Fermat Pedagogy – a method to help students make effective lecture notes

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Fermat Pedagogy

– a method to help students make effective lecture notes

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***Abstract*—We have developed and implemented a form of lecturing, which we call Fermat Pedagogy, that promotes an alternative form of student note-taking. We did a comparative study in a basic calculus course if about 300 students randomly divided into two groups, one taught with Fermat Pedagogy and control group.**

We used a diagnostic test and final test to collect data. We report both in the students thoughts about note-taking and also about the level of knowledge in both groups in the beginning and in the end of the course.

The main three opinions about note-taking that we find in the studentsresponse are that note-taking help to keep focus, that the notes provide an shorter text and but that note-taking makes it harder to listen.

The quantitative results indicates that Fermat Pedagogy can help the more motivated students to get a higher grade, but seem also have the effect that the weaker students do worse than they otherwise should have. These indications are not conclusive and need more studies to be verified. This may never the less make a foundation for a discussion on how we might look at our own lectures and how they are perceived, and documented, by our students.

***Index Terms*—Fermat pedagogy, higher education, mathematics, lecture notes**

I. Introduction

Many university teachers experience that their students focus so much on the careful note-taking during lectures, so that this task takes too much focus and the students then miss too much of the "current moment" in the lectures. There seems to be a belief that *"If I copy this correctly, I will understand the context when I go over the notes later"*. This is not always so easy, and there is also a deeply human wish to procrastinate hard tasks. On the other hand, many of us has experience how hard it is to remain focused if we don't have a pen in our hand. This has been supported in the literature, e.g. (Di Vesta and Gray, 1972).

Can the note-taking be developed to better enhance students learning? In search of that we in this study use an alternative form of lecturing, that we call Fermat Pedagogy, that supports margin notes written by the students directly in the course book. Our research question is: How will the Fermat Pedagogy affect the students learning?

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II. Background

One of the challenges in mathematical didactic is the assimilation of abstract concepts. The series of calculus courses today can be seen as a repetition of a few concepts such as limits, derivative and integrals, but where the repetition takes the students deeper and more abstract for each iteration (Lundh, 2010). Compare for example the progression from Riemann sums to Lebesgue integrals, or natural numbers to high dimensional complex numbers. This troublesome process does not only provide new knowledge but old knowledge will be updated. In other words, a sort of a de- and re-programming to set the table for an even more complex and refined milleu for the working memory to create context (Klingberg, 2008, 2011).

What is important for succeeding in learning first year university mathematics is a multidimensional problem (Bengmark et al., 2017). One of the essential factors are the students own actions, of which one is note-taking during lectures. The role of note-taking has historically gone from being a way to create your own course book to a way to support your memory and to give some supplements to the printed course book. Note-taking is considered as an important factor for success among the students but much less so by the lecturers (Anthony, 2000).

The authors experience as lecturers is that students sometimes get so caught up with note-taking that they are not able to also focus on the ideas that are presented. But we also appreciate that taking notes can help the student focus. So, to keep the note-taking as a way to keep focus, but at the same time avoiding unreflected board copying we have encouraged marginal notes as a golden middle way. Margin notes as such are far from a novelty, on the contrary, they have been around for centuries. Figure 1 exemplifies how margin notes was more of a rule than an exception in the old ages. The form of lecturing, described below, that supports the use of margin notes during lectures was therefor coined Fermat Pedagogy.

