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COMPARATIVE STUDY OF OPTIMIZATION SCHEMES IN MINERAL PROCESSING SIMULATIONS

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ABSTRACT

Modelling and simulations for mineral processing plants have been successful in replicating and predicting predefined scenarios of an operating plant. However, there is a need to explore and increase the potential of such simulations to make them attractive for users. One of the tools to increase the attractiveness of the simulations is through applying optimization schemes. Optimization schemes, applied on mineral processing simulations, can identify non-intuitive solutions for a given problem. The problem definition itself is subjective in nature and is dependent on the purpose of the operating plant.

The scope of this paper is to demonstrate two optimization schemes: Multi-Objective Optimization (MOO) using a Genetic Algorithm (GA) and Multi-Disciplinary Optimization (MDO) using an Individual Discipline Feasible (IDF) approach. A two stage coarse comminution plant is used as a case plant to demonstrate the applicability of the two optimization schemes. The two schemes are compared based on the problem formulations, types of result and computation time. Results show that the two optimization schemes are suitable in generating solutions to a defined problem and both schemes can be used together to produce complementary results.

KEYWORDS

Modelling, Simulations, Comminution, Mineral Processing, Multi-Objective Optimization, Genetic Algorithm, Multi-Disciplinary Optimization, Individual Discipline Feasible.

INTRODUCTION

The comminution process for a mineral processing plant consists of multiple crushing and classification stages and aims to produce fine material to liberate ore from the minerals. The modelling and simulation of such processes are well established using static process models (King, 2001; Napier-Munn et al., 1996). A recent development in the dynamic modelling for such processes has shown further increase in fidelity compared to the static process models (Asbjörnsson, 2015). The developed simulations can be utilized to design, operate and control mineral processing plants based on the requirements of users (Bhadani et al., 2017). Typically, the requirements in the operation of the comminution process are conflicting in nature, as for instance, reduction of power draw by process units, maximization of production of fine materials, and increasing utilization of the process units. These conflicting requirements leads to a Multi-Objective Optimization (MOO) problem to operate a mineral

processing plant. The MOO problems are normally referred to as non-linear problems and its fundamental understanding with respect to process behaviour is difficult to achieve.

Numerous studies have shown applications of Multi-Objective Optimization schemes in mineral processing simulations using Genetic Algorithms (GA) to solve optimization problems (Huband et al., 2006; Svedensten and Evertsson, 2005). However, a MOO problem can be solved by applying other mathematical schemes, such as Multi-Disciplinary Optimization (MDO) architecture. The exploration of the mathematical schemes have not been carried out to the full extent for mineral processing simulations. The mathematical and algorithmic development in the field of MOO holds potential to be applied in cross-disciplinary problems such as in mineral processing. The division of cross-disciplinary problems in mineral processing is subjective in nature and the problems depend on the part of the process which is being captured in the simulation.

The scope of this paper is to demonstrate and compare two optimization schemes: Multi-Objective Optimization (MOO) using a Genetic Algorithm (GA) and Multi-Disciplinary Optimization (MDO) using a distributed Individual Discipline Feasible (IDF) approach. The two optimization schemes are applied to an analytical model for a two stage coarse comminution process. The optimization is based on a trade-off between production of fine material versus power draw of the crushers. The two optimization schemes are compared based on problem formulation, result types and computation time. The paper first demonstrates a theoretical plant consisting of the two stage coarse comminution process and its modelling approach. The two optimization schemes applied to this plant are described followed by the section of results and discussion where the comparison of the two schemes is made.

