

THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

On Robust Steering Based Lateral Control of Longer and
Heavier Commercial Vehicles

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To my son, Sam

Abstract

Rapid growth in the transportation of goods has led to raised concerns about environmental effects, road freight traffic, and increased infrastructure usage. The increasing cost of fuel, and issues with congestions and gas emissions, make longer and heavier commercial vehicles (LHCVs) an attractive alternative to conventional heavy vehicles. However, one major issue concerning LHCVs is their potential impact on traffic safety. A typically dangerous behaviour happens during sudden evasive steering maneuvers, which causes amplified lateral motions in the towed units. These amplified motions can lead to the towed units' oscillation, large offtracking and, in a worst case scenario, cause rollover.

The main objective of this thesis is to develop robust steering-based controllers for improving the lateral performance of LHCVs at high speeds by suppressing unwanted amplified motions in the towed units. Robust control methods aim to achieve an adequate level of robustness against model uncertainties and disturbances, while at the same time satisfying the desired closed-loop system performance specifications. The proposed robust control syntheses are formulated based on an \mathcal{H}_∞ static output-feedback (SOFB), in which only one easily measurable state variable is required. As the measurement of the driver steering input is available, a combined version of SOFB and dynamic feed-forward (DFF) is also developed and several techniques for designing DFF are proposed. The control synthesis problems are solved by using linear matrix inequality (LMI) optimizations. The theoretical contributions of this research mainly lie in the derivation of a novel LMI condition for an integral quadratic constraint (IQC) on the states and also in the derivation of a set of new LMI conditions for the DFF design method. From a practical point of view, the proposed controllers are simple and easy to implement, despite their theoretical complexity.

The effectiveness of the designed controllers is verified through numerical simulations performed on linear vehicle models as well as high-fidelity vehicle models. The verification results confirm a significant reduction in yaw rate rearward amplification, lateral acceleration rearward amplification and high-speed transient off-tracking, thereby improving the lateral stability and performance of the studied LHCVs.

Keywords: Commercial vehicles, Rearward amplification, Static output feedback, Dynamic feed-forward, Robust control, LMI-based \mathcal{H}_∞ synthesis

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Maliheh Sadeghi Kati
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December, 2018

List of Publications

The content of this thesis is based on the following publications:

Paper A

Maliheh Sadeghi Kati, Hakan K ro lu and Jonas Fredriksson, "H_∞ Static Output Feedback Synthesis under an Integral Quadratic Constraint with Application to High Capacity Transport Vehicles," European Control Conference, 2016.

Paper B

Maliheh Sadeghi Kati, Hakan K ro lu and Jonas Fredriksson, "Robust Control of an A-double with Active Dolly based on Static Output Feedback and Dynamic Feed-forward," 14th International Heavy Vehicle Transport Technology Symposium, 2016.

Paper C

Maliheh Sadeghi Kati, Hakan K ro lu, Jonas Fredriksson and Mohammad Manjurul Islam, "Robust Static Output Feedback with Dynamic Feed-forward for Lateral Control of Long-combination Vehicles at High Speeds," under second revision in IEEE Transactions on Control Systems Technology.

Paper D

Maliheh Sadeghi Kati, Hakan K ro lu and Jonas Fredriksson, "Robust Lateral Control of Heavy Combination Vehicles under Moments of Inertia and Tire Cornering Stiffness Uncertainties," International Journal of Vehicle System Dynamics (in press), 2018.

Paper E

Maliheh Sadeghi Kati, Jonas Fredriksson, Bengt Jacobson and Leo Laine, "Gain-scheduled H_∞ Controller Synthesis for Actively-steered Longer and Heavier Commercial Vehicles," submitted to Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering.

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Maliheh Sadeghi Kati, "Definitions of Performance Based Characteristics for Long Heavy Vehicle Combinations", Technical Report, Chalmers University of Technology, 2013.

Nomenclatures

LHVC	long heavy vehicle combination
CHCV	conventional heavy combination vehicle
LHCV	long heavy combination vehicle, loner and heavier commercial vehicle
CHVC	conventional heavy vehicle combination
LMI	linear matrix inequality
IQC	integral quadratic constraint
A-double	tractor-semitrailer-dolly-semitrailer
EU-28	28 member countries of European Union
GHG	greenhouse gas
ETAC	European Truck Accident Causation
LQR	linear quadratic regulator
COG	center of gravity
PID	proportional-integral-derivative
FLC	fuzzy logic control
AHV	automated heavy vehicle
PATH	Partners for Advanced Transit and Highways
MRAC	model reference adaptive control
LTV	linear time-varying
LTI	linear time-invariant
Eurostat	European statistics
GVW	gross vehicle weight

GCM	gross combination mass
IEA	international energy agency
LCV	long combination vehicle
LHGV	long heavy goods vehicle
LVC	long vehicle combination
HCT	high capacity transport
PBC	performance based characteristics
PBS	performance based standard
EMS	European modular system
ETT	en Trave Till
RA,RWA	rearward amplification
HSTO	high speed transient offtracking
SOFB	static output feedback
DOFB	dynamic output feedback
DFE	dynamic feed-forward
GS	gain scheduled, gain scheduling
ST	semitrailer
CAT	center-axle trailer
DY	dolly
FFT	Fast Fourier transform
VTM	Volvo Transport Model, Virtual Truck Model
t	tonnes, tons
m	meters
deg	degrees
s	seconds

km	kilometres
freq, f	frequency
L	Lagrange function
T	total kinetic energy
U	total potential energy
q	generalized coordinate
Q	generalized external force
X_1	longitudinal position of the first vehicle unit
Y_1	lateral position of the first vehicle unit
φ_1	yaw angle of the first vehicle unit
θ	articulation angle
ω_z, r	yaw rate
a_y	lateral acceleration
a_{y11}	lateral acceleration of the first axle of the first vehicle unit
I_z	yaw moment of inertia
m	mass
F_{xk}	tire longitudinal force
F_{yk}	tire lateral force
r_y	position vector
δ	steering angle
δ_{driver}	steering angle of the driver
δ_{dolly}, δ_u	steering angle of the dolly
α_{ky}	lateral slip angle
C_{a_k}	tire cornering stiffness
\mathcal{M}_q	inertial matrix

\mathcal{C}_q	damping matrix
\mathcal{K}_q	stiffness matrix
\mathcal{B}_q	force distribution matrix related to external disturbances
\mathcal{H}_q	force distribution matrix related to control inputs
V_{x1}, v_{x1}, v_x	longitudinal velocity of the first vehicle unit
V_{y1}, v_{y1}	lateral velocity of the first vehicle unit
A	system matrix in the linear state-space model
B	disturbance input matrix in the linear state-space model
H	control input matrix in the linear state-space model
x	state vector
y	measurement signal
u	control input
w	disturbance signal
z	performance output
γ	\mathcal{L}_2 -gain performance
V	Lyapunov function
X, Y	Lyapunov matrix
K, K_{fb}	feedback gain
K_{ff}	feed-forward gain
\mathcal{T}_{zw}	transfer function from w to z
λ	weighting factor

Contents

	Page
Abstract	i
Acknowledgment	ii
List of Publications	iv
Nomenclatures	vii
1 Introduction	1
1.1 Background	1
1.2 Literature Review	5
1.3 Objectives	8
1.4 Contributions	8
1.5 Limitations	9
1.6 Outline	10
2 Longer and Heavier Commercial Vehicles	11
2.1 Introduction to Longer and Heavier Commercial Vehicles . .	11
2.2 Performance Based Standards	14
2.3 Vehicle Dynamic Modelling	16
3 Controller Synthesis based on LMI Optimization	23
3.1 Introduction to Linear Matrix Inequalities (LMIs)	23
3.2 \mathcal{H}_∞ Synthesis Methods	26
3.3 Robust Control Synthesis	29
4 Summary of Included Papers	33

4.1	Paper A: \mathcal{H}_∞ Static Output Feedback Synthesis under an Integral Quadratic Constraint with Application to High Capacity Transport Vehicles	33
4.2	Paper B: Robust Control of an A-double with Active Dolly based on Static Output Feedback and Dynamic Feed-Forward	34
4.3	Paper C: Robust static output feedback with dynamic feed-forward for lateral control of long-combination vehicles at high speeds	34
4.4	Paper D: Robust Lateral Control of Long-combination Vehicles under Moments of Inertia and Tire Cornering Stiffness Uncertainties	35
4.5	Paper E: Gain-scheduled \mathcal{H}_∞ Controller Synthesis for Actively-steered Longer and Heavier Commercial Vehicles	35
5	Concluding Remarks and Future research Directions	37
5.1	Concluding Remarks	37
5.2	Recommendations for Future Work	38
	Appendix	39
	References	43
	Appended Papers	52
A	\mathcal{H}_∞ Static Output Feedback Synthesis under an Integral Quadratic Constraint with Application to High Capacity Transport Vehicles	53
1	Introduction	55
2	Problem Formulation	57
3	Sufficient Solvability Conditions for Problem 1	58
4	Application to the Lateral Control of a High Capacity Transport Vehicle	63
5	Concluding Remarks	70
B	Robust Control of an A-double with Active Dolly based on Static Output Feedback and Dynamic Feed-forward	73
1	Introduction	75
2	Vehicle Model	76
3	Control Design	80
4	Simulation Results	85

5	Conclusions	88
C	Robust Static Output Feedback with Dynamic Feed-forward for Lateral Control of Long-combination Vehicles at High Speeds	91
1	Introduction	93
2	Combined static output feedback and dynamic feed-forward control design	97
3	Application to the lateral control of the A-double combination vehicle	111
4	Example designs and simulations	119
5	Conclusion	127
D	Robust Lateral Control of Long-combination Vehicles under Moments of Inertia and Tire Cornering Stiffness Uncertainties	137
1	Introduction	139
2	Vehicle Models	141
3	Robust Controller Synthesis	147
4	Controller Design and Simulations	153
5	Summary and Conclusions	164
E	Gain-scheduled \mathcal{H}_∞ Controller Synthesis for Actively-steered Longer and Heavier Commercial Vehicles	175
1	Introduction	177
2	Investigated Heavy Vehicles	179
3	Vehicle Models	180
4	Controller Synthesis	185
5	Simulation results	189
6	Concluding Remarks	199

Chapter 1

Introduction

This chapter provides the background, literature review and objectives for this thesis. It also states the limitations, main contributions and outline of the thesis.