III. Method

The selected course was a course in basic calculus given in the intersection between secondary and tertiary education for almost 300 students. The students were randomly divided into two groups that got separate lectures for the cause of limited lecture hall capacity. One of the groups was randomly

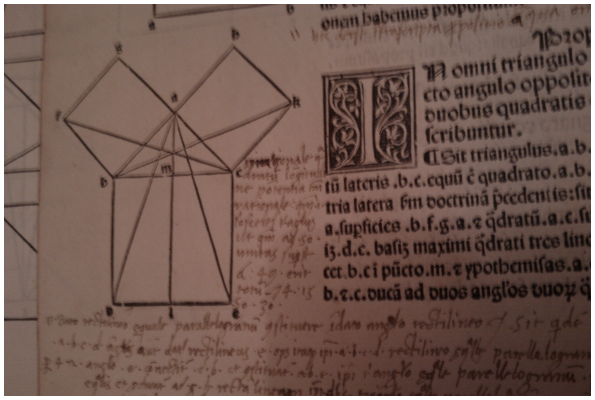


Fig. 1. This is an example of an older margin note from Euclid's Elements, where the active reader on neatly Latin wrote down his supplementary figures, comments and thoughts about the Pythagorean Theorem.

chosen to be the group taught using Fermat Pedagogy and hence called the Fermat group, while the other group became the control group.

Two sets of data was collected, student answers to a diagnostic test and to the final exam.

Diagnostic text

Although the two groups had over one hundred students each, we can not assume that the ability and knowledge should be equally distributed between the two groups when the course begins. In order to investigate the previous knowledge, and set a base line for the evaluation at the end of the experiment, a diagnosis test was given the first week of the course (Fermat group n=153, control group=95).

Attached to the diagnosis test, there were two open questions[1] to let the students described there also their note-taking habits, strategies and thoughts about it. The reason for adding these two questions was two fold. First of all we wanted to see if our formulation of the problem of the the common note-taking habits today was identified as a problem also for the students themselves. A second reason was to see if our thoughts of implementing the Fermat pedagogy was in harmony with the comments we would get, and if not, we got a chance to adjust them.

Final exam

The same final test was given to both groups (Fermat group n=125, control group n=121). The students had four hours to complete the exam which consisted of eight questions. The scope and the content was similar to what exams usually look like in this course, i.e. the questions were not design specially for this experiment. The first questions on the final exam could be answered by applying one of the procedures taught in the course. After that there were some tasks with solutions that needed several separate steps. Finally there was also some tasks where the students had to come up with the right idea to solve the problem.

[1]Why do you make lecture notes as you do? Are you happy with it or do you think it would be more efficient to do it some other way?

IV. The teaching

Both groups had the same number of lectures and exercise sessions and were both taught by the same teacher. The lectures for both groups were held three times a week and were 90 minutes long, divided into two part by a 15 minutes break. Furthermore the schedule included two exercise sessions per week per student.

The differences in teaching between the two groups are now explained.

Implementation of Fermat Pedagogy

The concept was conceived, named and piloted in a calculus course given in a team teaching form by the two authors in the fall of 2014. [2]

Fermat pedagogy is defined a s t he l earning environment created when the lecturer uses the course literature in such a way that the students do not need to feel stress to document everything that is presented on the board, or through the projector, but can instead focus on the content in the lecture and to formulate spontaneous question during the lecture. In order to keep the focus and boost the learning process, the students are encourage to keep their pens in their hands to make margin notes, illustrations and to underline. This will also make the course literature to their "own".

By using our experience from the fall of 2014, and the written comments we got form the students diagnostic test, we chose to implement the Fermat pedagogy by encouraging the student in the Fermat group to following five imperatives:

1. Keep focus — Hold your pen — Bring the book: To sit back and listen is pleasant, but there is an overwhelming risk that the thoughts starts to wander away a bit easier and quicker on the lectures. By holding a pen in hand ready write down key words, draw sketches in the margin and make under-, or more commonly, over-linings, the focus will be more easily kept and the connection between the book and the lecture will be much stronger.

2. Keep your eyes on the board and interact in the moment with the lecturer: By not copying that is written on the board, time is freed that can instead be used to capture the moment and more actively follow the lecturer's reasoning.