MODELING OF A TWO STAGE COARSE COMMUNITION PLANT

A two stage coarse comminution plant design was chosen to demonstrate a conflicting trade-off existing in mineral processing plants and the plant layout is shown in Figure 1. The plant is intended to generate fine materials below 20 mm. The material from the primary crushing source (0-250 mm) is first fed into the first one-deck screen (*S1*) where two streams of material are generated. The first stream of finer material (*PF1*) is transferred into a bin and the oversize material stream (20-250 mm) is fed to the first crusher (*C1*). The product of *C1* is fed into a two-deck screen which generates three material streams. The oversized material (60+ mm) is recirculated back to the bin, the material size (20-60 mm) is fed to the second crusher (*C2*) and the fine material stream (*PF2*) is put into the bin. The product from *C2* is screened in a one-deck screen (*S3*) and generates two material streams. The oversize stream (20+ mm) is recirculated back to *C2* and the fine stream (*PF3*) is collected in the bin. To restrict the design space for the above plant, the process is varied by changing two variables (*CSS1* and *CSS2*) which are the Closed-Side Settings for the two crushers *C1* and *C2* respectively. The power draw of the two crushers (*PC1* and *PC2*) are measured. There are two main conflictive objective functions for the plant: maximization of the production of the fine materials and minimization of the power draw in the crushers.

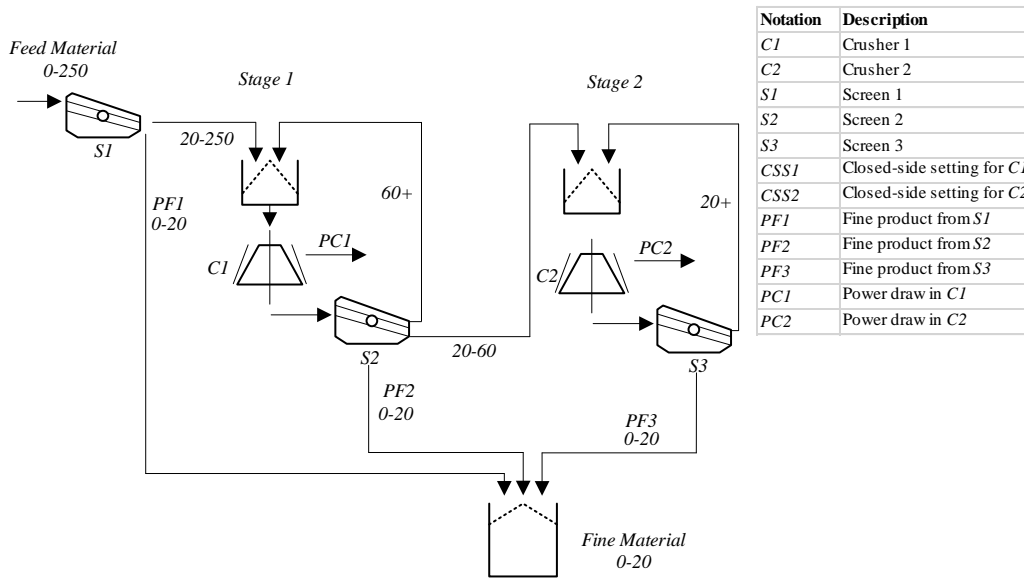


Figure 1. Layout of two stage coarse comminution plant.

The modelling of the plant is carried out in a MATLAB/Simulink environment based on the approach described by Asbjörnsson (Asbjörnsson, 2015). The process is considered as a continuous process and each unit model includes derivatives for mass m and properties γ of the material with respect to time t (Eq. 1 and 2). The crusher model is a Whiten crusher model (Eq. 3) (Whiten, 1972) while the screen model is represented with a Reid-Plitt efficiency curve (Eq. 4) (Reid, 1971). The material flow between the crushers and the bins are regulated using a PI controller.

$$\frac{dm(t)}{dt} = (\dot{m}_{i,in}(t) - \dot{m}_{j,out}(t)) \quad (1)$$

$$\frac{d\gamma_i(t)}{dt} = \frac{\dot{m}_{i,in}(t)}{m(t)} (\gamma_{i,in}(t) - \gamma_i(t)) \quad (2)$$

$$p = [I - C][I - CB]^{-1} f \quad (3)$$

$$E_i = 1 - e^{(-\ln 2(x_i)^m)} \quad (4)$$

The power draw (W) in the crushers are calculated using the Bond equation (Eq. 5) (Bond, 1952), where P_{80} and F_{80} are 80% sizes in μm of the product, and feed respectively, W_i is Work Index in kWh/t .