1.1 Background

Total goods transport activities in the 28 member countries of European Union (EU-28) were estimated to be about 3516 billion tonne-kilometres in 2015 which has increased by 23.6 % compared to 1995. The share of road transport as the biggest contributor accounted for 49 % of this total, intra-EU maritime transport for 31.6 %, rail transport for 11.9 %, inland waterways transport for 4.2 %, oil pipelines for 3.3 % and intra-EU air transport for 0.1 % of the total. These statistics focus only on intra-EU transport, not transport activities within the rest of the world [1].

The transport sector consumed about one third of the energy demand in the EU-28 and was responsible for 23.5 % of total greenhouse gas (GHG) emissions (including international aviation) in 2015. The largest source of the GHG emissions produced by transport sectors in 2015 was from road transport with a share of 72.9 %. Of these road freight transport emissions, 61.5 % was contributed by cars, while 25.9 % came from heavy-duty vehicles and busses. Hence, the entire transport sector, and particularly road freight transport, has been identified for further environmental and overall efficiency improvements for a sustainable future in Europe [1, 2]. In addition, significant improvements within the transport sector are expected in the area of safety, reducing accidents and fatalities. According to the 2005 European Truck Accident Causation (ETAC) study [3], based on the 624 accidents in-

volving heavy goods vehicles (Vehicle Gross Weight (VGW) >3.5 tonnes) in seven European countries, 303 people were killed and 774 injured over a period of 2.5 years (2004-2006). The findings of this study revealed that heavy goods vehicles were the main cause of 25 % of these accidents. The annual report of Community database on Accidents on the Roads in Europe (CARE) [4] revealed that heavy goods vehicles (VGW > 3.5 tonnes) were responsible for 520 out of total 26132 road fatalities occurred in the EU-28 in 2015.

To enhance the efficiency, safety and environmental performance of transport system, the European Union has recognised the urgent need for developing a vision and setting targets. It is expected that setting targets leads to more effective and realistic programmes and resource allocation. The European Commission's 2011 Transport White Paper has provided an ambitious vision for the future of EU transport in which a reduction of at least 60 % of GHG emissions is targeted by 2050, compared to 1990 levels. The White Paper also sets the goal of zero road fatalities and serious injuries in road transport by 2050 (Vision Zero) [5].

In order to meet the long-term 60 % GHG emission reduction target, one of the most effective ways is to increase the size (height, length and weight) of heavy goods vehicles. The increased payload per vehicle is expected to reduce transport costs, the number of trips and also energy intensity per unit of payload, and consequently reduce GHG emissions. Unlike most other countries in Europe where only conventional heavy vehicles with the maximum length of 18.75 m, maximum weight of 40 tonnes (44 tonnes for combined transport) and height of 4 m are permitted (according to the European directive 96/53/EC), heavy vehicles up to 25.25 m in length and 60 tonnes in weight are permitted in Sweden and Finland, with trials underway in some other EU member countries (Norway, Denmark, Netherlands, Belgium and Germany). However, some other countries outside Europe have allowed longer and heavier vehicles for many years, like Brazil, Canada, Australia, New Zealand, Mexico, South Africa and the USA.

The potential environmental, economic and practical impacts of implementing longer or/and heavier commercial vehicles (LHCVs) in road transport have been widely investigated in many countries (e.g. [6–13]). In a report prepared by Woodroffe and Ash, the use of LHCVs instead of the common non-LHCVs truck (45 feet) in Alberta has been resulted in 29 % saving in transportational costs, 32 % in fuel consumption and GHG emissions, 44 %

reduction in vehicle-kilometer travelled and 40 % decreased road wear [6]. In another study conducted by a Swedish transport institute, the performance of LHCVs (25.25 m) were compared with two commonly-used EU heavy vehicles (18.75 m and 16.5 m). The findings of this study indicated that the use of LHCVs leads to a reduction in the number of trips by 32 % and consequently reduced fuel consumption and GHG emissions by 15 %. In addition, the total operational costs will drop by 23 % [7].

Road safety is the most controversial issue of LHCVs. There are contrasting opinions about the road safety of LHCVs among different groups of specialists and scientists. On the one hand, proponents argue that the introduction of LHCVs means fewer vehicles on roads and reduced congestion, and consequently a smaller exposure to road accident risk. In other words, the probability of traffic accidents increases with vehicle-kilometers travelled and since the vehicle-kilometers travelled are reduced, the number of accidents are expected to decline [14]. On the other hand, opponents disagree, stressing that LHCVs are more likely to be involved in accidents due to their larger size, if compared with conventional commercial vehicles [15]. In a simulation-based study conducted by Glaeser and Ritzinger [8], it has been shown that LHCVs have worse performance in terms of rearward amplification of lateral acceleration in rear trailers, rear trailer off-tracking at high speeds and also poor low-speed maneuverability. Even so, when considering these issues, it can not be concluded that LHCVs are significantly unsafe, but it is recommended that their use should be limited to the roadways that are geometrically sufficient to accommodate such vehicles. In addition, Steer et al. [10] conclude that there is no evidence that LHCVs increase safety risk, but increased length and weight may increase the severity of accidents. They also mention that introducing LHCVs would bring an overall improvement in road safety due to the reduction in vehicle-km.

The relationship between the features of LHCVs and road safety issues are investigated in [14, 16]. Based on the findings of these studies, traffic congestion and road safety are highly influenced by the main features of LHCVs such as heavy weight, long length, poor manoeuvrability, and low dynamical stability. On the contrary, in a study by Aurell and Wadman [17], it is argued that LHCVs have in general better dynamic stability than shorter heavy vehicles. Although so far, there is no empirical evidence showing that LHCVs are significantly more dangerous than conventional heavy vehicles. Despite the slight discordance in the conclusions of these studies, they have shown that

with improved vehicle designs, it is possible to overcome the safety-related problems of LHCVs in such a way that they can perform as well or better than currently-used heavy vehicles.

Main dynamical instability modes associated with heavy commercial vehicles are known as trailer swing, jackknifing, trailer lateral oscillation and rollover [18], see Figure 1.1. Trailer swing occurs when the wheel of the trailer (the towed unit) are locked up and the trailer start to swing out. This is more likely to happen on slippery/poor-conditioned roads, specially when the trailer is empty or lightly loaded. Jackknifing happens when the rear wheels of the lead unit are locked up due to improper and hard braking, or slippery or poor-conditioned roads. Trailer lateral oscillation happens due to instabilities in yaw motions of the towed units mainly because of sudden evasive driver steering. Rollover occurs due to roll instabilities in the center of gravity of the vehicle and results in wheels lift-off the ground. Main factors involved in rollover crashes are high speed, high centre of gravity, improper braking and excessive lateral force.

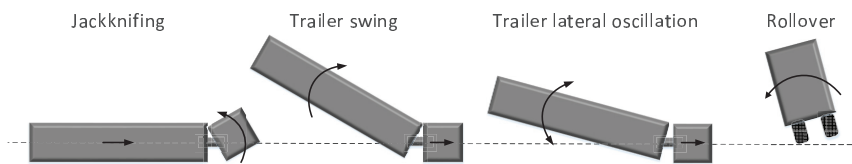


Figure 1.1: Most common instability modes; yaw instability (jackknifing, trailer swing and trailer lateral oscillation) and yaw/roll instability (rollover)

Therefore, in this regard, there is a crucial need for innovative technical solutions that improve the high-speed stability and low-speed maneuverability of LHCVs in order to promote the use of LHCVs and ease the concerns about the impact of LHCVs on traffic safety as well as damages to road infrastructure.

One way to improve low-speed maneuverability and high-speed stability would be to steer the axles of the towed vehicle units in LHCVs. Hence, in order to make the towed unit steerable, several active and passive steering techniques have been proposed. To solve the problems of manoeuvrability at low speed, a number of passive steering systems have been developed (e.g. [19–21]). Such systems steer some axles on the towed unit according to a simple geo-

metrical relationship or force/moment balance. The passive steering systems are able to reduce tire and road wear and improve low-speed steady-state manoeuvrability. However, they have a detrimental effect on high-speed performance, leading to high-speed yaw instability and increased rearward amplification and poor maneuverability. All passive steering systems found to date are locked at high speeds. Moreover, the passive steering systems are designed to work in steady-state circular motion. They do not generally provide the correct steering inputs for transient manoeuvres [22, 23].

To overcome these problems at high speeds, an active steering system can be used instead of passive steering systems. Such systems can offer viable solutions for both low-speed maneuverability and high-speed stability of LHCVs. Therefore, different studies have been carried out to develop control strategies for active steering of heavy vehicles (e.g. [24–31]). Active steering systems work similar to passive steering systems by steering the rear axles. However, the steering angle of each axle is no longer dependent on a simple geometric relationship or force balance. They usually consider the vehicles current states to calculate the required steering angles to be performed on the rear axles. The focus of this thesis is on active steering of LHCVs to improve their dynamic behavior by attenuating lateral oscillation of the towed units at high speeds. The next section summarizes some of existing studies on steering-based control of heavy vehicles.

1.2 Literature Review

The use of active steering systems has shown a remarkable potential in improving maneuverability and lateral performance of heavy commercial vehicles. The active steering of heavy vehicles has been widely investigated to improve the low-speed manoeuvrability (e.g. [25–27, 32, 33]), high-speed lateral stability (e.g. [28, 29, 31, 34–36]), and both the maneuverability and lateral stability (e.g. [30, 37–43]).

The most common approach used in the active steering systems is linear quadratic regulator (LQR) algorithms (e.g. [28–30, 34, 36, 37, 39, 42].) Unlike other current systematic control design syntheses, like \mathcal{H}_∞ , classical LQR controller might suffer from poor robustness when the system is exposed to parameter variations and is out of its nominal condition (e.g. [44–46]). In other words, the success of the LQR-based controllers is dependent on the accuracy of linear vehicle models.

