited in its range of how it can move, as it will only be able to slow down. It is much more stable to set an actual set point that should be held by the controller, however for debottlenecking and plant optimization in the simulation environment it is a practical approach as there is minimal risk associated with running the simulation.

Another aspect that is present in both Figure 6.7 and 6.8 is when the controller is set up in a setpoint slaving fashion as described in Section 5.3, the MPC will calculate set points and send to the local SISO loops. If the control loops are very slow, which is the case with the HPGR-set point, it can be observed that the solid red line is below the red dashed line. The dashed line is the value that is being calculated by the MPC, and the solid line is the output of the SISO-loop. If this distance or lag between these two is too large then the controller can become ineffective and possibly unstable. The SISO-loops have not been retuned. However if the circuit was controlled with an MPC on a daily basis the loop speed (controller gains) could probably be increased, and this effect minimized.

6.3 Circuit modeling

From Paper D, the robustness properties, and the controllability were explored for the circuit developed within this research, which was presented in Section 5.2. Firstly in Figure 6.9 and 6.10, the time response is shown for when there are no introduced variations and for 5% periodic variation for the crusher capacity and split ratio over the screen. In Figure 6.9 there are a few on/off of the interlocks and a little oscillation, in the beginning, however thereafter the circuit moves to a steady state in all variables, this is due to the fact that the circuit capacities are balanced (production rate of sub-circuit 1 is equal to sub-circuit 2). When variations are introduced as in Figure 6.10, the response is not at steady state anymore and an erratic interference pattern appears, especially for the flows, as the interlocks turn these on and off. It can be concluded that a balanced circuit could be stable if there are no variations present, however as soon as there are variations, there is a chance

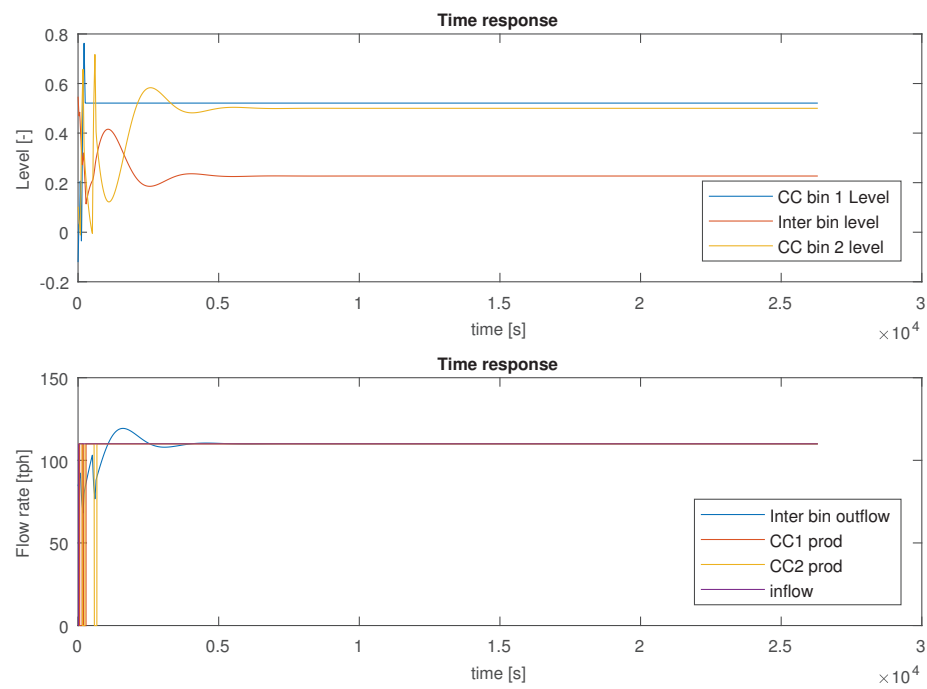


Figure 6.9: No introduced variations.