$$W = W_i \left(\frac{10}{\sqrt{P_{80}}} - \frac{10}{\sqrt{F_{80}}} \right) \quad (5)$$

The first crusher ($C1$) is configured as a medium coarse chamber for CS -type Sandvik crushers, while the second crusher ($C2$) is configured as a medium fine chamber for CH -type Sandvik crushers. In order to understand the problem setup, the plant was simulated with respect to individual design variables. Figure 2 a shows changes in the production of different sizes of material by changing $CSS1$ of the crusher ($C1$). It can be noted that with increasing $CSS1$, the production of fine material decreases in

stage 1 of the plant while the production of coarse material (20-60 mm) and the recirculating load (60+ mm) increases. On similar grounds, Figure 2 b shows the behaviour of the second crusher (C2) with respect to CSS2 and there is a decrease in production of fine material in stage 2 of the plant. For the same change in parameters, the power draw in C1 gradually decreases with an increase in CSS1 as shown in Figure 2 c. It is also interesting to see that the power draw in the crusher (C2) gradually increases as the feed to C2 is increased. Similarly, there is a decrease in power draw for the crusher (C2) as shown in Figure 2 d. It is observable that the power draw in C1 is relatively higher than the power draw in C2, which is dependent on capacity and size of the crushers and also on the work load distribution between the two crushers.