A number of control systems have been proposed based on combined feed-forward and proportional feedback [31] and proportional integral derivative (PID) control (e.g. [38,40,43]) algorithms and have been successfully applied to various vehicle handling and stability control. Undoubtedly conventional P, PD and PID controllers are by far the most frequently used controllers in different engineering applications due to their simplicity and ease of online re-tuning. Yet, the selection of the controller parameters (three parameters P, I, D to tune) does not give optimum values of the controller directly. In addition these controllers may not guarantee the robustness of the system. Compared with the aforementioned conventional PID and LQR control methods, fuzzy logic control (FLC) method (e.g. [25, 27, 28]) has its advantages in dealing with systems with large uncertainties. Nevertheless in spite of its practical success, it is hard to program and prior knowledge of the system is required to model the fuzzy system. In addition, it is a very time-consuming design and there is not a standard analytical procedure for designing, analyzing its stability and tuning it.

Most of the works discussed above are not considering robust stability and robust performance in the controller design procedure. The majority of the lateral control approaches for heavy vehicles are based on exact system model in which it is assumed that all parameters and variables of the considered vehicle model for control designs are known and measurable or can be estimated. Consequently, the applicability of these control systems are restricted by the accuracy of vehicle units' parameters and may exhibit significant performance degradation or even instability in the presence of parametric uncertainties such as mass, yaw moment of inertia, position of center of gravity and tire cornering stiffness coefficient.

To the author's best knowledge, very few publications are available in the literature that address the issue of directly synthesizing robust lateral control of heavy vehicles in presence of parametric uncertainties and un-modeled nonlinear dynamics, especially in the case of active-steering based control. Much of the early research on robust synthesis of heavy vehicles has been conducted over 20 years ago by the California PATH (Partners for Advanced Transit and Highways) program (under project TO4201/MOU385). The main goal of this project was to implement and test previously developed lateral control algorithms for the automated lane following in the context of automated highway systems, and enhance robustness and performance of lateral controllers. For instance, in [47], a robust \mathcal{H}_∞ loop shaping for lateral con-

trol of automated heavy vehicles (AHVs) was designed in order to cope with the model uncertainties such as vehicle longitudinal velocity, road adhesion coefficient and cargo loads in the trailer.

In the 2003 PATH report [48], five different types of robust controllers against un-modeled nonlinear dynamical uncertainties and parametric uncertainties have been designed and experimentally implemented for lateral control of AHVs; four nonlinear controller (a sliding mode control, a nonlinear robust feedback linearization controller, a nonlinear loop-shaping controller, an adaptive robust control) and one linear controller (a linear robust feedback controller with a feedforward compensation). It was observed that all the proposed controllers provide robustness against uncertainties in tire cornering stiffness and location of center of gravity due to varying loads, and also un-modeled dynamics such as roll, pitch, bounce, and suspension dynamics. However, as the uncertainty increases, the performance of the sliding mode controller severely degrades due to the inevitable high control gain. It was also concluded that the implementation of nonlinear controllers are much more complicated than that of linear controllers. In [49], a linear parameter varying controller is designed to incorporate velocity dependence of the vehicle dynamics in the control design of automated lane keeping for heavy vehicles. It should be noted that in all above mentioned studies by the PATH program, only the front wheels of the tractor are steerable for the tractor-semitrailer type of heavy vehicles.

In a study done by Wang [50], a model reference adaptive control (MRAC) strategy is developed for active trailer steering system control. The MRAC technique has been used to improve the robustness of the active trailer steering system with respect to vehicles parametric variation and varied operating conditions such as the variations of vehicle longitudinal velocity and trailer payload. The main vehicle researched in this work is a double-trailer heavy vehicles (B-train), which is the most commonly used across Canada for goods transportation. In [51] Ni et al. proposed an LMI-based LQR method for active trailer steering of an A-double vehicle in which the vehicle longitudinal velocities and the time constant of the steering actuator model are considered as uncertain parameters. In the proposed controllers, it is assumed that all the state variables are perfectly measured, which is not the case in reality and not all the state variables are available or easily measurable.

1.3 Objectives

The main objective of the research presented in this thesis is to develop active steering-based control strategies for improving the high-speed lateral performance and stability of LHCVs, with a primary focus on a prospective LHCV denoted as the A-double (tractor-semitrailer-dolly-semitrailer). The proposed controllers should be designed in such a way that the controlled vehicle not only maintains robust stability in the presence of parameter uncertainties and un-modeled dynamics but also achieves a desirable level of robust performance.

1.4 Contributions

Several robust steering-based control strategies are proposed for LHCVs. The synthesis problem in this thesis is formulated as an \mathcal{H}_∞ -type design problem and can conveniently be solved by using linear matrix inequality (LMI) optimizations.

The main contributions of this thesis are as follows:

- The proposed controllers ensure robust stability and performance in the face of model uncertainties such as steering actuator model parameters, cornering stiffness of tires and yaw moment of inertia of trailers.
- The developed controllers steer the axles of selected towed vehicle unit in a way to suppress undesired amplified motions, namely yaw rate rearward amplifications, towards the towed vehicle units.
- The controller syntheses are formulated based on a static output feedback (SOFB), which uses information from only one articulation angle that is relatively easy to measure.
- A novel LMI condition is derived that ensures an integral quadratic constraint (IQC) on the states.
- Since the measurement of the driver steering is available, combined SOFB with dynamic feed-forward (DFF) has been considered and several alternatives for designing the DFF controller are proposed.

- To deal properly with tire cornering stiffness uncertainties, a linear time-varying (LTV) tire model is suggested, in which the cornering stiffness is considered as a time-varying uncertain parameter.
- Since the resulting state-space representation of the system is rationally dependent on uncertain moment of inertia parameters, a descriptor-type representation of the system is hence employed to avoid the rational dependency. Accordingly, a synthesis method is developed to deal with the systems with the descriptor form representation.
- A gain-scheduled control synthesis is developed by considering the longitudinal velocity as the scheduling parameter for a selected set of LHCVs.

The performance of the proposed controllers are verified by performing numerical simulation on linear vehicle models as well as high-fidelity vehicle models in both frequency and time domains.

1.5 Limitations

The results of this thesis are subjected to the following limitations.

- The proposed controller syntheses are only verified by using high fidelity vehicle models, but not experimentally with a test vehicle.
- The driver role is limited only to a steering input in the performed simulations and a simple driver model is used for modelling the frequency content of the driver steering for control synthesis purposes.
- In this thesis only steering actuators for control of the vehicle dynamics are considered; other actuators such as braking actuators and active suspension components are excluded.
- The controller syntheses are designed based on a simplified single-track linear vehicle model without accounting for frame flexibility, roll, pitch or bounce dynamics.
- There is always measurement noises and other disturbances in addition to the driver input entering the system in an unpredictable way which are not mentioned in this thesis.

1.6 Outline

Chapter 2 provides the background on longer and heavier vehicles, related performance measures and the linear vehicle model used in this thesis. Chapter 3 describes an overview of the theoretical material that is required for the proposed controller syntheses of the next chapters. Chapter 4 summarizes the scientific contributions of the appended papers at the end of the thesis. Finally, Chapter 5 provides some concluding remarks and suggestions for further work.

Chapter 2

Longer and Heavier Commercial Vehicles

In this chapter, first a brief introduction is given on the background of Longer and Heavier Commercial Vehicles (LHCVs). Afterwards, some important performance based standards (PBSs) used for evaluating heavy commercial vehicles are presented. In the end, the linear vehicle model used for the design of the proposed controllers is briefly described.

2.1 Introduction to Longer and Heavier Commercial Vehicles

Heavy goods vehicles operating in international and national road networks in the EU must comply with certain rules on weights and dimensions that are set by Council Directive 96/53/EC in 1996 [52]. In most European countries, the permitted length of a goods vehicle is restricted to a maximum of 16.5 m in length for standard articulated heavy vehicles (e.g. a tractor-trailer) and 18.75 m for road trains (e.g. a rigid truck or tractor-trailer pulling a drawbar trailer). The maximum authorized weight is set to 40 tonnes, except for 40-foot (13.6 m) ISO containers operating in intermodal transports (combined transports) which are allowed a maximum weight of 44 tonnes. Two examples of existing European heavy vehicles are depicted in Figure 2.1. However a number of countries, including Brazil, Canada, Australia, New Zealand, Mexico, Africa and the USA, allow even longer and heavier combinations than those used in Europe.

The Directive 96/53 EC [52] and the European Modular System (EMS) [53] also give the possibility to use longer combination vehicles for each member country in the European Union on the condition that the vehicles are formed by the established EU modules and operate only in national transports. Modular combinations are flexible and may have a varying number of modular



Figure 2.1: Examples of European heavy vehicles

units combined in different order as can be seen in Figure 2.2.

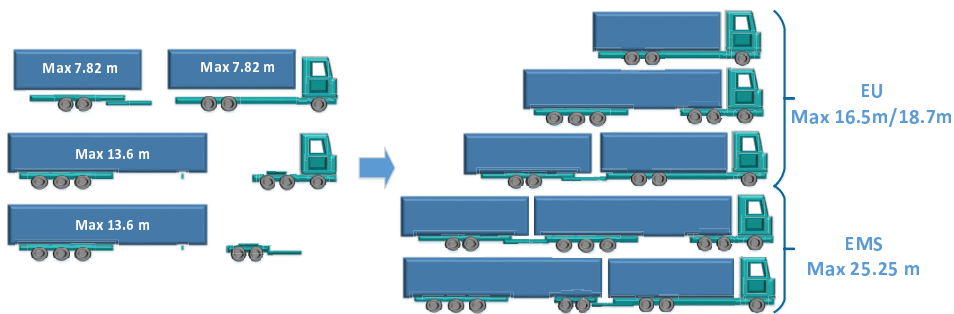


Figure 2.2: European and EMS heavy vehicles [53]

Since 1970, Sweden and Finland are allowed to use modular combination vehicle each carrying one short module (7.82-m long) and one long module (13.6-m long). This leads to a maximum permitted length and weight of 25.25 m and 60 tonnes respectively [17]. Figure 2.3 shows two commonly used EMS vehicles in Sweden. Since 2008, 25.25 m/60 tonnes vehicles are also allowed on the major road network in the Netherlands and on selected main roads in Denmark and Norway [54, 55].