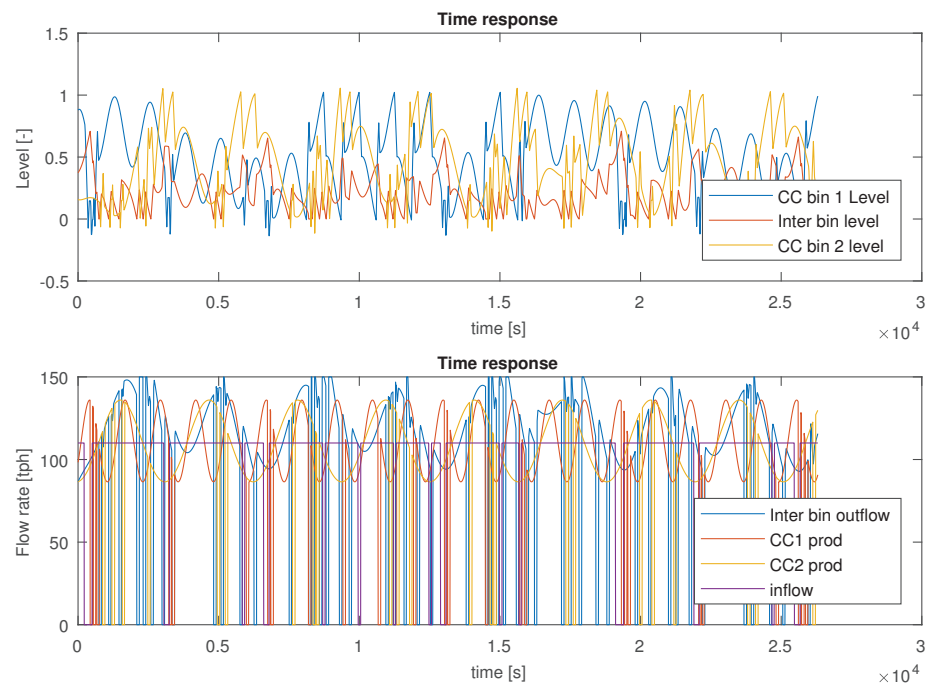


Figure 6.10: Crusher capacity and screening split varied with 5%

of ending up in an interference pattern as in Figure 6.10.

In Figure 6.11 the circuit in Paper D is simulated without any variations in two cases. For Figure 6.11 (a), the bin levels have been varied in size, and the three surfaces correspond to different initial filling of the bins. The lower the initial filling, the more sensitive it will be, however without any variation it is insensitive to the sizes of the bins when the circuit is balanced. For Figure 6.11 (b), the circuit is unbalanced in the sense that the capacity of the crusher in sub-circuit one is different from the capacity of the crusher in sub-circuit two. The conclusion from Figure 6.11 (b) is that as long as the downstream circuit is the bottleneck, the loss in production rate is minimal, however if there is under capacity in the upstream and overcapacity downstream the circuit can loose up to 30% of its performance. This theory needs to be extended and further formalized in future work as it will have implications on plant design and operation in the industry.

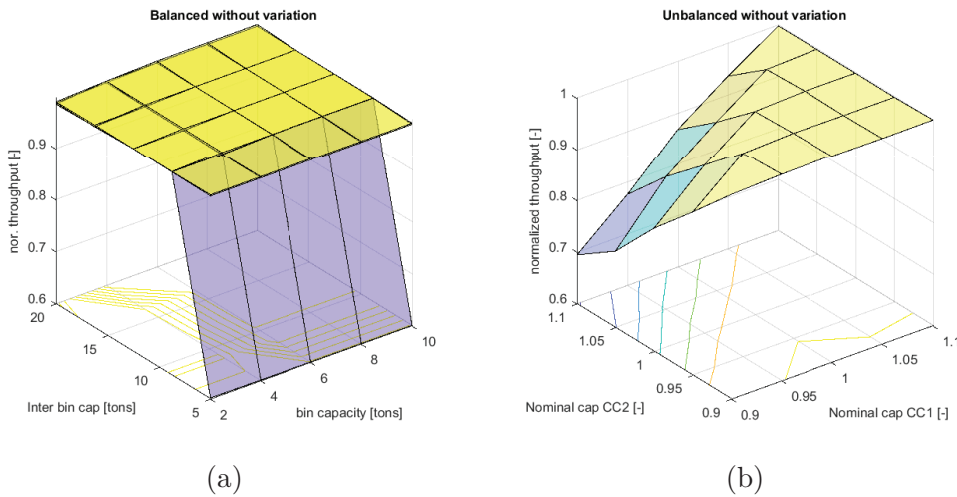


Figure 6.11: Simulation results from the circuit without variations, for (a) the bin sizes are varied and for (b) the balancing of the circuit is shifted to see the effect on the circuit performance.