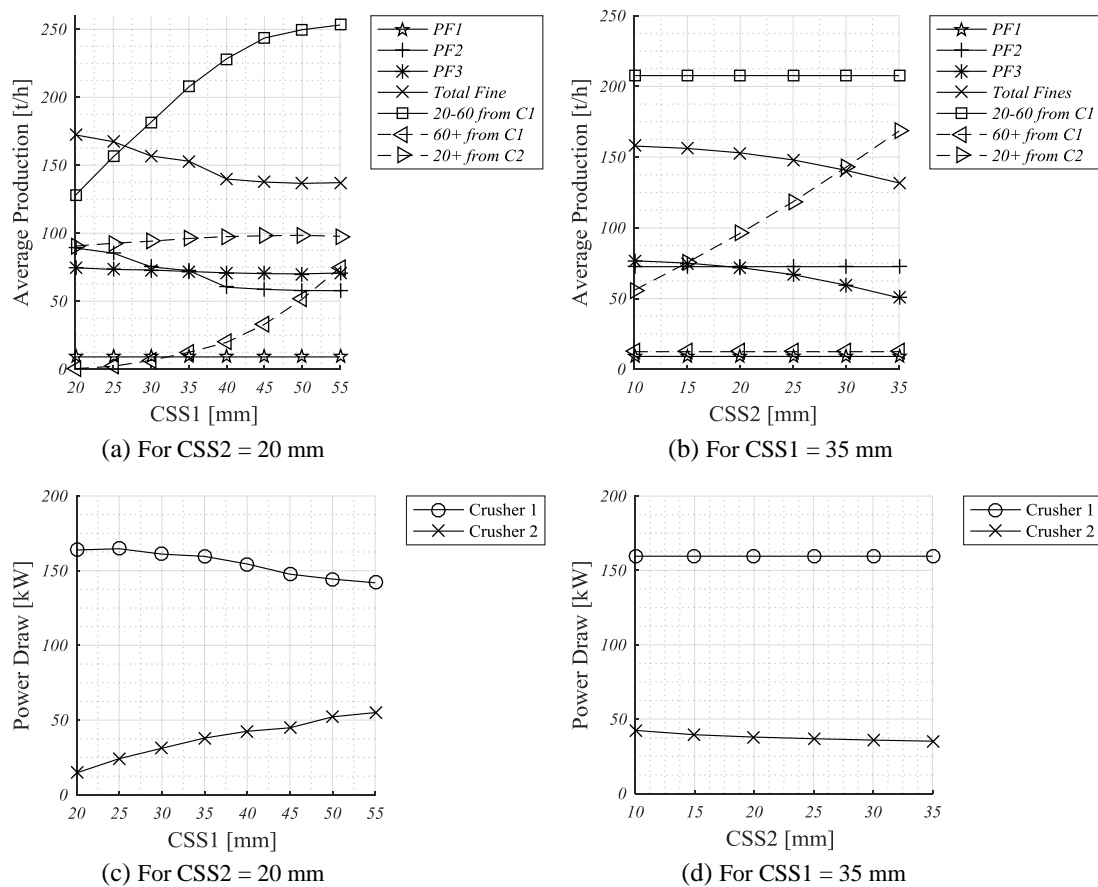


Figure 2. Effect in the production of material and the power draw by the crushers on changing closed-side settings of the crushers in the two stage coarse comminution plant.

OPTIMIZATION SCHEMES

A solution to the MOO problem can be achieved through multiple optimization schemes, depending on the problem formulation (Papalambros and Wilde, 2017). The optimization problem formulation is subjective in nature and it depends on the choice of users and the relevance to reality. As

already stated, the optimization problem formulation is based on two main objective functions: maximization of production of the fine material and minimization of the power draw by the crushers. This section describes the two optimization schemes applied to the considered case plant.

MOO using GA

Genetic Algorithm is a heuristic based algorithm and is developed from inspiration of natural evolution processes. The GA is normally applied to the objective functions that are non-linear and stochastic in behaviour (Kramer, 2017). In this case, the two-stage comminution process can be considered as a complex problem with non-linear behaviour. The MOO problem formulation is shown in Figure 3. The two stage comminution process is considered as a black-box model. The system optimizer parses the design variables (x) to the process simulation. The simulation returns the output variable (y) to the system optimizer and this process is repeated until the convergence criteria is achieved. The MOO problem is used to produce a Pareto-front using GA to illustrate the trade-off between the two objectives. The choice of the objective functions and problem formulations are critical in generating the relevant results using this approach. The choice of the solution is dependent on the user and the relevance to practical applications.

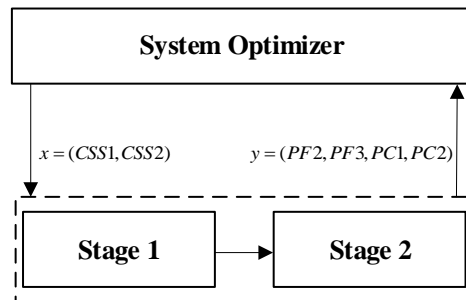


Figure 3. The MOO problem formulation for the two-stage comminution process for GA.

The optimization problem formulation for the GA is shown below.

$$\begin{aligned}
 & \textbf{Multi-Objective Optimization} \\
 & \min f1, f2 \\
 & f1 = \sum_{j=2}^3 -w_j PF_j(x, y), f2 = \sum_{j=1}^2 v_j PC_j(x, y) \\
 & w.r.t \rightarrow x, y \\
 & x = \{CSS1, CSS2\} \\
 & y = \{PF2, PF3, PC1, PC2\} \\
 & w_j = \{3, 1\}, v_j = \{1, 1\} \\
 & s.t.: x_{lb} = \{20, 10\}, x_{ub} = \{55, 30\}
 \end{aligned}$$

Weight factors w_j and v_j are added to the objective functions to include the significance of the work load distribution between the two stages of the comminution plant.

The general algorithm used for solving the MOO problem using a GA is shown below (Kalyanmoy, 2001; Kramer, 2017).