LHCVs refer to modular combination vehicles which are even longer and heavier than the ones currently permitted in Sweden, see Figure 2.4. Since 2009 on-road trials are being conducted in Sweden to investigate the potential economical and environmental benefits of LHCVs. One of the projects is so-called "ETT" (En Trave Till) project [56] which was first proposed by Skogforsk (The Forestry Research Institute in Sweden) in 2006. The on-road trial of the first ETT vehicle with a total length of 30 m and a maximum weight of 90 tonnes was started in 2009. The ETT vehicle was tested within



(a) Truck+semitrailer



(b) Tractor+semitrailer+center axle trailer

Figure 2.3: Examples of EMS heavy vehicles in Sweden

timber haulage industry over more than three years in the northern part of Sweden. The results of using LHCVs showed both transport costs and CO₂ emissions were reduced by 20 % if compared to the use of regular 24 m/60 tonnes timber trucks. No negative impact on road safety was observed and road wear was not increased as the weight was distributed over more axles.



(a) Truck+Double center axle trailer (DUO-CAT)



(b) Tractor+semitrailer+dolly+semitrailer (DUO2, A-double)

Figure 2.4: Examples of longer EMS heavy vehicles on trials in Sweden

Another interesting project so-called "Duo2" started in 2010 was initiated by Volvo AB, DB Schenker and the Swedish Transport Administration along with several other companies [57]. The purpose of the project was to investigate the potential benefits of transporting cargo loads more efficiently on the existing road network. Two double-trailer vehicles, as shown in 2.4, were considered to transport general cargos between Göteborg and Malmö under two years. The combination vehicles used in this project are 32 m long with a maximum weight of 80 tonnes. The Duo2 concept led to a substantial re-

duction of 27 % in terms of distance converged and 8 % reduction in the fuel consumption and CO₂ emissions compared to the current transport structure [58].

One major issue concerning LHCVs is their potential impacts on traffic safety that is the most controversial issue. Road safety performance of LHCVs depends on their technical features such as power train and braking systems capability, lateral dynamical stability, manoeuvrability and etc. By introducing LHCVs as a part of future transportation, there is a need to ensure that they are performing within specific boundary conditions. Defining proper technical characteristics, denoted as Performance Based Standards (PBSs) for LHCVs would assist to create operational requirements by which LHCVs will be allowed to operate in the road network with less negative road safety impacts. In the next section, a comprehensive list of safety related PBSs is introduced and some of these PBSs that are used in this thesis are briefly explained.

2.2 Performance Based Standards

Heavy vehicles are traditionally regulated by tightly-defined prescriptive regulations and rules on their mass and dimensions which provides little space for innovations. In fact, the prescriptive regulations are detailed and inflexible rules that are generally only indirectly related to the desired vehicle performance. On the other hand, the Performance Based Standard (PBS) scheme offers the heavy vehicle industry the possibility to achieve higher safety, productivity and sustainability than current prescriptive regulations through innovative and optimized design of heavy vehicles.

Under the PBS approach, performance measures are utilized to specify the performance required from the vehicle. The PBS scheme has been implemented in New Zealand, Australia, and Canada [59,60,60–62]. Safety related standards included in PBS scheme can be divided into two main groups, longitudinal and lateral characteristics. The PBSs are listed as follows [61, 63]:

- Longitudinal measures: startability, gradeability, acceleration capability, stopping distance, down-grade holding capability,
- Lateral measures: rearward amplification (RA), high speed transient offtracking (HSTO), high speed offtracking, swept path width, high

speed steady-state offtracking, yaw damping coefficient, straight line offtracking, lateral clearance time, steady-state rollover threshold, deceleration capability in a turn

Startability and gradeability characteristics indicate the ability of the vehicle combination to start from rest on an up-grade and maintain speed on an up-grade, respectively. Acceleration capability reflects the vehicle's ability to clear intersections and rail crossings etc. These three characteristics are power train- and tire-related characteristics. Stopping distance and down-grade holding capability characteristics concern braking system. Steady-state rollover threshold, yaw damping ratio and deceleration capability in a turn are vehicle combination's characteristics reflecting the vehicle lateral stability.

Rearward amplification (RA), high speed offtracking and straight line offtracking indicate the trailers's dynamic characteristics. Swept path width is concerning the vehicle combination manoeuvrability and insuring that the vehicle safely manoeuvres around corners. Compared to Australian PBSs, some new characteristics are added to the above list such as stopping distance, down-grade holding capability, lateral clearance time, high speed steady-state offtracking and deceleration capability in a turn (for further information see [63]).

While making a sudden lateral movement in a heavy vehicle, each unit in the combination experiences different lateral acceleration which is amplified towards the rearmost unit of the vehicle as shown in Figure 2.5. The RA is used to quantify this dynamical behaviour of heavy vehicles. The RA is defined as the ratio of the maximum value of the motion variable of interest (e.g. yaw rate or lateral acceleration) of the worst excited towed vehicle unit to that of the first vehicle unit during a specified manoeuvre at a certain friction level and constant speed. Lower values of the RA indicates better lateral performance of heavy vehicles [61, 63].

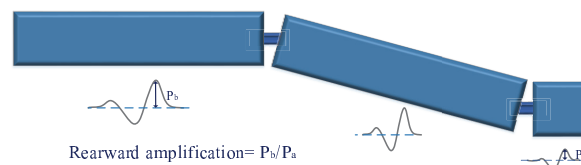


Figure 2.5: Illustration of RA

When an articulated heavy vehicle is negotiating with a turn or performing a lane-change at high speeds, there is a tendency for the rear axles to sway outside of the front axle's path. This tendency to sway outward is called high speed offtracking or outboard offtracking and can be either determined in a steady-state turn or in a lane change maneuver; the latter is called as high speed transient offtracking (HSTO). The HSTO is defined as the lateral deviation between the path of the front axle of the lead unit and the path of the rearmost axle in the towed vehicle unit in a lane change maneuver, as shown in Figure 2.6. A high value of this overshoot might lead to collision with the road objects or other vehicles especially when the lane width is narrow and traffic flow on the road is high [31].

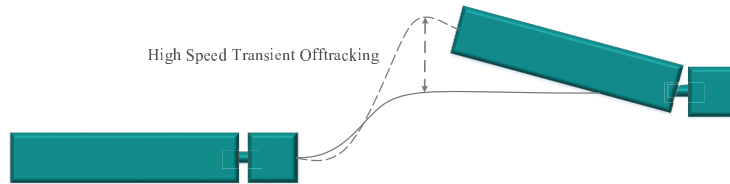


Figure 2.6: Illustration of HSTO

In this thesis, the RA and HSTO measurements are used to evaluate the high-speed lateral performance of candidate LHCVs.

2.3 Vehicle Dynamic Modelling

Vehicle models and tire models should capture the most relevant dynamics and reflect reality. Both linear and nonlinear models are commonly used. However, linear models are well suited for control design purposes, while nonlinear models are more often used to verify the controllers and evaluate them. The linear models have a limited operating range due to assumptions and simplifications that are made when they are designed. In this thesis, the simplest linear vehicle model is used which has two degree-of-freedom representing the lateral and yaw motions of the lead vehicle unit and one degree of freedom for each towed vehicle unit representing its yaw motion. The nonlinear high-fidelity vehicle models used in this thesis are denoted as Volvo Transportation Models (VTM) and are developed and well-tested against numerous test data by Volvo Group Trucks Technology [64–67].

To derive the equations of motion of the linear vehicle model for multi-unit vehicles, there are usually two kinds of forces applied to the system that should be considered: the given forces generated by the actuators and the constraint forces resulted from the interaction between the vehicle units. The methods used in the derivation of the linear models are based on either the Newtonian approach or the Lagrangian approach. The Newtonian approach is more practical if all given forces and constraint forces acting on the vehicle units are known, and the Lagrangian approach is more convenient if the potential and kinetic energies of the units are known. Despite their differences, they are equivalent to each other and result in equivalent descriptions of the dynamics. The Lagrangian mechanics formulation has the advantage over the Newtonian mechanics because it can eliminate the coupling constraints and forces at the coupling joints between the vehicle units that are usually unknown. In addition, the number of equations are fewer in the Lagrangian approach [68]. Therefore in this thesis, the Lagrangian approach is employed in order to derive the equations of motion. In the derivation of the linear vehicle model, the longitudinal velocity v_x is considered as a constant value and all angles are assumed to be small.

The Lagrangian approach is briefly described here. The development of this approach is based on defining the Lagrange function $L(q, \dot{q})$ in terms of generalized coordinates q and their time derivatives \dot{q} . The Lagrangian is defined as the difference between the total kinetic energy T and the total potential energy U of the system, i.e. $L(q, \dot{q}) = T(q, \dot{q}) - U(q)$. The Lagrangian equations are then obtained by differentiating the Lagrange function $L(q, \dot{q})$ with respect to the generalised coordinates q_i and their time derivatives as

$$\frac{d}{dt} \frac{\partial L(q, \dot{q})}{\partial \dot{q}_i} - \frac{\partial L(q, \dot{q})}{\partial q_i} = Q_i, \quad i = 1, \dots, n_q, \quad (2.1)$$

where n_q is the number of generalized coordinate and Q_i represent the generalized external forces associated with the corresponding generalised coordinates q_i . The vector of generalized coordinates for n-unit vehicles is formed as

$$q(t)^T = [Y_1 \ \varphi_1(t) \ \theta_1(t) \ \theta_2(t) \ \dots \ \theta_j(t)], \quad j = 1, \dots, n - 1, \quad (2.2)$$

where Y_1 and φ_1 are the lateral displacement and the yaw angle of the COG of the lead unit, respectively. The remaining elements are the articulation angles between the attached units, whereas θ_j is the articulation angle between the units j and $j + 1$. The schematic diagram of the linear vehicle model for a 3-unit vehicle is depicted in Figure 2.7, where the driven axles of the lead

unit and the axles of the towed units are lumped together into a single axle in the middle position of each unit's axle group.