3. Read the book: In recent years, we have noticed that the course book is gradually descending as the source for knowledge acquisition. Instead, the studies are increasingly being built on distributed weekly study notes, shared lecture presentations, instructional videos on youtube, etc. It is soon a minority of students who spend time reading the coherent text in the course book. In order for Fermat Pedagogy to work, one has to give the textbook the time it needs, unless otherwise, there will be no own coherent notes, neither your own nor

[2]Pierre Fermat's so called last theorem, that the Diophantine equation $x^n + y^n = z^n$ does not have any solution for integers $n > 2$, made a famous note that he had found a wonderful proof, but unfortunately the margin was to narrow to let him write down the proof. "*Hanc marginis exiguitas non caperet.*"

the lecturer's. The book is thus placed in focus. We also want to point out the value that the course book was in this study sufficiently t hin a nd l ight t o b e f ully m obile t o b e brought to the lectures. Unfortunately, the more common American calculus books are as thick and heavy as bricks.

4. Make the book your own: Many of today's students have not written in books during their previous study time. We therefore encourage students at this course to break this taboo and really put their markings on the sides, splash the pages with coffee drops and take command of the white inviting margins. We also saw that a few students during the course cleverly expanding the margins by inserting Post-it notes on the pages.

5. Practice a lot, but start slow avoid misunderstandings: By borrowing the exercise strategy from the music studies (Griffin, 2 013), w e w ant t o g ive t he i nitial p resentation of the new concepts at such a slow pace and with multiple perspectives to avoid misinterpretations from the beginning. The lecturer then becomes a companion in the pages of the course book to support and ensure that the image is not only correct, but also multi-faceted.

The above five p oints w ere h ighlighted t o t he s tudents in the beginning of the course, and also at later occasions. All the lectures followed to course book closely. Throughout each lecture the lecturer made sure that the student continuously were updated on where in the book the present material was found. The students were encouraged to have that page in the book open on their desk and follow the development. The lectures exclusively included examples, theories and their proofs from the book. This to make it possible for the student to abstain from taking notes. The lecturer sometimes used book page photos on his iPad to show how one could make drawings, under-linings and notes in the book. See the example in Figure 2.

Implementation in the control group

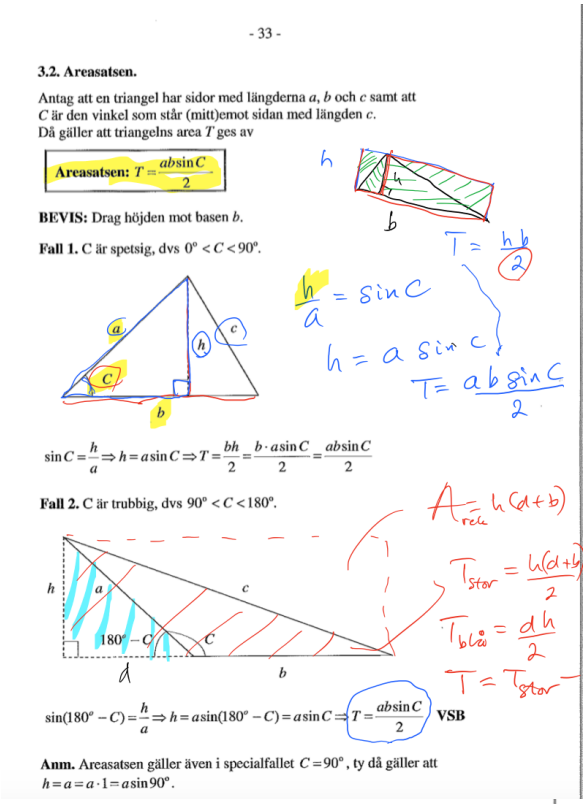
The student in the control group were not presented with the five imperatives described above. No guidelines for note-taking were discussed. The content of the lectures for the control group was exactly the same as in the Fermat group even if it they included much less references to the material in the book. The lecturer used examples and argument that were not to be found in the book.

V. Results

Here we report in the result from the two sources of data. The diagnostic test included two questions about the students note-taking. We start with reporting about our findings on that.