Algorithm:

Input- Design variable x

Output- Pareto front for multiple-objective functions ($f1^*$, $f2^*$) and optimized variable set (x^*)

0: Initiate population

Repeat

Repeat

0.1: Crossover

0.2: Mutation

0.3: Fitness computation

Until → Population complete

1: Selection of parental population

Until → Termination condition

MDO using a distributed IDF

The MDO architecture is a representation of the arrangement of various sub-disciplines involved in an engineering system and it has been used to achieve global optimization for complex engineering problems (Martins and Lambe, 2013). A distributed IDF approach within the MDO allows division of the system into two or more levels of optimization problem. The two stage comminution process in this case can be considered as an engineering system optimization problem. To illustrate the application of the distributed IDF approach, the system is divided into a two level design optimization problem: the system optimization problem and the individual sub-process optimization problem. The system design optimization problem aims to minimize the power draw from the crushers while the individual sub-process optimization problem aims to maximize the fines generated in each sub-process.

Figure 4 shows the application of the distributed IDF approach to the two stage coarse-comminution plant. In each main iteration, the system optimizer is subjected to minimize the power draw of the two crushers by using two sets of variables. The two sets of variables are the design variables (x_1) consisting of the two closed-side settings as well as duplicate copies of the same design variable (x_2). In each sub-process optimization, the sub-optimizer aims to maximize the production of fine material and uses local design variables and a duplicate copy of other sub-process design variables to produce local optima points. These local optima points are then used for calculation of the objective function in the main iteration. The individual sub-process optimizations can be solved in parallel. The system optimizer maintains the consistency between the design variables and the duplicate copy of the design variables by an addition of consistency constraints in the problem definition. The solution is iterated until the system optimizer has converged. This approach considers that the optimization problem can be decoupled into multiple optimization problems and the application of a gradient-based algorithm can solve such problem formulation. In this case, the gradient based algorithm, sequential quadratic programming, was used to solve the optimization problem.

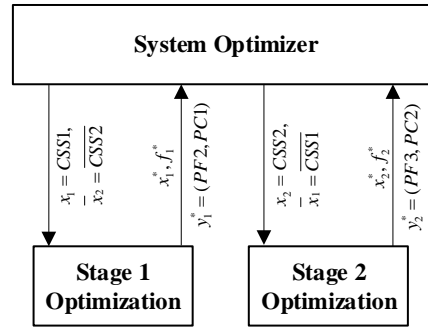


Figure 4. The MDO problem formulation for two-stage comminution process using IDF formulation

The bi-level optimization problem formulation for the IDF approach is shown below:

<i>System Optimization</i>	<i>Stage 1 Optimization</i>	<i>Stage 2 Optimization</i>
$\min f = \sum_{j=1}^2 PC_j(x, \bar{x}, y)$	$\min f1$	$\min f2$
$w.r.t \rightarrow x, \bar{x}, y$	$f1 = -PF2(x_1, \bar{x}_2, y_1)$	$f2 = -PF3(x_2, \bar{x}_1, y_1)$
$x = \{(CSS1)_1, (CSS2)_2\}$	$w.r.t \rightarrow x_1, \bar{x}_2, y_1$	$w.r.t \rightarrow x_2, \bar{x}_1, y_1$
$\bar{x} = \{(\overline{CSS1})_1, (\overline{CSS2})_2\}$	$x_1 = (CSS1)_1, \bar{x}_2 = (\overline{CSS2})_2$	$x_2 = (CSS2)_2, \bar{x}_1 = (\overline{CSS1})_1$
$y = \{(PF2, PC1)_1, (PF3, PC2)_2\}$	$y_1 = (PF2, PC1)_1$	$y_1 = (PF3, PC2)_1$
$s.t. : \ x - \bar{x}\ = 0$	$s.t. : x_{1lb} = (20), x_{1ub} = (55)$	$s.t. : x_{2lb} = (10), x_{2ub} = (30)$
$x_{lb} = \{20, 10\}, x_{ub} = \{55, 30\}$		
$\bar{x}_{lb} = \{20, 10\}, \bar{x}_{ub} = \{55, 30\}$		

The general algorithm for solving the problem is shown below (Bhadani et al., 2018; Martins and Lambe, 2013).