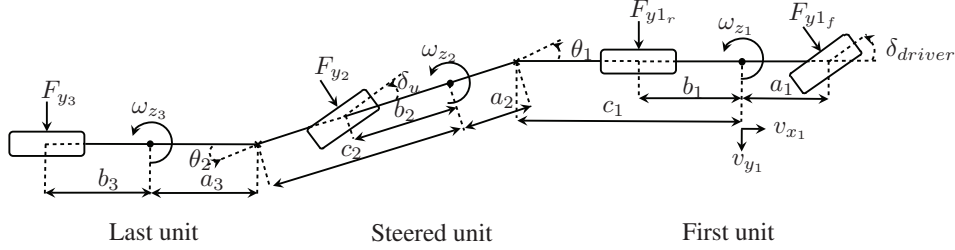


Figure 2.7: Simplified bicycle model of a 3-unit vehicle

Since only the planar motion is considered in the linear model, the potential energy is set to zero (i.e. $U = 0$). The kinetic energy is then calculated as the total sum of the translational and rotational kinetic energies of the system. The kinetic energy of the system with n units is thus obtained as

$$T = \frac{1}{2} \sum_{j=1}^n (m_j v_j^2 + I_{zj} \omega_{zj}^2), \quad j = 1, \dots, n, \quad (2.3)$$

where v_j is the translational velocity, ω_{zj} the yaw rate (yaw velocity), m_j the mass and I_{zj} the yaw moment of inertia of the vehicle unit j . The generalized forces Q_i are given by

$$Q_i = \sum_{k=1}^{n_k} F_k \frac{\partial r_k}{\partial q_i}, \quad (2.4)$$

where n_k is the number of forces and F_k 's are the tire forces with the position vector r_k . In the bicycle vehicle model, it is assumed that the axles in each axle group are combined together in the center of the axle group, therefore F_k 's actually mean the tire forces on each axle. The force F_k has two components; longitudinal force F_{xk} and lateral force F_{yk} . Since it is assumed that the vehicle is travelling with a constant longitudinal velocity, the longitudinal acceleration is thus zero. Consequently, the longitudinal tire forces are equal to zero (i.e. $F_{xk} = 0$).

To construct the equations of motion of the vehicle, it is often more convenient to transform the velocities in the global (inertial) frame to the vehicle-fixed frame. The coordinates in the vehicle-fixed frame is more useful, since

these coordinates are more likely to be available by either direct measurements or estimations in the real vehicle. To this end, the following transformation matrix is performed

$$\mathbf{R}_2(\phi) = \begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{bmatrix}. \quad (2.5)$$

The tire forces including the lateral and longitudinal forces are expressed as

$$F_k^T = [F_{x_k} \ F_{y_k}] \mathbf{R}_2(\varphi_k + \delta_k), \quad k = 1_f, 1_r, 2, \dots, n, \quad (2.6)$$

where δ_k is the steering angle for the steered axles and it is zero for unsteered axles. The subscripts 1_f , 1_r , 2 , and n denote the front axle of the first vehicle unit, the rear axles of the first vehicle unit, the axle group in the second vehicle unit and the axle group in the last vehicle unit, respectively.

A linear time-invariant (LTI) tire model is often used in this model assuming that the tire behaves linearly up to a certain slip angle. Hence, the lateral tire force (F_{y_k}) on each axle is considered as a linear function of the lateral slip angle (α_{y_k}) and defined as

$$F_{y_k} = C_{\alpha_k} \alpha_{y_k}, \quad k = 1_f, 1_r, 2, \dots, n, \quad (2.7)$$

where the proportionality constant C_{α_k} is the sum of the cornering stiffness of the tires on each axle group and is determined by the slope of F_{y_k} versus α_{y_k} curve at $\alpha_{y_k} = 0$. The lateral slip angle is described by

$$\alpha_{y_k} = -\arctan\left(\frac{v_y}{v_x}\right) + \delta_k, \quad k = 1_f, 1_r, 2, \dots, n, \quad (2.8)$$

where δ_k is the steering angle of the steered axles and for the first axle of the lead unit is equal to δ_{driver} , nonzero and to be designed for the steered axles in the towed units, and zero for the un-steered axles.

The position vectors r_k are expressed relative to the position vector $r_1 = [X_1, Y_1]$ defined at the center of gravity (COG) of the first vehicle unit as

$$\begin{aligned} r_{1_f} &= r_1 + \begin{bmatrix} a_1 & 0 \end{bmatrix} \mathbf{R}_2(\varphi_1), \\ r_{1_r} &= r_1 - \begin{bmatrix} b_1 & 0 \end{bmatrix} \mathbf{R}_2(\varphi_1), \\ r_2 &= r_1 - \begin{bmatrix} c_1 & 0 \end{bmatrix} \mathbf{R}_2(\varphi_1) - \begin{bmatrix} (a_2 + b_2) & 0 \end{bmatrix} \mathbf{R}_2(\varphi_2), \\ r_k &= r_{k-1} - \begin{bmatrix} (a_k + b_k) & 0 \end{bmatrix} \mathbf{R}_2(\varphi_k), \quad k = 3, \dots, n, \end{aligned} \quad (2.9)$$

where a_k , b_k and c_k in each unit are the distance between the COG and the front coupling joint, the distance between the COG and the center axle group

and the distance between the COG and the rear coupling joint, respectively. Since there is no front coupling joint in the first unit and the rear coupling joint in the last unit of the vehicle, therefore a_1 for the first unit is the distance between the COG and the front axle and c_n and b_n for the last units are equal, see Figure 2.7. φ_k denotes the experienced yaw angle in the vehicle unit k .

The translational velocities of different units used in equation (2.3) are defined in the corresponding COG of each vehicle unit and are calculated as

$$v_k = \frac{dr_k}{dt}, \quad k = 1_f, 1_r, 2, \dots, n, \quad (2.10)$$

whereas v_k is the time derivative of r_k defined in (2.9). The yaw rate ω_{zk} and yaw angle φ_k of the vehicle unit k are expressed in the coordinate system of the first unit ($\omega_{z1} = \dot{\varphi}_1$) as

$$\begin{aligned} \omega_{zj} &= \omega_{z1} + \dot{\theta}_1 + \dots + \dot{\theta}_k, \\ \varphi_{zj} &= \varphi_{z1} + \theta_1 + \dots + \theta_k. \end{aligned} \quad (2.11)$$

By using the small-angle approximation, the following simplifications can be applied: $\sin(\theta) \approx \theta$, $\cos(\theta) \approx 1$ and $\arctan(\theta) \approx \theta$. As the last step of the derivation of the model based on Lagrangian formulation, one needs to insert the equations (2.4) and (2.3) in (2.1) with the considered positions, velocities and forces. Finally, the following equation is obtained

$$\mathcal{M}_q \ddot{q}(t) + \mathcal{C}_q \dot{q}(t) + \mathcal{K}_q q(t) = \mathcal{B}_q \delta_u(t) + \mathcal{H}_q \delta_{driver}(t). \quad (2.12)$$

The input signal $\delta_u \in \mathbb{R}^{n_u}$ is the control input vector and $\delta_{driver} \in \mathbb{R}^{n_d}$ is the driver steering input, respectively. In our case, the considered control input δ_u is the steering angles to be designed and applied to the axles of the steerable vehicle units. The matrix $\mathcal{M}_q \in \mathbb{R}^{n_q \times n_q}$ is the inertial matrix containing all of the inertial information of the system which is usually symmetric and positive definite, $\mathcal{C}_q \in \mathbb{R}^{n_q \times n_q}$ is the damping matrix and is a velocity-dependent matrix, $\mathcal{K}_q \in \mathbb{R}^{n_q \times n_q}$ is the stiffness matrix, $\mathcal{B}_q \in \mathbb{R}^{n_q \times n_u}$ and $\mathcal{H}_q \in \mathbb{R}^{n_q \times n_d}$ are the force distribution matrices related to the external disturbance forces and the control input forces applied to the system.

Now considering the state vector as $x_q^T \triangleq [q^T \quad \dot{q}^T]$, the state-space model of the system of (2.12) in can be written as follows:

$$\underbrace{\begin{bmatrix} \dot{q}(t) \\ \ddot{q}(t) \end{bmatrix}}_{\dot{x}_q(t)} = \underbrace{\begin{bmatrix} 0 & I \\ -\mathcal{M}_q^{-1} \mathcal{K}_q & -\mathcal{M}_q^{-1} \mathcal{C}_q \end{bmatrix}}_{\mathcal{A}} \underbrace{\begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix}}_{x_q(t)} + \underbrace{\begin{bmatrix} 0 \\ \mathcal{M}_q^{-1} \mathcal{B}_q \end{bmatrix}}_{\mathcal{B}} \delta_u(t) + \underbrace{\begin{bmatrix} 0 \\ \mathcal{M}_q^{-1} \mathcal{H}_q \end{bmatrix}}_{\mathcal{H}} \delta_{driver}(t). \quad (2.13)$$

Thanks to the structure of the matrix \mathcal{K}_q , two states Y_1 and φ_1 are removed from x_q to obtain the state-space model to be used in the controller design. As a result, the state vector of the vehicle model $x \in \mathbb{R}^{n_x}$ is formed as

$$x = [\theta_1 \dots \theta_n \ v_{y1} \ \dot{\varphi}_1 \ \dot{\theta}_1 \dots \dot{\theta}_n]^T. \quad (2.14)$$

By removing the relevant row blocks from all matrices and also the relevant column blocks from \mathcal{A} , \mathcal{B} and \mathcal{H} , the dynamics of the system are expressed as

$$\dot{x}(t) = A x(t) + B \delta_u(t) + H \delta_{driver}(t), \quad (2.15)$$

where $A \in \mathbb{R}^{n_x \times n_x}$ is the state matrix, $H \in \mathbb{R}^{n_x \times n_d}$ is related to the disturbance inputs and $B \in \mathbb{R}^{n_x \times n_u}$ represents the input matrix. The obtained state-space description is used for the purpose of synthesizing controllers throughout this thesis. The reader is referred to [69, 70] for more mathematical details on the derivation of the linear vehicle model.

Chapter 3

Controller Synthesis based on LMI Optimization

This chapter provides an introduction to the use of linear matrix inequalities (LMIs) in the analysis and synthesis of the control systems used in this thesis. Some basic materials and techniques used in this chapter are briefly introduced in Appendix A.

3.1 Introduction to Linear Matrix Inequalities (LMIs)

Linear matrix inequalities (LMIs) and LMI techniques have recently emerged as powerful analysis and control design techniques to a wide variety of engineering problems. LMIs are an important class of convex optimization problems which are numerically tractable problems. Many engineering optimization problems can be translated into LMI problems and be solved numerically using recently developed interior-point algorithms in an efficient and practical manner [71]. The basic idea of the LMI method is to translate or approximate a given analysis or synthesis problem into a convex optimization problem with linear objective and linear inequality constraints. In general, there are two main cases subjected to the study of LMIs; feasibility problem and optimization problem [72–74].