Students thoughts about note-taking

Comments from this could be divided into three categories: take notes to keep focus, get an alternative course text, and comments about the difficulty t o w rite a nd l isten a t t he same time. There was also comments that did not fit w ith the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3.2. Areasatsen.

Antag att en triangel har sidor med längderna a, b och c samt att C är den vinkel som står (mitt)emot sidan med längden c. Då gäller att triangelns area T ges av

Areasatsen: $T = \frac{ab\sin C}{2}$

BEVIS: Drag höjden mot basen b.

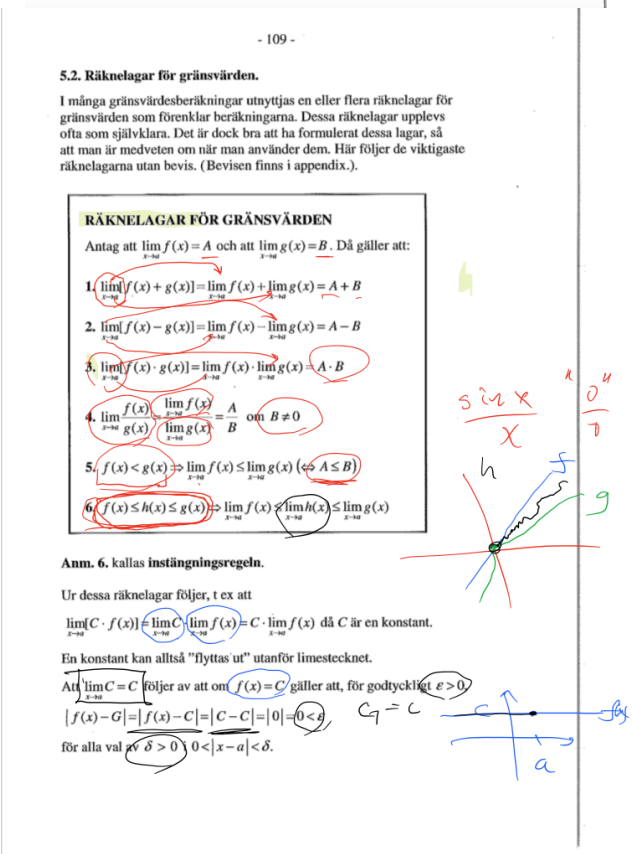
Fall 1. C är spetsig, dvs $0° < C < 90°$.

$$\sin C = \frac{h}{a} \Rightarrow h = a\sin C \Rightarrow T = \frac{bh}{2} = \frac{b \cdot a\sin C}{2} = \frac{ab\sin C}{2}$$

Fall 2. C är trubbig, dvs $90° < C < 180°$.

$$\sin(180° - C) = \frac{h}{a} \Rightarrow h = a\sin(180° - C) = a\sin C \Rightarrow T = \frac{ab\sin C}{2}$$ **VSB**

Anm. Areasatsen gäller även i specialfallet $C = 90°$, ty då gäller att $h = a = a \cdot 1 = a\sin 90°$.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5.2. Räknelagar för gränsvärden.

I många gränsvärdesberäkningar utnyttjas en eller flera räknelagar för gränsvärden som förenklar beräkningarna. Dessa räknelagar upplevs ofta som självklara. Det är dock bra att ha formulerat dessa lagar, så att man är medveten om när man använder dem. Här följer de viktigaste räknelagarna utan bevis. (Bevisen finns i appendix.).

RÄKNELAGAR FÖR GRÄNSVÄRDEN

Antag att $\lim_{x\to a} f(x) = A$ och att $\lim_{x\to a} g(x) = B$. Då gäller att:

1. $\lim_{x\to a}[f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x) = A + B$
2. $\lim_{x\to a}[f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x) = A - B$
3. $\lim_{x\to a}[f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = A \cdot B$
4. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{A}{B}$ om $B \neq 0$
5. $f(x) < g(x) \Rightarrow \lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$ $(\Leftrightarrow A \le B)$
6. $f(x) \le h(x) \le g(x) \Rightarrow \lim_{x\to a} f(x) \le \lim_{x\to a} h(x) \le \lim_{x\to a} g(x)$

Anm. 6. kallas **instängningsregeln**.