If variations are introduced for the crusher capacity and screening split, the results in Figure 6.12 are obtained. In Figure 6.12 (a) and (b) are simulations of how the throughput is affected by the different bin sizes, The difference between (a) and (b) is the type of variations that is present, in (a) it is white noise and in (b) the variation is periodic. The purple, blue and yellow surfaces correspond to different initial bin levels. In Figure 6.12 (b), the performance starts to decrease at an intermediate bin capacity lower than 10 tons, which is visible by looking at the contour lines, however for Figure 6.12 (a) this happens at about 7 tons. The conclusion is that the periodic variation has a more severe effect on the performance. For the bin before the crushers in each sub-circuit the difference between (a) and (b) seems to be very small. These phenomena will need further investigation in order to make the conclusions more rigid and extend their validity.

Finally, in Paper D, the "push" strategy was evaluated, in other words trying to push in as much material as possible into the circuit. The results of this are shown in

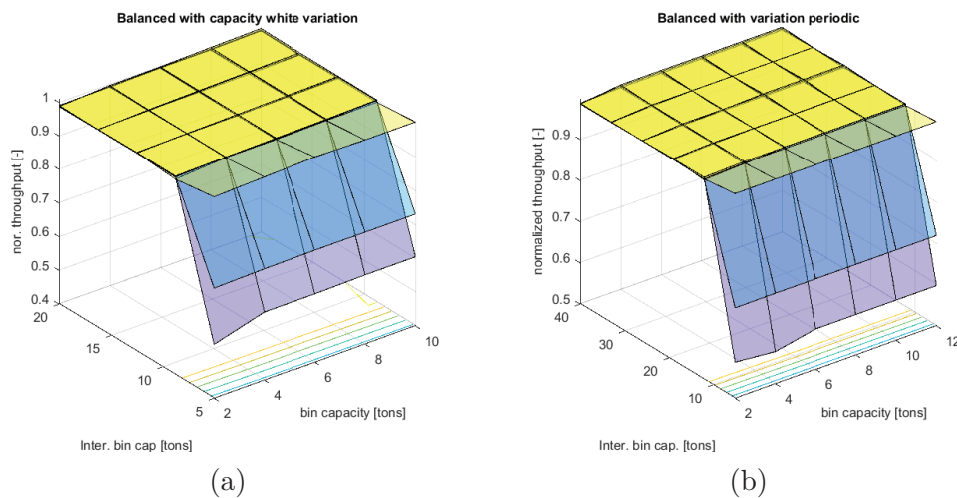


Figure 6.12: Simulation results for the throughput with different bin sizes when variations are present, in (a) the variations are white noise and in (b) the variations are periodic. In both cases each sub-circuit have a balanced capacity.

Figure 6.13. The available feed to the circuit from Paper D is varied between for 60% to 140%, and the circuit throughput is recorded for 8 hours. If the circuit is underfed it performs very poorly as it goes on and off, clearly showing the interlock strategy implemented is not sufficient. As the input is increased the performance gets better and better, and between 90-100% the input is the same as the performance. If the input is increased further, due to the way choke fed crushers work in a circuit and act as throttle valves, it does not help to increase the input as the crusher will throttle down the performance. With a better implemented control strategy, the lower input ranges can be handled sufficiently, as shown by Asbjörnsson [1]. However, it is not possible to push material through a cone crusher circuit; the only way to get more throughput is to increase the capacity of the crusher itself.

From Paper E and by Johansson [21], evaluations between model accuracy and plant data have been made for larger circuit models where they are intended to be used for simulations and predictions of circuit performance. The error between the plant data and the model prediction was calculated and aimed to be as low as possible. In Paper E it is shown that an entire circuit can be described with an error below 10%. In order to be able to trust the dynamic process models that are being developed this is a tool to show how well the model describes reality. In Figure 6.14 an example of a signal logged in a SCADA-system plotted together with a model prediction from a dynamic model running with the same inputs as the real plant had for the same time period. Indications of that the model is describing the actual process can be identified in the graph if the patterns of the lines are similar, not only looking at the value but also if the plant and the model are moving in the same direction. For the example in Figure 6.14, the prediction follows many of the trends seen in the process data, however not all of trend. Especially for the start and stop sequences the model deviates more extensively from the real process.