3.1.1 Feasibility problem

In these kinds of problems, the interest is only on testing whether there exists a feasible solution that renders the considered LMI constraint satisfied. In this case, the LMI constraint is a convex constraint on a vector $x \in \mathbb{R}^n$ of the

form

$$F(x) := F_0 + \sum_{i=1}^n x_i F_i \prec 0, \quad (3.1)$$

where x is the vector of decision variables and $F_i \in \mathbb{R}^{m \times m}$ are given real symmetric matrices. The inequality in (3.1) means that the vector of the decision variables x should render the symmetric matrix $F(x)$ negative definite. In other words, the largest eigenvalue of $F(x)$ should be negative. The feasibility problem is testing the existence of the vector x such that the inequality in (3.1) holds. The problem is feasible if there exists a solution x , otherwise it is said to be infeasible.

As a very important property, multiple LMI constraints can be regarded as a single LMI since

$$F_1(x) \prec 0, F_2(x) \prec 0, \dots, F_n(x) \prec 0, \quad (3.2)$$

is equivalent to

$$F(x) = \text{diag}(F_1(x), F_2(x), \dots, F_n(x)) \prec 0, \quad (3.3)$$

where $F(x)$ denotes the block-diagonal matrix with $F_1(x), F_2(x), \dots, F_n(x)$ on its diagonal. As well-known, a block-diagonal symmetric matrix is negative (semi) definite if and only if its diagonal blocks are negative (semi) definite.

As an example, consider the problem of determining the asymptotic stability of an autonomous dynamical system given as

$$\dot{x}(t) = Ax(t), \quad (3.4)$$

where $x \in \mathbb{R}^{n_x}$ is the state vector and $A \in \mathbb{R}^{n_x \times n_x}$ is the system matrix. In the beginning of 1890, Lyapunov showed that the system (3.4) is asymptotically stable if there exists a quadratic Lyapunov function $V(x) = x^T P x$ with a Lyapunov matrix as $P = P^T \in \mathbb{R}^{n_x \times n_x} \succ 0$ such that $dV(x)/dt \prec 0$ along the trajectories of (3.4). Equivalently, the system is asymptotically stable if and only if there exists a symmetric matrix P such that

$$\begin{aligned} A^T P + P A &\prec 0, \\ P &\succ 0. \end{aligned} \quad (3.5)$$

Obviously, this inequality is nothing else than an LMI feasibility problem. The Lyapunov inequality is clearly in the form of (3.2) and can be casted to

a single LMI as in (3.3) given as

$$\begin{bmatrix} -P & 0 \\ 0 & A^T P + P A \end{bmatrix} \prec 0. \quad (3.6)$$

It is easy to show that the Lyapunov inequality in (3.5) is equivalent to the feasibility of the LMI constraint in (3.6) [72–74].

3.1.2 Optimization problem

A Semidefinite Programming (SDP) problem is a convex optimization problem of the form

$$\begin{aligned} & \text{minimize } c^T x := \sum_{i=1}^n c_i x_i \\ & \text{subject to } F(x) := F_0 + \sum_{i=1}^n x_i F_i \prec 0. \end{aligned} \quad (3.7)$$

This problem involves the determination of minimum $c^T x$ over all $x \in \mathbb{R}^n$ that satisfy the given LMI constraints. The feasibility problem can be viewed as a special case of the optimization problem (3.7) for $c = 0$. The SDP is a very natural generalization of linear programming (LP), where the componentwise inequalities between vectors are replaced by matrix inequalities [75].

As an example, consider calculating the \mathcal{H}_∞ norm of an LTI system (see Appendix A) described as

$$\begin{cases} \dot{x}(t) = Ax(t) + Hw(t), \\ z(t) = Cx(t) + Gw(t), \end{cases} \quad (3.8)$$

where $x \in \mathbb{R}^{n_x}$ is the state vector, $w \in \mathbb{R}^{n_w}$ the external disturbance input and $z \in \mathbb{R}^{n_z}$ the performance output. The system matrices A , H , C and G are constant matrices of appropriate dimensions. The \mathcal{H}_∞ norm of this system can be found by solving the following optimization problem for $P \succ 0$

$$\begin{aligned} & \text{minimize } \gamma \\ & \text{subject to } \begin{bmatrix} A^T P + P A & P H & C^T \\ H^T P & -\gamma I & G^T \\ C & G & -\gamma I \end{bmatrix} \prec 0. \end{aligned} \quad (3.9)$$

Many control problems can be defined as optimization problems with LMI constraints, for instance \mathcal{H}_∞ -type static state-feedback and static output-feedback syntheses that will be discussed in the sequel.

3.2 \mathcal{H}_∞ Synthesis Methods

\mathcal{H}_∞ control theory was designed to reduce modeling errors and unknown disturbances in a system, while providing quantifiable optimization of large scale multi-variable problems. The general setting of the \mathcal{H}_∞ synthesis problem can be stated as follows.

Given a linear time-invariant (LTI) plant P described with a state-space description

$$\begin{cases} \dot{x}(t) &= Ax(t) + Hw(t) + Bu(t), \\ z(t) &= Cx(t) + Gw(t) + Du(t), \\ y(t) &= Sx(t) + Rw(t), \end{cases} \quad (3.10)$$

where the system signals are: $x \in \mathbb{R}^{n_x}$ the state vector, $w \in \mathbb{R}^{n_w}$ the external disturbance input, $u \in \mathbb{R}^{n_u}$ the control input, $z \in \mathbb{R}^{n_z}$ the performance output, $y \in \mathbb{R}^{n_y}$ the measurement output. In general, the control input is generated as

$$u(t) = Ky(t), \quad (3.11)$$

where K is the feedback gain to be determined. With the plant P and the controller K , the closed-loop system admits the realization

$$\begin{cases} \dot{x}(t) &= \underbrace{(A + BKS)}_{A_{cl}} x(t) + \underbrace{(H + BKR)}_{H_{cl}} w(t), \\ z(t) &= \underbrace{(C + DK S)}_{C_{cl}} x(t) + \underbrace{(G + DK R)}_{G_{cl}} w(t). \end{cases} \quad (3.12)$$

The aim would be to have:

- **Internal stability:** for $w = 0$, the state vector of the closed loop system in (3.12) tends to zero as time goes to infinity.
- **\mathcal{H}_∞ performance:** the \mathcal{H}_∞ norm of the closed-loop transfer function from w to z (i.e. $\|T_{wz}\|_\infty = \|C_{cl}(sI - A_{cl})^{-1}B_{cl} + D_{cl}\|_\infty$) is minimized for all stabilizing K .

Then, the \mathcal{H}_∞ synthesis problem is formulated as follows:

Problem 1 *Given a linear time-invariant system P as in (3.10), find a feedback gain matrix K such that the closed-loop system (3.12) is stable and the following objective is satisfied for all $w(\cdot)$ with $0 < \|w\|_2 \triangleq \sqrt{\int_0^\infty w(t)^T w(t) dt} < \infty$ when $x(0) = 0$:*

$$\|z\|_2 < \gamma \|w\|_2. \quad (3.13)$$

In this expression, the scalar γ represents the level of guaranteed \mathcal{L}_2 -gain performance and is typically desired to be minimized. On the other words, the γ value is the desired upper bound on the worst-case energy gain from w to z . Indeed, the performance objective of (3.13) is an \mathcal{H}_∞ constraint on the transfer function $\mathcal{T}_{zw}(s) \triangleq \hat{z}(s)/\hat{w}(s)$, where \hat{w} represents the Laplace transformation of w . Indeed, condition (3.13) can be equivalently expressed as

$$\|\mathcal{T}_{zw}\|_\infty \triangleq \sup_{\text{Re}\{s\}>0} \|\mathcal{T}_{zw}(s)\| < \gamma, \quad (3.14)$$

where $\|\cdot\|$ represents the maximum singular value.

In the sequel, the solution of the \mathcal{H}_∞ synthesis problem is provided based on two types of controller, namely, static state-feedback and static output-feedback. With state feedback, all states (e.g., $y = x$) of the system are assumed to be available for use by the controller, whereas with output feedback, a set of output variables (e.g., $y = Sx + Rw$) related to the state variables are available.

3.2.1 \mathcal{H}_∞ state-feedback synthesis

In the synthesis based on state-feedback framework, all the system's states are assumed to be available for measurement. Therefore, the state-space description of the system will be a special case of the system (3.10), in which the measurement output y is equal to the state vector x (i.e. $S = I$ and $R = 0$). Consequently, the control input will be formed as

$$u(t) = Kx(t), \quad (3.15)$$

where $K \in \mathbb{R}^{n_u \times n_x}$ is the control gain matrix to be designed. Obviously the closed-loop system with the state-feedback control input can be characterized as

$$\begin{cases} \dot{x}(t) &= (A + BK)x(t) + Hw(t), \\ z(t) &= (C + DK)x(t) + Gw(t). \end{cases} \quad (3.16)$$

As a results of the bounded real lemma (see Appendix A), the controller stabilizes the system and ensures that the \mathcal{H}_∞ -norm of the closed-loop system is

less than γ if and only if there exists a matrix variable $X = X^T \in \mathbb{R}^{n_x \times n_x} \succ 0$ for which

$$\begin{bmatrix} (A + BK)^T X + X(A + BK) & XH & (C + DK)^T \\ H^T X & -\gamma I & G^T \\ (C + DK) & G & -\gamma I \end{bmatrix} \prec 0. \quad (3.17)$$

As can be seen, the LMI condition depends non-linearly on the decision variables K and X , which makes the LMI condition a bilinear matrix inequality (BMI). In order to arrive at an LMI condition, a congruence transformation (see Appendix A) is applied to (3.17) with block diagonal matrix $\text{diag}(Y, I, I)$ in which $Y = X^{-1} \succ 0$. This leads to the following solution for the Problem 1.

Lemma 1 *There is a solution to Problem 1 if and only if there exists a matrix variable $Y = Y^T \in \mathbb{R}^{n_x \times n_x} \succ 0$ for which*

$$\mathcal{N} = \text{He} \begin{bmatrix} AY + BN & H & 0 \\ 0 & -\frac{\gamma}{2}I & 0 \\ CY + DN & G & -\frac{\gamma}{2}I \end{bmatrix} \prec 0, \quad (3.18)$$

where $\text{He } \mathcal{M} \triangleq \mathcal{M} + \mathcal{M}^T$ and $N = KY$.