Algorithm:

Input- Design variable x

Output- Optimized variable x^* , objective function f^*

0: Initiate system optimizer iteration

Repeat

1: Compute sub-process objective and constraints

For each sub-process i , **do**

1.0 Initiate sub-process optimization

Repeat

1.1 Evaluate sub-process i

1.2 Compute sub-process i objective and constraints

1.3 Compute new design point for sub-process $(i+1)$

Until 1.3 \rightarrow Optimization i has converged

End for

2. Compute new system design points

Until 2 \rightarrow System optimization has converged

RESULTS AND DISCUSSION

The two optimization schemes produced relevant results for the operation in the mineral processing simulations. Figure 5 a shows the Pareto front between the two objectives defined in the MOO problem. The corresponding solution points for the design variables are shown in Figure 5 b. It can be noted that the MOO method using GA generates a wide range of solutions. The user can select the most competitive solution depending on the judgement in relation to the reality. In this case, the solutions obtained at the boundary values of the design variable (e.g., $CSS1 = 20, 55$) are not suitable solutions. This can be interpreted from practical aspects such as that the capacity of the crusher will decrease if operated at the lower bound, and the crusher will produce higher recirculating material if operated at the upper bound (see Figure 2 a).

It can be observed from Figure 5 c, d compared to Figure 5 a, b that the weight of $PC1$ in function $f1$ is changed from a value of 3 to 5 (increasing the importance of the power draw in Stage 1 compared to Stage 2), a different set of solutions is obtained. This particular feature of weighing the objectives helps in exploring the problem design region. The choice of the weight is dependent on the process and the importance of the particular function with respect to other competing functions.

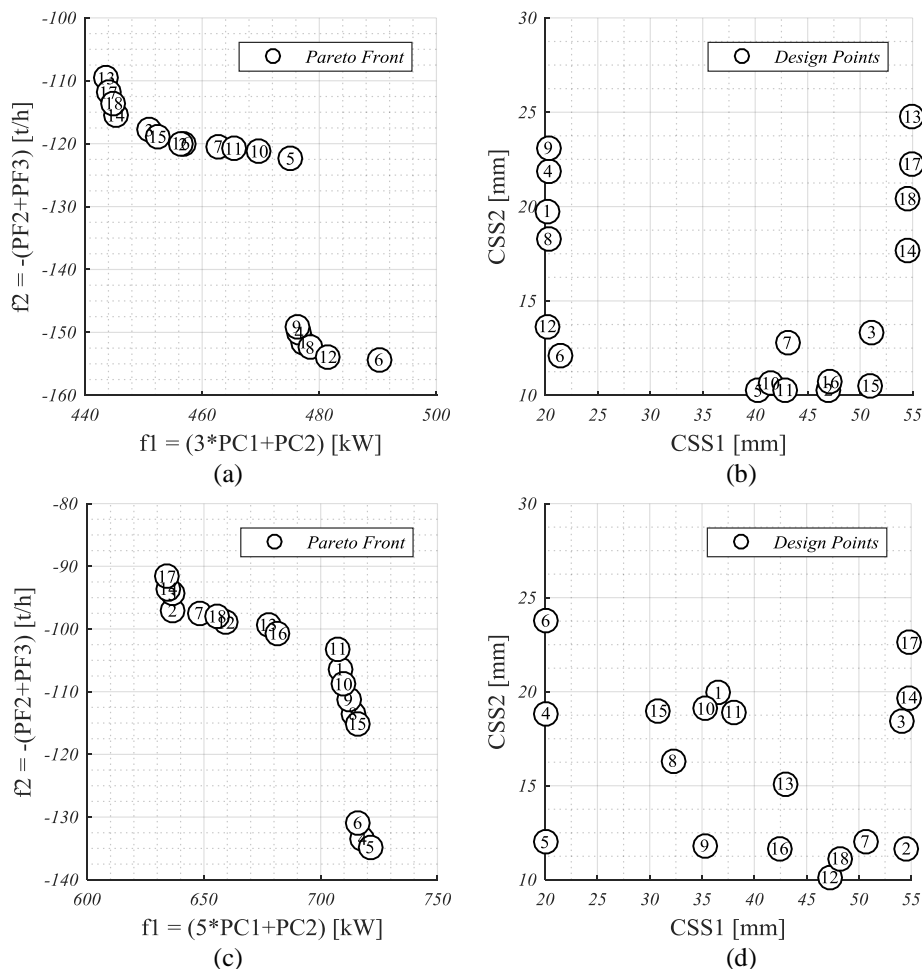


Figure 5. Pareto-fronts and solution points of the MOO problems using GA.