Ur dessa räknelagar följer, t ex att

$\lim_{x\to a}[C \cdot f(x)] = \lim_{x\to a} C \cdot \lim_{x\to a} f(x) = C \cdot \lim_{x\to a} f(x)$ då C är en konstant.

En konstant kan alltså "flyttas ut" utanför limestecknet.

Att $\lim_{x\to a} C = C$ följer av att om $f(x) = C$ gäller att, för godtyckligt $\varepsilon > 0$

$|f(x) - G| = |f(x) - C| = |C - C| = |0| = 0 < \varepsilon$

för alla val av $\delta > 0$ $0 < |x - a| < \delta$.

Fig. 2. Two examples of an interactive demonstration, using an iPad and the projector, on how one could make lecture notes. Quite a bit of emphasis was put on saying that it is not only ok, but even good to scribble in the books. As clearly illustrated here, it does not have to look pretty, the important thing is to add things that help your understanding or your memory.

three themes that we classified as 'other'. Here follows a few comments for these categories.

To keep focus: "I am pretty lazy and easily distracted, it would probably be better if I took more notes."

"I have a hard time to concentrate for long periods of time. By writing down everything I force myself to concentrate. When I can concentrate anyway, I learn more just by listening since it is easier to understand that way."

"I think I learn better by writing down & take notes at the same time I listen."

To get an alternative/shorter text: "[I] think it is better to get down as much as possible, so I can go back and understand the notes later. [I] try to write down in words what is done as much as I have time for — learn more easily if I can read a text and see the connections with the examples at the same time. "

"I am content. My own notes are easier to understand than what is written in the book."

"I want to go back and check what was brought up under the lectures, and it is easier to remember from the material being used as examples than to only check the book. This makes the book to more of a secondary source than if I relied less on the notes."

"I believe that what is written on the black board is important. I believe that the lecturer has to show all we have to know (that was at least the case with the physics). Thus I will get the most important with me back home as well!"

"The teacher should go over everything on the lectures that is needed to manage the exercises. Then I know if I write down EVERYTHING on the lectures, I can always go back to my notes and check what got stuck on and see how to move on. Therefore, I write down EVERYTHING!"

"I learn more when I write myself, since I can then easily add own comments etc."

Difficult to write and listen at the same time: "It does not feel effective [to take notes], [I] listen too little. Too much focus to write down everything."

"It is sometimes hard to listen when I write and [I] have unfortunately a hard time understand how mathematics work by reading. Regardless if it is my own notes or the book."

"I am not completely happy since I don't always have time to write down everything I would like. I think it is more effective to write down key words or write faster."

"The problem with this type of teaching is in my opinion that you are so occupied to write that you don't get time to understand it. Stop now and then and be quite so we get time to copy and then listen without being occupied by writing. That would make us understand. It is completely impossible to understand well if you have to concentrate on other things at the same time. "

Other comments: "Earlier experience of note-taking have not helped me in my studies. I think it depends more on the note-taking technique than the actual information. Thus I try to find new methods to write notes. But at the moment, I have not found it."

"I am content, otherwise I would do my note-taking differently."

"'Thinking with the hand'
write first, think afterwards ...
I am happy."

Results on students development.

In Figure 4 there are two histograms of normalized exam results. We note that the breakdowns have almost the same average, approximately 26 points, but the Fermat group (the blue bars) has a more rectangular distribution than the (orange) control group. The standard deviation for these two distributions are 11.9 for the Fermat group and 9.7 for the control group respectively. Through an F-test we can reject the null hypothesis that the two groups have the same variance with a p-value of 0.03.