By solving the optimization problem and obtaining the matrix variables N and Y , the state-feedback gain matrix K can be easily constructed through $K = NY^{-1}$.

It should be noted that in the state feedback techniques, it is required either to have the measurement of every system's state some of which might be expensive or even impossible to be measured, or to use the observer-based controllers which makes the implementation task expensive and hard.

3.2.2 \mathcal{H}_∞ static output-feedback synthesis

In many applications, having access to the full state information is quite uncommon situation and usually the information available for feedback purposes consists of a reduced set of the states or a linear combination of the states. In this context, static output-feedback (SOFB) controllers represent a very interesting option due to their conceptual simplicity and ease in practical implementation. Therefore, in this section the solution of the \mathcal{H}_∞ synthesis problem is provided based on the SOFB.

Let us now consider the \mathcal{H}_∞ synthesis problem for the system (3.10). The aim of the SOFB control problem is to find a constant matrix gain $K \in \mathbb{R}^{n_u \times n_y}$ defined by the control law given as in (3.11) that stabilizes the closed-loop system and guarantees that the \mathcal{H}_∞ -norm of the transfer matrix \mathcal{T}_{zw} is less than γ . By inserting (3.11) in (3.10), the closed-loop description of the system can be presented as in (3.12).

In order to solve Problem 1, the approach proposed in [76] is used, in which sufficient solvability conditions expressed in the form of dilated LMIs. The basic idea behind the dilated LMIs is to not use the Lyapunov matrix X in the construction of the feedback gain and as a result this will lead to effectively decreasing conservatism in robust output feedback synthesis. It is hence assumed a feedback gain matrix as

$$K = NW^{-1}, \quad (3.19)$$

where $N \in \mathbb{R}^{n_u \times n_y}$ and invertible matrix $W \in \mathbb{R}^{n_y \times n_y}$ are two new matrix variables to be determined. In this fashion, the control input is decoupled from the Lyapunov matrix X . The relevant solution of Problem 1 is summarized as follows:

Lemma 2 *There is a solution to Problem 1 if there exist matrix variables $0 \prec X = X^T \in \mathbb{R}^{n_x \times n_x}$, $W \in \mathbb{R}^{n_y \times n_y}$ and $N \in \mathbb{R}^{n_u \times n_y}$ for which*

$$\mathcal{N} = \text{He} \begin{bmatrix} -\phi W & \phi(SX - WS) & \phi R & 0 \\ BN & AX + BNS & H & 0 \\ 0 & 0 & -\frac{\gamma}{2}I & 0 \\ DN & CX + DNS & G & -\frac{\gamma}{2}I \end{bmatrix} \prec 0, \quad (3.20)$$

where $\phi \in \mathbb{R}_+$ is an arbitrary (and yet fixed) scalar.

The SOFB gain matrix is then computed as in (3.19). The reader is referred to [76] for further details and derivations.

3.3 Robust Control Synthesis

Most control engineering problems can be solved by using classical control methods. It is possible to obtain perfectly satisfactory performance in many engineering applications using just PID controllers, particularly when the system is almost linear and its mathematical model is precise. But there

is always a risk of modelling errors and uncertainties. These modelling errors and uncertainty can arise from real parameter variations, unmolded or incorrectly modeled dynamics, and neglected nonlinearities and external disturbances. To solve these types of control problems, there is a need for more powerful tool than the classical control. In this context, robust control theory is developed as an extension to optimal and modern control methods. Robust control methods aim to achieve robust performance and/or stability in the presence of bounded modelling errors and uncertainties [77].

In order to synthesize a robust controller, the ideas and methods presented in the previous section can be extended to uncertain parameter-dependent (time-invariant as well as time-varying) systems and can be used for synthesizing constant as well as parameter-dependent controller gains that ensure robust performance. Let assume a parameter-dependent system described by

$$\begin{cases} \dot{x}(t) &= A(\delta)x(t) + H(\delta)w(t) + B(\delta)u(t), \\ z(t) &= C(\delta)x(t) + G(\delta)w(t) + D(\delta)u(t), \\ y(t) &= S(\delta)x(t) + R(\delta)w(t), \end{cases} \quad (3.21)$$

where the system matrices depend on an uncertain parameter vector $\delta \in \Delta$ and Δ denotes a compact (i.e. closed and bounded) uncertainty set. In robust synthesis problems for such systems, the uncertain parameter vector (and if relevant its derivative) is assumed to take values from a compact region denoted as Δ . The synthesis LMIs in the previous section clearly become parameter-dependent when dealing with uncertain parameter-dependent systems, i.e. $\mathcal{N}(\delta) \prec 0, \forall \delta \in \Delta$. Here the uncertain parameters are assumed to be time-variant parameters.

As a result, the parameter-dependent LMIs are required to be satisfied over the whole uncertainty region. This eventually leads to infinitely many LMIs, i.e. one LMI for each point of the uncertainty region. In order to formulate tractable optimization problems, finitely many LMI conditions are needed to ensure that the parameter-dependent LMIs are satisfied over the whole uncertainty region. This is done by employing a so-called relaxation scheme [78] that is suitable to use for the particular type of parameter dependency (e.g. affine, quadratic, general polynomial, or rational) in the LMI conditions.

As is employed in our design methods in this thesis, it is briefly described here how problems can be formulated in the simplest case of affine parameter dependence. Let us represent the uncertain parameter vector as $\delta = [\delta_1 \delta_2 \cdots \delta_q]^T \in \mathbb{R}^q$. Assume that δ is time-invariant and each δ_i is assumed

to take values from a known finite interval $[\underline{\delta}_i, \bar{\delta}_i]$. This implies that the uncertainty region (represented as Δ) is identified as a hyper-rectangle called a parameter box:

$$\Delta \triangleq \{\delta \in \mathbb{R}^q : \underline{\delta}_i \leq \delta_i \leq \bar{\delta}_i, i = 1, 2, \dots, q\}. \quad (3.22)$$

The set of extreme points (i.e. the vertices of this parameter box) are represented as

$$\Delta_v \triangleq \{\delta \in \mathbb{R}^q : \delta_i \in \{\underline{\delta}_i, \bar{\delta}_i\}\} = \{\delta^1, \delta^2, \dots, \delta^{2^q}\}. \quad (3.23)$$

Let us now consider an LMI that has affine parameter dependence as follows

$$\mathcal{N}(\delta) = \mathcal{N}_0 + \mathcal{N}_1\delta_1 + \dots + \mathcal{N}_q\delta_q \prec 0, \forall \delta \in \Delta. \quad (3.24)$$

Since the parameter dependence is affine and the uncertainty region is polytopic, it is necessary and sufficient to impose the LMI condition only on the vertices, i.e.

$$\mathcal{N}(\delta^j) \prec 0, j = 1, \dots, 2^q. \quad (3.25)$$

Thanks to this equivalence, infinitely many LMIs in (3.24) are replaced by the 2^q LMI conditions in (3.25). It should be noted that the matrix variables that are not used in the computation of the controller parameters can be chosen to depend on all uncertain parameters (i.e. measurable and un-measurable). If there are some uncertain parameters that are measurable during online operation, in order to have a potentially less conservative design, one can choose the matrix variables used in the calculation of the controller gain to have dependence on these parameters. In this case, the resulting controller will be parameter-dependent and hence can be scheduled during online operation.

Chapter 4

Summary of Included Papers

This chapter provides a short summary of the appended papers that constitute the base for this thesis. Papers A-D are focused on improving the lateral performance of an A-double heavy vehicle by active steering of the dolly axles at high speeds. Paper E is focused on synthesizing a gain-scheduled controller design for a selected set of longer and heavier commercial vehicles (LHCVs). All the syntheses are performed via linear matrix inequality (LMI) optimizations.

4.1 Paper A: \mathcal{H}_∞ Static Output Feedback Synthesis under an Integral Quadratic Constraint with Application to High Capacity Transport Vehicles

This paper proposes an LMI-based approach for \mathcal{H}_∞ static output feedback (SOFB) synthesis under a novel integral quadratic constraint (IQC) on the states. The main idea of imposing the IQC constraint is to minimize the energy to energy gain of states. As a particularly relevant application, the proposed approach is considered for minimizing the yaw rate rearward amplification (RA) in order to enhance the lateral performance of the A-double. The RA is defined as the peak to peak gain of the yaw rates. Hence by applying the derived IQC on the yaw rates, the RA values of the towed units are indirectly reduced. The verification results confirm significant reductions in the yaw rate RA and high speed transient off-tracking (HSTO) in the last semitrailer.

4.2 Paper B: Robust Control of an A-double with Active Dolly based on Static Output Feedback and Dynamic Feed-Forward

In this paper, a robust controller synthesis based on SOFB combined with dynamic feed-forward (DFF) is presented. The controller is designed to ensure an \mathcal{H}_∞ performance objective in the face of parametric uncertainty in the dolly steering actuator. The steering actuator is simply modeled by a first order filter characterized by a time constant and a transport delay which are assumed to be uncertain in a known bounded range. It is observed that the lateral performance of the A-double is improved significantly when the DFF from the driver steering angle accompanies the static feedback from the articulation angles if compared to the case in which DFF is not applied.

4.3 Paper C: Robust static output feedback with dynamic feed-forward for lateral control of long-combination vehicles at high speeds

The focus of this paper is on a controller synthesis based on SOFB combined with DFF in order to ensure robustness against cornering stiffness uncertainty in the tire parameters. In this study, the cornering stiffness coefficients are treated as time-invariant parameters with known lower and upper bounds. Two alternative DFF designs are proposed which are applicable to known systems as well as uncertain parameter-dependent systems. The theoretical novelty of this paper mainly lies in the derivation of a set of new sufficient LMI conditions for the first DFF design method in order to guarantee the required performance objectives. The second DFF method is adapted from previous relevant works by including a weighting filter acting as a simple driver model. In this approach, only the measurement of lead unit's steering angle and one articulation angle are required for the DFF and the SOFB controllers, respectively. The results are verified using a high-fidelity vehicle model and confirm a significant reduction in the yaw rate RA, and HSTO as a byproduct, thereby improving the lateral stability and performance of the A-double during sudden lane change manoeuvres.