Figure 6 represents the convergence curve and solution points for the MDO problem using the distributed IDF approach. The design variables and the duplicate copies of the design variables converged to a single solution point. The solution obtained with the IDF approach are $CSS1 = 44.38$ mm and $CSS2 = 10$ mm which is comparable to the range of solutions obtained using the GA ($CSS1 = 40$ to 50 mm and $CSS2 = 10$ to 15 mm, see Figure 5 b). A brief remark between the two optimization schemes are presented in Table 1.

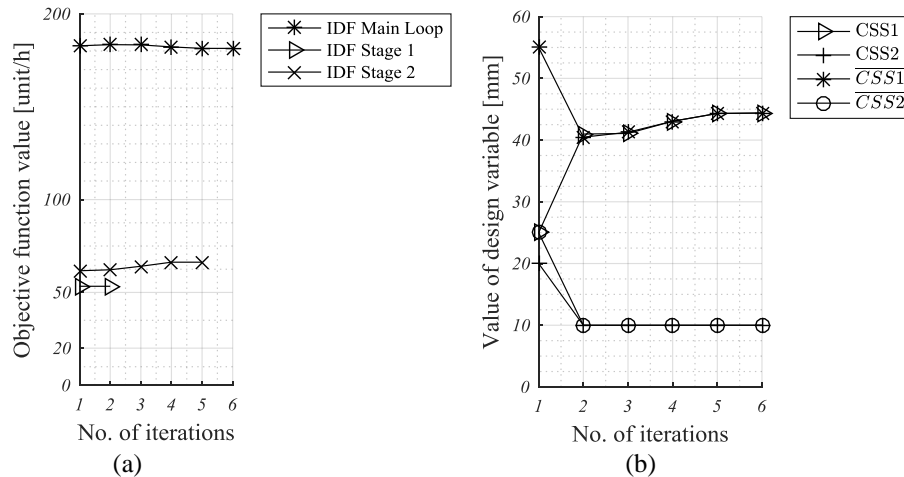


Figure 6. Convergence curve and solution points of the MDO problem using the distributed IDF formulation.

Table 1 – Comparison between the two optimization schemes

	MOO using GA	MDO using distributed IDF
Problem Formulation	Weighted sum approach to formulate comprehensive optimization objectives.	Can be decoupled into two or more levels of optimization problems.
Result Types	Pareto-front highlighting the spectrum of solutions. Choice is based on reasoning of the solution space.	Balanced solution between system optimization and sub-process optimization.
Computation Time	High. Dependent on algorithm settings such as population size and generation.	Low. Dependent on the initial start point of the algorithm.

CONCLUSIONS

The two optimization schemes: Multi-Objective Optimization (MOO) using the Genetic Algorithm (GA) and Multi-Disciplinary Optimization (MDO) using the Individual Discipline Feasible (IDF) approach are demonstrated using a two stage coarse comminution plant. Both the schemes are suitable in producing reasonable and comparable results, and can be used together to complement the results. The MOO scheme using GA is suitable for exploring the solution space while the MDO scheme using IDF approach is suitable for identifying the balancing point of the system. The two schemes differs in terms of problem formulation since the results obtained using GA are sensitive to the weights added

in the MOO problem formulation. The behaviour of the two optimization schemes needs to be studied with an increased number of design variables to be able to further compare the computational performance. Further development in the optimization objective functions are needed to explore the capabilities of the mineral processing simulations.

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