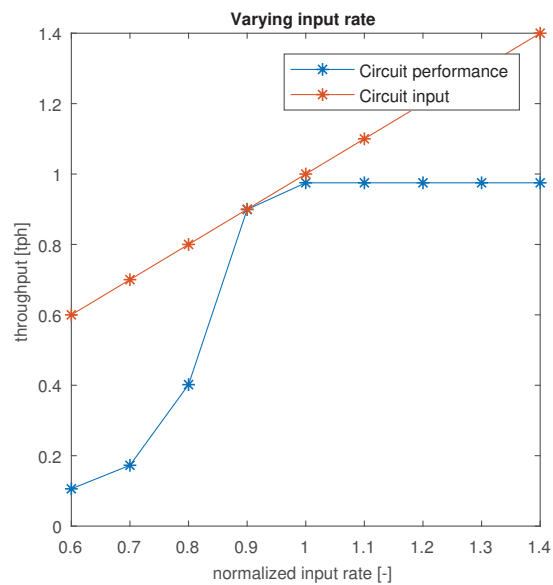


Figure 6.13: The relationship between the circuit input rate and the circuit performance for the investigated circuit from Paper D. Below 0.9 in input rate there is an interlock effect and above 1 the circuit is crusher limited.

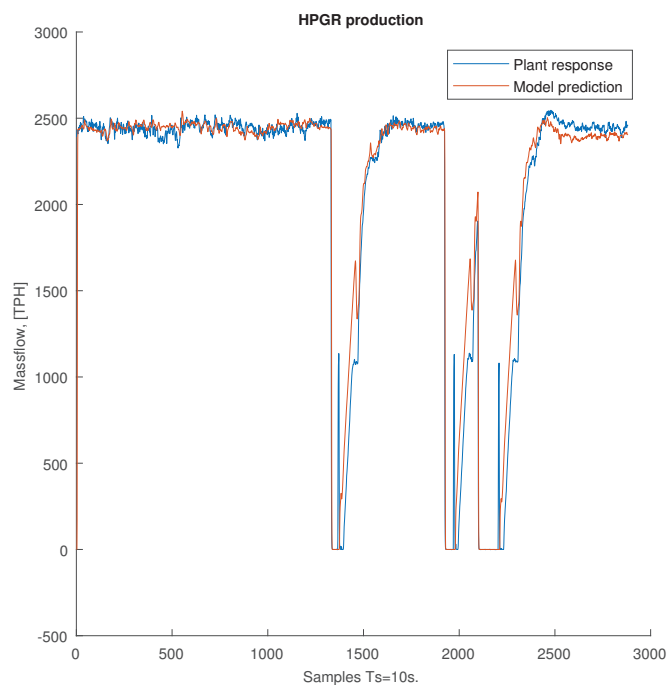


Figure 6.14: Model predictions plotted with actual plant data from Paper E as a way to quantify how well the model captures the plant behaviour.

Chapter 7

Discussion

In this chapter, the results and outcomes of the research are discussed along with the answers to the research questions. Finally directions for future work within this research are given

The research presented in this thesis circles around modeling and understanding of how to model and how to apply the models in different scenarios. The three main areas of modeling that are touched upon and where the front line of the research is pushed forward include:

- High fidelity models
- Fast process models
- control models

The models are all aimed to be time dynamic and have predictive capability. The minerals processing and aggregates industry have long relied on steady-state models for plant designs. When the margin on profit is getting smaller, minerals processing operators are looking for opportunities to improve their processes, where an integral part will be the comminution and classification plants. To fully understand and improve operations, tools such as time dynamic modeling will be needed. The way this type of research will move forward is by problem oriented challenges and by developing solutions for specific needs. Apart from the need for new models and implementation, there will also be a need for the skills needed to simulate and interpret the results from time dynamic models. The amount of generated data is far larger than from a steady-state simulation, which will require new skills to process the data and extract valuable information from it.

Physical modeling will always be an alternative to data-driven models in the future and especially in some cases. If the model structure needed to describe the unit is not possible to implement in the controller, one may need to resort to a linearized model with updated coefficients. Data-driven modeling will most likely profoundly impact the world we live in, as the quantities of data collected grow day by day. This will be the case in minerals processing as well, with most certainty. However, it

is essential to pay attention to assumptions made and not blindly trust all models and the data. All models come with inaccuracies and process knowledge will still be required in order for successful implementation and qualitative understanding.