4.4 Paper D: Robust Lateral Control of Long-combination Vehicles under Moments of Inertia and Tire Cornering Stiffness Uncertainties

In this paper, the synthesis of a robust steering-based controller is presented for the A-double. The main purpose of this controller is to achieve robust stability and robust performance in the presence of uncertainties in the cornering stiffness of the tires and the moments of inertia of the semitrailers. In order to have a more accurate tire model than the one used in the previous paper and capture some important un-modeled tire dynamics, a linear time-varying (LTV) tire model is used, in which the cornering stiffness is considered as a time-varying uncertain parameter. In addition, the moments of inertia enter rationally in the system matrices of the linear vehicle model. Therefore, in order to avoid this rational dependency, a descriptor-type representation of the system is employed which is more convenient to work with, compared to the standard state-space description of the system. The controller synthesis is then formulated as an LMI-based \mathcal{H}_∞ -type SOFB for the systems that have affine parameter dependence in the descriptor form. The controller uses information from the second articulation angle which is easily measurable. The driver steering input is also taken into account by including a static feed-forward in the formulation. The effectiveness of the controller is evaluated in both frequency and time domains. The simulation results obtained from a high-fidelity vehicle model show a significant improvement in the high-speed lateral performance of the A-double that can be achieved by the controller in presence of parametric uncertainties.

4.5 Paper E: Gain-scheduled \mathcal{H}_∞ Controller Synthesis for Actively-steered Longer and Heavier Commercial Vehicles

This paper proposes an LMI-based design technique that allows to systematically design gain-scheduling (GS) controllers for high speed lateral control of a selected set of heavy vehicles; A-double, A-triple, truck-dolly-semitrailer and truck-double center-axle trailer. The proposed GS controller synthesis with the vehicle longitudinal velocity as the GS parameter guarantees both stability and performance, and avoids the troublesome interpolation step used in the classical GS design approach. The controller steers the axles of the selected towed units of each vehicle to enhance its lateral stability and performance at high speeds. By active steering of the selected towed units, a

significant reduction is observed in the yaw rate RA, as well as reduction in the lateral acceleration RA and HSTO, as byproducts.

Chapter 5

Concluding Remarks and Future research Directions

This chapter contains some concluding remarks and also some ideas for potential future works.

5.1 Concluding Remarks

In multi-unit heavy vehicle combinations, each vehicle unit in the combination might amplify the yaw rate or the lateral acceleration of the unit ahead of it in severe maneuvers at high speeds. High amplification would clearly lead to higher risks of accidents and rollover. Therefore, it is desired to limit or decrease this amplification, that can be quantified by measuring the rearward amplification (RA), to a desired level to meet some performance based standards. By active steering of the towed units at high speeds, the required lateral force for yaw motion regulation is generated earlier before the increase in the yaw rates. In fact, the RA is suppressed by preemptive action. Furthermore, since the lateral forces are generated due to the imposed steer angles (rather than the side slip of the tires), the side slip angle of the towed units and consequently off-tracking will also be decreased significantly without degrading the vehicle's maneuverability.

It should be noted that there is usually a trade-off relationship between the low-speed maneuverability and high speed lateral stability, which means that what improves the low-speed performance is likely to degrade the high speed performance and vice versa. Hence, the proposed controllers should be turned off at low speeds.

5.2 Recommendations for Future Work

In this study, several robust control syntheses are proposed for improving the yaw and lateral dynamic performance of LHCVs (mainly the A-double). However, the rollover issues are not included in this research completely, but the developed controller could prevent the roll instabilities occurred due to high lateral acceleration. The functionality of the proposed controller synthesis could be extended by adding the roll dynamics to the vehicle linear model and roll enhancement objectives to the controller synthesis formulation.

Additionally, the integration of the steering and braking controllers should be investigated for potential further improvements.

The developed controllers are based on the assumption that all the axles of the steered vehicle unit are steered equally in the same direction. Further efforts could be dedicated to investigate the advantage of using individual steering at each single axle of the steered vehicle unit.

The driver interaction with the developed controllers is included by considering a simple linear model describing the frequency content of the driver steering in lane change maneuvers. A more advanced linear driver model could be considered in related future works.

As a challenging direction that is particularly relevant for the considered application, the problem in which the IQC constraint is replaced with a peak-to-peak constraint on the states can be considered.

Furthermore, the performance of the control strategies developed in this thesis should be validated using the experimental vehicle.

Appendix A

Preliminaries

Relevant to this thesis, some basic materials and techniques have been presented in this chapter that has been used in the appended papers and also in Chapter 3.

A.1 Schur complement lemma

The Schur complement lemma is very useful since it may convert quadratic matrix inequalities that appears regularly in many control problems (e.g. Lyapunov and Riccati inequalities) into LMIs. According to the Schur complement lemma, the negative-definite square block matrix

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \prec 0. \quad (\text{a.1})$$

is equivalent to the following nonlinear inequalities

$$\begin{cases} R \prec 0 \\ Q - SR^{-1}S^T \prec 0 \end{cases} \quad (\text{a.2})$$

$$\begin{cases} Q \prec 0 \\ R - S^TQ^{-1}S \prec 0 \end{cases} \quad (\text{a.3})$$

where $Q = Q^T$ and $R = R^T$. In other words, the set of nonlinear inequalities in (a.2) and (a.3) can be represented as the LMI in (a.1). A proof of the Schur complement lemma can be found in [79].

A.2 Congruence transformation

A congruence transformation of a square matrix $X = X^T$ is in fact a mapping $X \mapsto Y^T X Y$ with a square nonsingular matrix Y . By applying the congruence transformation, the definiteness property of X doesn't change, i.e.

$$X \prec 0 \text{ if and only if } Y^T X Y \prec 0. \quad (\text{a.4})$$

This transformation leaves also the number of negative/zero/positive eigenvalues of X invariant. The Schur complement lemma and congruence transformation are often the key to transform nonlinear matrix inequalities into LMIs.

A.3 Generalized plant concept

The proposed control syntheses throughout this thesis make use of a closed-loop control system with a general configuration such as the one given in Figure a.1. The subsystem P , called the generalized plant, is assumed to be given and contains the plant, actuators, weights, and uncertainties if there is any. The second subsystem K is referred as the controller and its design must guarantee internal stability of the closed-loop system and make it behave in a desired manner. Interpretations for the various signals depicted in Figure a.1

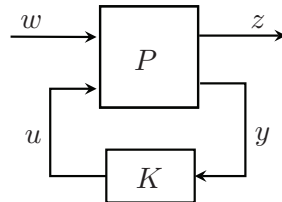


Figure a.1: Block-diagram of general closed-loop configuration

are as follows:

- Disturbance inputs (w) contain all exogenous signals (possibly containing disturbance, reference and measurement noise)
- Performance/controlled outputs (z) are usually signals that should be rendered small, such as the tracking error or control effort.

- Control inputs (u) represent the control inputs to be designed by the controller
- Measured outputs (y) comprise the plant measured outputs accessible to the controller

Most control system problems can be represented with this generalized configuration.

A.4 \mathcal{H}_∞ performance

A popular performance measure of a stable linear time-invariant (LTI) system is the \mathcal{H}_∞ norm of its transfer function. Suppose an LTI system described as

$$\begin{cases} \dot{x}(t) &= Ax(t) + Hw(t), \\ z(t) &= Cx(t) + Gw(t), \end{cases} \quad (\text{a.5})$$

where $x \in \mathbb{R}^{n_x}$ is the state vector, $w \in \mathbb{R}^{n_w}$ the external disturbance input and $z \in \mathbb{R}^{n_z}$ the performance output. The system matrices A , H , C and G are constant matrices of appropriate dimensions. The transfer function of the system from w to z is given by $T_{zw}(s) = C(sI - A)^{-1}H + G$. Since it is assumed that the system is asymptotically stable, T_{zw} is bounded for all $s \in \mathbb{C}$ with positive real part. The \mathcal{H}_∞ norm of the system is given by

$$\|T_{zw}\|_\infty \triangleq \sup_{\text{Re}\{s\} > 0} \sigma_{\max}(T_{zw}(s)) < \infty, \quad (\text{a.6})$$

where $\|\cdot\|$ represents the maximum singular value. In words, the \mathcal{H}_∞ norm of a transfer function is the supremum of the maximum singular values of the frequency response of the system. The \mathcal{H}_∞ norm measures the system input-output gain for finite energy [73, 74].

A.5 Integral quadratic constraint

In some applications, it is desired to enforce certain objectives on transfer functions from states to states, rather than external disturbance to states as in (a.6). Such requirements correspond to integral quadratic constraints (IQC) on the states. In fact, it is a constraint expressed in terms of integrals of norm squares of the signals as follows:

$$\|p\|_2 \leq \sigma \|q\|_2. \quad (\text{a.7})$$

where q and p are states or a function of states of the system (a.5). As derived in [80], an equivalent frequency domain constraint is expressed as

$$\|\mathcal{T}_{pq}(j\omega)\| \leq \sigma, \quad \forall \omega \geq 0, \quad (\text{a.8})$$

where $\mathcal{T}_{pq}(s) \triangleq \mathcal{T}_{pw}(s)\mathcal{T}_{qw}^{-1}(s)$ and w is the external disturbance input to the system.

It should be noted that there is also the IQC approach that is more commonly known in the literature, in which integral quadratic constraints are also involved. The same terminology is used in this thesis to avoid potential confusion. However, the IQC framework is used to facilitate a systematic and efficient stability and performance analysis for a wide variety of uncertain dynamical systems via LMIs [81].

A.6 Bounded Real Lemma

The bounded real lemma converts the \mathcal{H}_∞ performance condition given as $\|\mathcal{T}_{zw}\|_\infty \prec \gamma$ to an equivalent LMI condition.

Now using the bounded real lemma, the system (a.5) is asymptotically stable and $\|\mathcal{T}_{zw}\|_\infty \prec \gamma$, if and only if, there exists a positive symmetric matrix $X = X^T \succ 0$ such that

$$\begin{bmatrix} A^T X + X A & X H & C^T \\ H^T X & -\gamma I & G^T \\ C & G & -\gamma I \end{bmatrix} \prec 0, \quad (\text{a.9})$$

where γ is a positive scalar value. The matrix $X \in \mathbb{R}^{n_x \times n_x}$ is usually called as Lyapunov function matrix [72–74].

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