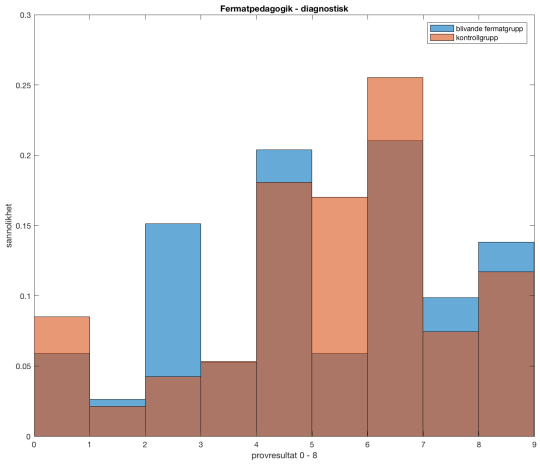

Fig. 3. The result from the diagnosis test of the two groups, where the Fermat group is illustrated with blue bars and the control with orange. A Kolmogorov-Smirinov test showed that the is similar in that the two results can not be assumed to come from two different distributions (p=0.46). However, the control group had a slightly higher average, 4.8 vs 4.7 and a bit lower variance 4.9 vs 5.4, although not significant as mentioned above.

VI. Discussion

We conclude that even if these students are beginners at the university their answers, to the questions about note-taking on the diagnostic test, most of them realize that it is a problem to balance the completeness versus the understanding of the note-taking. As a bit of a surprise, we also noted that there are students who believe that everything that can be tested on the exam has to pass through the black board first. Some also find comfort in the shortness of the lecture notes in comparison to the book.

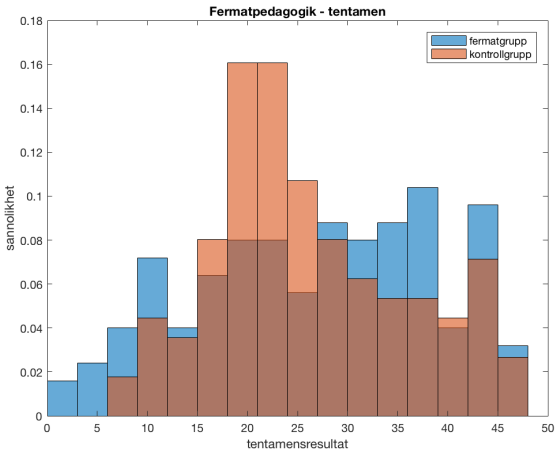

Fig. 4. The result of the written exam at the end of the course. The Fermat group are the blue bars and the control the orange. Note that the distribution of the Fermat group seem to be more rectangular shaped, i.e. both more low results and more top grades.

Concerning the quantitative findings we see that the Fermat group has a significantly higher variation, as can be seen in Figure 4. If this is the result of Fermat Pedagogy need further investigation[3]. Are there student in the Fermat group that got a grade 4 (30-39 points) on the final exam, that would have only reached a grade 3 (20-29 points) if taught as in the control group? And perhaps even more important, could it be an affect of the Fermat Pedagogy that some students that otherwise got a grade 3 now only got a F (below 20 points)?

There are some aspects of Fermat Pedagogy that make it plausible to believe that it can influence stronger and weaker student very differently. For example, the Fermat Pedagogy is designed to make the student actively and directly focus on the ideas and processes presented in the lectures, rather than on writing notes. This is probably particularly helpful for student that are on track and that are able to follow the argument in real time, while they are presented, and less so for students that lack necessary knowledge. On top of that Fermat Pedagogy could easily invoke less ambitious students in a false belief that: "He just goes through what's in the book, so I can read tomorrow instead."

We conclude by noticing that it would be interesting to see further research done on the affects of students note-taking and how the teacher can to help the student to develop effective strategies concerning note-taking. More specifically we wonder if another, similar study also would show that the strong students benefit from Fermat Pedagogy. Furthermore it would be interesting to see if Fermat Pedagogy actually is disadvantageous for weaker students, and if so, what note-taking strategy is more helpful for them?

[3]For example: We do not know how the students communicated or informally moved between the groups.

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