The idea behind Paper D was to approach the operations problem from the opposite side. The method researchers usually apply in minerals processing includes utilizing a rigorous testing program of ore responses and sampling and then back fitting a static model to a single data set. The reasoning in Paper D was the right opposite, namely to use as little info and testing as possible, guess or estimate the variations and introduce them in a simulation model that decently describe the circuit or process but utilize the computer to simulate all possible variations to understand what the possible outcomes are. The method demonstrated will be developed further and is envisioned to have an impact especially for understanding in order to avoid designing ill behaving plants.

In Table 7.1 there is a list of current developed unit models with in this research and at different levels. As mentioned previously different models have different areas of applications and it is important to be able to distinguish between them and how they can be used.

Table 7.1: Summary of models on different application levels

Model	High fidelity	Fast	Control model
HPGR	x	x	x
Jaw crusher	x		
Conveyor		x	x
Screen		x	x
Bin/storage unit		x	x
Cone crusher		x	

Future work will aim to improve the model library to include more equipment models on all levels.

7.1 Answers to the research questions

RQ1: How can high fidelity fundamental models of minerals processing equipment be developed in order to handle machinery of force conditioned type?

Answer: A crushing machine such as the HPGR, where the hydraulic pressure dictates how much the material bed is compressed and ultimately reduced in size, it is of interest to know the compression ratio for a certain hydraulic pressure. In Paper B, the force from the hydraulic system is used to solve for the equation of motion for the roller over time. By calculating the position of the roller the force conditioned machine has been translated to a compressive device. The position of the roller can be used to calculate

the compression ratio the material sees, which is what previous particle size prediction methods by Evertsson [12] need to calculate particle size.

RQ2: How can fundamental models of minerals processing equipment be developed in order to handle machinery with fast dynamic behavior?

Answer: In both Paper A and B, the residence time in the crusher is short, and the way to handle this has been to locally within the model increase the sampling frequency and do multiple calculations per iteration. In Paper A, it is resolved to describe a material packet as it passes through the crusher. In Paper B, the equation for solving for the capacity and position of the roller is discretized at 400 Hz, so for each iteration of the model the position of the roller and the forces are solved for 400 times.

RQ3: How can a minerals processing plant's degree of robustness be studied?

Answer: Minerals processing plants process natural types of brittle material, such as rock, which comes with natural variations that affect the process. In Paper D, fast models and control logic were used to study a circuit's sensitivity to variations and how variations and materials storage units sizes affect the ability to cope with variations. The approach was to use a large simulation test plan, which was possible as the simulation models used were very fast.

RQ4: What consequences do robustness studies have on plant design?

Answer: One of the outcomes is that a plant design inherently should have robust properties, in other words not be sitting on an unstable equilibrium point. To be able to cope with the variations present adjustability is needed, for example be possible to balance the load between multiple sub-circuits. Another outcome was that too small storage units within the circuit can cause instability and interlocking effects. These conclusions should be further formalized and a more comprehensive list, similar but longer than the one in presented in Paper D.

RQ5: How can models based on fundamental principles be used to improve plant control?

Answer: Two areas of improvement have been demonstrated in this research, first in Paper C, the understanding of what primary response different unit operations have can be gained by developing fundamental models. The models within the controller rely on understanding of how the equipment works for successful implementation. Secondly, by having a high fidelity model available plant control can be tested off-line with the model, which was done in both Paper C and E. Apart from being able to develop the controller off-line it is also an action that reduces risk as when the solution is deployed on the real plant as it is already 90-95% tuned, and most bugs have been found. A high fidelity process models allows for tuning,

for example, PID-based SISO-loops on the plant in a green field project before the plant exists.

RQ6: What methods are used for moving between the high fidelity modeling domain and the control modeling domain?

Answer: In order to develop control models understanding of both the control methods and the phenomena being described is needed. The choice of control method may depend on what is being attempted, in Paper C linear MPC was used that has requirements on the model structure. Secondly as discussed in Paper D, different units have different first-order responses; trying to use these first-order responses is a good start for making a control model. Other methods could be linearization or a structure where the model is updated depending on where it is in its operating range.

7.2 Future work

The future work within this research will focus on the following:

- Further develop the capabilities with dynamic modeling with high fidelity models of machines and processes that have a wide range of applications.
- Formalize and further investigate the robustness of plant operation and design.
- Develop and test a digital process twin of a minerals processing circuit with update of models and AI capabilities.
- Explore and apply control methods for modeling and controlling process resources to minimize their use.

Apart from the above listed points, validation tests of both models and controllers will be on the agenda. The validation and implementation will need to be done with industry collaboration, which also secures that the research is utilized and can be benefited from.

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